Quantum-to-Classical Transition with Single-Photon–Added Coherent States of Light

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Single-photon–added coherent states are the result of the most elementary amplification process of classical light fields by a single quantum of excitation. Being intermediate between a single-photon Fock state (fully quantum-mechanical) and a coherent (classical) one, these states offer the opportunity to closely follow the smooth transition between the particle-like and the wavelike behavior of light. We report the experimental generation of single-photon–added coherent states and their complete characterization by quantum tomography. Besides visualizing the evolution of the quantum-to-classical transition, these states allow one to witness the gradual change from the spontaneous to the stimulated regimes of light emission.

A coherent state |ω⟩ is the closest analog to a classical light field and exhibits a Poisson photon number distribution with an average photon number |ω|². Coherent states have relatively well-defined amplitude and phase, with minimal fluctuations permitted by the Heisenberg uncertainty principle. On the contrary, a Fock state |n⟩ is strictly quantum-mechanical and contains a precisely defined number (n) of quanta of field excitation, hence its phase is completely undefined.

Photon-added coherent states (|1⟩) are the result of successive elementary one-photon excitations of a classical coherent field, and they occupy an intermediate position between the Fock and the coherent states. They are obtained by repeated (m times) application of the photon creation operator ˆa† on a coherent state (|n⟩, m being a normalization constant and m an integer) and reduce to the limit Fock or coherent states for a → 0 or m → 0, respectively. Quite differently from the so-called displaced Fock states, where a coherent state is used to displace a number state [for example, by mixing the two fields upon a highly reflecting beam splitter (2)], photon-added coherent states can be roughly obtained as obtained from the displacement of a coherent state operated by a Fock state. Indeed, one easily finds that all the |n⟩ terms with n < m are missing in the expansion of the states |n, m⟩ in the Fock basis, and that all the elements of the corresponding density matrix are essentially displaced toward higher indices ρ_{i,j} → ρ_{i+1,j+1} leaving all the elements with i,j < m void.

In the case of a single quantum of field excitation (n = 1), the single-photon–added coherent states (SPACSs) read as

|1⟩ = \frac{\hat{a}^\dagger |ω⟩}{\sqrt{1+|ω|^2}} (1)

Unlike the operation of photon annihilation, which maps a coherent state into another coherent state (that is, a classical field into another classical field), a single-photon excitation of a coherent state changes it into something quite different. In general, the application of the creation operator ˆa† changes a completely classical coherent state into a quantum state with a varying degree of nonclassicality that becomes more evident the smaller the initial amplitude of the |ω⟩ state. In the extreme case of an initial vacuum state |0⟩, a single excitation event transforms it into the very nonclassical single-photon Fock state |1⟩, which exhibits negative values of the Wigner function (3, 4). The Wigner function is a quasi-probability distribution (3–7), which fully describes the state of a quantum system in phase space (either the position-momentum space for an harmonic oscillator or, equivalently, the space spanned by two orthogonal quadratures of the electromagnetic field for a single-mode state of light, as in this case) in the same fashion as a probability distribution (nonnegative by definition) characterizes a classical system. The negativity of the Wigner function is indeed a good indication of the highly nonclassical character of the state (Fig. 1).

We report the experimental generation of SPACSs and their complete tomographic analysis, which unveils the nonclassical features associated with the excitation of a classical coherent field by a single light quantum. Parametric down-conversion in a nonlinear crystal is the basis for the production of the desired states (Fig. 2). Here one high-energy pump photon can annhilate into two photons that obey the global energy and momentum conservation laws and thus have lower energies and are normally emitted into symmetrically oriented directions, also called the signal and idler modes. When no other field is injected in the crystal, spontaneous parametric down-conversion takes place, starting from the input vacuum field, and pairs of entangled photons with random (but mutually correlated) phases are produced. In order to generate SPACSs, one has to inject a seed
coherent field $|\alpha\rangle$ into the signal mode of the parametric amplifier, and the conditional preparation of the target state takes place every time that a single photon is detected in the correlated idler mode. If the parametric gain is kept sufficiently low, which is always the case in our experimental situation, the final output state can be approximated as

$$|\psi\rangle = \left(1 + g\hat{a}^\dagger\hat{a}\right)|\alpha\rangle|0\rangle + |\alpha\rangle|0\rangle + \left(1 + g\hat{a}^\dagger\hat{a}\right)|\alpha\rangle|1\rangle$$

(2)

(where $g$ is a gain constant with $|g| \ll 1$, and the output signal mode will contain the original coherent state most of the times, except for the few cases when the state $|1\rangle$ is detected in the idler output mode; these relatively rare detection events project the signal state into the SPACS $|\alpha,1\rangle$ (Fig. 2B), which corresponds to the stimulated emission of one photon in the same mode of $|\alpha\rangle$. The absence of a seed coherent field leaves with the usual expression for the spontaneous process, so that, by studying the evolution of the quantum state while the amplitude $\alpha$ gradually increases from zero, one can actually witness the smooth transition from the spontaneous to the stimulated regimes of light emission, with the transformation of an initial purely quantum state (the single-photon Fock state) into a classical coherent one. This is accompanied by the birth of a well-defined phase and can be described in more visual terms as the transition from the particlelike to the wavelike behaviors of the electromagnetic field.

The same state as described by Eq. 2 has recently been generated (8) to produce arbitrary superpositions of zero- and one-photon states. In that case, however, the conditioning was performed upon the detection of a single photon in the same mode $|1\rangle$ of the input coherent state, so that the final state was completely different from the ones investigated here and, for low $\alpha$ values, of the form $\left(|\alpha\rangle|0\rangle + g|1\rangle\right)$. The injection of a single photon instead of a coherent state as a seed for conditional parametric amplification has also been investigated (9) and experimentally demonstrated (10, 11) with the amplification to the $|2\rangle$ Fock state in the context of quantum cloning.

The primary light source for the experiment is a mode-locked laser whose pulses are frequency-doubled to become the pump for degenerate parametric down-conversion in a type-I beta-barium borate (BBO) crystal. In this configuration, the output photons share the same linear polarization and exactly the same wavelength, corresponding to twice that of the pump. In order to nonlocally select a pure state on the signal channel, idler photons undergo narrow spatial and frequency filtering before being detected by a single-photon counting module (12–15). The weak seed coherent state $|\alpha\rangle$ is obtained by controlled attenuation of a small portion of the laser emission, which is fed into the signal mode of the nonlinear crystal and is then directed to a 50% beam splitter. Here it is overlapped with a second (intense) coherent state (again obtained from a portion of the original laser pulses), which is spatially and temporally matched to the conditionally prepared SPACS and serves as the local oscillator (LO) for homodyne measurements, which are performed with a recently developed high-frequency time-domain technique (4, 16, 17).

Balanced homodyne detection (18–20) allows the measurement of the electric field quadratures of an unknown state as a function of the relative phase $\theta$ imposed between such a state and the reference LO. By performing a series of homodyne measurements on equally prepared states, it is possible to obtain the probability distributions $p(x, \theta)$ of the quadrature operator $\hat{x} = \frac{1}{\sqrt{2}} (\hat{a}^\dagger e^{i\theta} + \hat{a} e^{-i\theta})$ and, given a sufficient number of quadrature distributions for different values of $\theta$, one is able to reconstruct the density matrix elements and the Wigner function of the field state under study (7, 17).

Figure 3 shows a sequence of reconstructed Wigner functions for increasing values of the seed coherent field amplitude $\alpha$. The first one (Fig. 3A), obtained with a blocked input, corresponds to the single-photon Fock state $|\alpha\rangle$. Because of the finite efficiency (measured to be about 59% in the present set of measurements) in the detection apparatus, a significant mixture with the vacuum state is present, which clearly allows classically impossible negative values to be reached around the center of the circularly symmetric (because of the undefined value of the phase)
Although the SPACS Wigner function eventually becomes entirely positive for sufficiently high values of the seed amplitude, the excitation of an otherwise classical coherent state by a single photon leaves a measurable mark of nonclassicality in the field quadrature statistics (\(J\)). In particular, whereas the original coherent state has equal fluctuations in the different quadratures independently from its amplitude, the one-photon--excited state exhibits a smaller uncertainty (a squeezing) in one of the quadratures and larger fluctuations in the orthogonal one. One can interpret this as a reduction in the intensity noise associated with the excitation by a perfectly defined number of quanta, which also increases the phase noise because of the lack of phase information intrinsic to Fock states. Whereas Fig. 4 clearly shows the amount of maximum quadrature squeezing for different values of the coherent seed amplitude, the effect on the intensity and phase noise is also evident from Figs. 1 and 3C. Here the reduced intensity fluctuations appear in the decreased width along the radial direction, whereas the increase in the phase noise is indicated by the appearance of the ringlike wings along the tangential direction of the Wigner distribution.

The ability to experimentally investigate the elementary action of the bosonic creation operator on a classical state is of interest both as a tool to take a closer look at such fundamental events in quantum physics and as a starting point for the investigation of the fuzzy border that separates the quantum and classical regimes of light behavior, with natural extensions toward even more “exotic” quantum entities, such as Schroedinger’s cat states \((21)\).

**References and Notes**

17. Materials and methods are available as supporting material on Science Online.
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**Supporting Online Material**

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**Materials and Methods References**

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