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Experimental realization of creation and annihilation operators and direct proof of the bosonic commutation relation

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Summary. — The recent development of experimental techniques to implement the creation and the annihilation operators in the optical domain has found great interest for quantum information manipulation and processing. Quantum states of light obtained by the application of alternated sequences of the creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ to a thermal state of light depend on the order in which the two operations are performed. The non-commutativity of the bosonic operators, $\hat{a}\hat{a}^\dagger \neq \hat{a}^\dagger\hat{a} \neq 1$, has been experimentally verified in terms of density matrixes and Wigner functions, which are reconstructed by quantum homodyne tomography. Recently, a setup has been proposed to directly and completely prove the bosonic commutation relation where a single-photon interferometer is used to create coherent superpositions of two alternated sequences of operators, such as $[\hat{a}\hat{a}^\dagger \pm \hat{a}^\dagger\hat{a}]$. Some of the latest experimental results are discussed.

PACS 03.65.Ta – Foundations of quantum mechanics; measurement theory.
PACS 42.50.Ar – Photon statistics and coherence theory.
PACS 42.50.Xa – Optical tests of quantum theory.

1. – Introduction

Quantum information processing, allowing the realization of classically impossible communication and computational tasks, is one of the most promising applications of quantum mechanics nowadays [1]. In particular, quantum optics is one of the most suitable fields where these tasks can be efficiently realized [2-4]. Here, the low interaction of the photons (at near-infrared frequencies) with the environment allows the realization and manipulation of robust quantum systems traveling at the speed of light. The recent development of experimental techniques to implement the creation and annihilation operators in the optical domain has found great interest for quantum information manipulation and processing [5,6]. In this paper we illustrate the manipulation of quantum information by single-photon operations. This gives us the first opportunity to directly verify quantum commutation rules by exploiting single-photon interferences. In the following we give a brief overview of the single-photon addition and subtraction operations, then we show the experimental setup used for the test of commutation rules and, finally, we discuss the experimental results and draw our conclusions.

2. – Single-photon addition and subtraction to/from a state of light

It is well known that the annihilation $\hat{a}$ and creation $\hat{a}^\dagger$ operators operate on a number state $|n\rangle$ (or Fock state) in a single mode of the radiation field by decreasing or increasing this number by exactly one unit

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \tag{1}$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{2}$$

The experimental application of $\hat{a}^\dagger$ and $\hat{a}$ is often referred to as single-photon "addition" and "subtraction" even if they act quite differently from the conventional deterministic addition and subtraction of objects from an ensemble [7]. Indeed, the creation and annihilation operators work in a probabilistic way, *i.e.* the probability of success is proportional to the number of particles originally present in the state. Accordingly, for a general state described by the density operator $\hat{\rho}$ the application of the $\hat{a}^\dagger$ operator not only shifts the excitation number distribution by one unit towards higher values, but also re-shapes it increasing the weight of terms containing a higher excitation number. On the other hand, the action of the $\hat{a}$ operator depends on the input state $\hat{\rho}$ as it is discussed in [8]. For instance, when the annihilation operator takes action on a thermal state the resulting photon number distribution is shifted toward lower energy values and the probability of the highly excited terms increases since the subtraction probability grows with the excitation number as dictated by eq. (1).

The effect of photon addition and subtraction on different light states has been discussed for a long time but their experimental demonstration has been achieved only very recently. The single-photon addition scheme was demonstrated by using the process of parametric down-conversion in a nonlinear crystal pumped by a strong laser field [5, 9], whereas the subtraction of a single photon from a traveling field was achieved by a high-transmission beam-splitter [6]. These basic quantum operations have been applied to several classical and nonclassical light states (such as coherent, thermal and squeezed states), and very recently the possibility to combine them in sequences [7] and coherent superpositions [10, 11] has been demonstrated, allowing the first direct proof of the commutation rules.

Single-photon addition – The controlled addition of a single photon can be realized by the conditional stimulated parametric down-conversion in a nonlinear optical crystal as shown in the schematic picture of fig. 1A. When only a pump laser beam interacts in a $\chi^{(2)}$-nonlinear crystal, a pump photon at frequency ω_p may spontaneously decay into two quantum mechanically entangled photons traveling along different directions, usually called signal and idler, with frequencies ω_s and ω_i such that they conserve energy ($\omega_p = \omega_s + \omega_i$) and momentum ($\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i$). When one of the two spontaneously emitted photons is detected along a particular direction with a given energy, the spatial properties and the energy of the other are also unambiguously determined. From first-order perturbation theory, the two-photon output state from the parametric down-converter

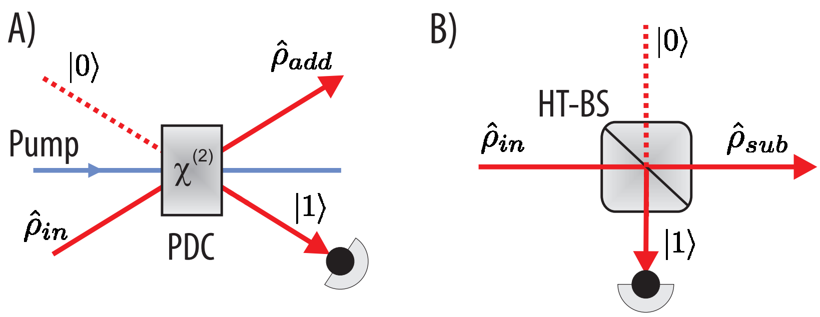

Fig. 1. – Schemes for the conditional implementation of single-photon addition (A) and single-photon subtraction (B). The detection of a single-photon heralds the preparation of the photon-added or photon-subtracted state.

described by the Hamiltonian $H = ig(\hat{a}_s^\dagger \hat{a}_i^\dagger - \hat{a}_s \hat{a}_i)$ is given by

$$|\psi\rangle \approx |0\rangle_s |0\rangle_i + g\hat{a}_s^\dagger \hat{a}_i^\dagger |0\rangle_s |0\rangle_i, \tag{3}$$

where the parametric gain of the crystal is assumed to be $g \ll 1$ in order to neglect higher-order terms. Thus the vacuum is mostly present at the output of the parametric down-conversion crystal (PDC), and a pair of entangled photons is emitted with a very low probability proportional to $|g|^2$. The gain g depends on the crystal length, on its nonlinear coefficient, and on the intensity of the pump beam; such parameters can be experimentally controlled.

If, instead of the vacuum $|0\rangle_s$, a seed light state $|\varphi\rangle_s$ is injected into the crystal, stimulated emission occurs into the signal mode, and the output state from the crystal is given by

$$|\psi\rangle \approx \left[1 + g\left(\hat{a}_s^\dagger \hat{a}_i^\dagger + \hat{a}_s \hat{a}_i\right)\right] |\varphi\rangle_s |0\rangle_i = |\varphi\rangle_s |0\rangle_i + g\hat{a}_s^\dagger |\varphi\rangle_s |1\rangle_i. \tag{4}$$

Then, the detection of the idler photon collapses the state of eq. (4) into the single-photon-added version of the seed state, *i.e.* $\hat{a}_s^\dagger |\varphi\rangle_s$. For a general input state of light described by the density operator $\hat{\rho}_{\text{in}}$, after the detection of a single photon in the idler channel the state in the signal mode is given by $\hat{\rho}_{\text{add}} = |g|^2 \hat{a}^\dagger \hat{\rho}_{\text{in}} \hat{a}$ with $g \ll 1$ (for details see [9]). It has been demonstrated that the photon addition operation converts any classical state of light into a nonclassical and non-Gaussian one [12].

Single-photon subtraction – Photon subtraction is implemented by reflection of a single photon by a high-transmission beam splitter (HT-BS). The detection of a single photon in the reflected path collapses the HT-BS–output state into the single-photon–subtracted version of the input field as shown in fig. 1B. If the transmission T of the beam splitter is very close to unity, then the output state after detection of one photon in the reflected beam is given by $\hat{\rho}_{\text{sub}} = \theta^2 \hat{a} \hat{\rho}_{\text{in}} \hat{a}^\dagger$ with $\theta \ll 1$, where θ is connected to the beam splitter transmissivity by $T = 1 - \sin^2(\theta)$ (for details see [13]).

Thermal light state generation – In this paper we concentrate on the results of the action of creation and annihilation operators on a thermal state of light such that the density matrix of the input state is described by

$$\hat{\rho}_{\text{in}} = \hat{\rho}_{\text{th}} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} |n\rangle\langle n|, \tag{5}$$

representing a mixture of number states with mean photon number equal to $\bar{n}$. In the experimental setup we use a pseudo-thermal light source obtained by inserting a rotating ground glass disk (RD) in a portion of the laser beam in order to avoid the technical problems connected to the handling of a true high-temperature thermal source [14, 15].

3. – Quantum state reconstruction by homodyne tomography

Since the first demonstration of quantum state reconstruction using homodyne tomography achieved by the group of Raymer [16] several new quantum states of light have been characterized in the recent years [17,18,5,19-21,7,11]. The state of light before and after the application of the chosen quantum operation is firstly measured by means of a technique widely used in the optical domain, called homodyne detection [22]. It consists in mixing the signal state to be analyzed with a reference beam called local oscillator (LO) by means of a 50–50 beam splitter. The two outputs of the beam splitter (BS in fig. 5) are detected by proportional photodiodes, then the two photocurrents are subtracted from each other and the difference signal is integrated over a proper time interval. The result is the measurement of the signal electric field quadratures $\hat{x}_\theta = \hat{x}\cos\theta + \hat{y}\sin\theta$ as a function of the relative phase θ imposed between the LO and the signal, where the two orthogonal field quadratures $\hat{x}$ and $\hat{y}$ are defined as $\hat{x} = (\hat{a}+\hat{a}^\dagger)/2$ and $\hat{y} = i(\hat{a}^\dagger - \hat{a})/2$ and $[\hat{x}, \hat{y}] = i/2$. They represent the position and momentum operators for a harmonic oscillator which describes a single-mode state of light [23]. From a complete set of homodyne measurements $\{x_\theta : \theta \in [0-\pi]\}$ it is then possible to perform tomographic reconstruction and thus retrieve the quantum state (for a complete review on quantum homodyne tomography see [24]), since one can reconstruct the density matrix and Wigner function of the analyzed state [25].

The density matrix elements of the analyzed state expressed in the number basis $\rho_{nm} = \langle n|\hat{\rho}|m\rangle$ can be obtained from the homodyne data by using the so-called *pattern functions* method [26,27] or the maximum likelihood estimation procedure introduced by Banaszek *et al.* [28]. Along this work we used the latter method which gives the density matrix that most likely represents the measured homodyne data. Firstly, one builds the likelihood function contracted for a density matrix truncated to M diagonal elements (with the constraints of Hermiticity, positivity and normalization), then the function is maximized by an iterative procedure [29, 30] and the errors on the reconstructed density matrix elements are evaluated using the Fisher information [30, 31].

In quantum optics it is often useful to represent the quantum state of light using quasi-probability distributions [23, 32, 33] which fully describe the state of a quantum system in phase space, in the same fashion as a probability distribution (non-negative by definition) characterizes a classical system. For instance, the P-function was introduced by Nobel laureate Roy Glauber in the 1960s [34, 35] and describes a quantum state in terms of coherent states. However, from an experimental point of view, the Wigner function [36], defined as

$$W(x,y) = \frac{2}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}\xi\, e^{i4y\xi} \langle x-\xi|\hat{\rho}|x+\xi\rangle \tag{6}$$

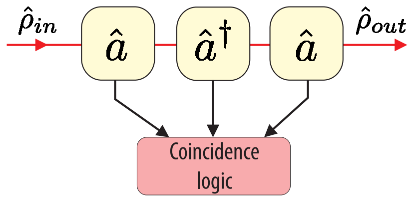

Fig. 2. – Combination of three quantum operations: single-photon addition between two single-photon subtraction stages. By the right combination of "cliks" from the detectors it is possible to select the desired $\hat{a}\hat{a}^\dagger$ or $\hat{a}^\dagger\hat{a}$ sequences.

and introduced in the early days of quantum mechanics, is the most useful quasi-probability distribution since it can be directly connected to the homodyne measurements (as demonstrated by Vogel and Risken [37], the marginal distributions of the Wigner function are connected to the distributions of the quadrature measurements). The negativity of the Wigner function is a good indication of the highly nonclassical character of the state.

The Wigner function can be retrieved from the quadrature distributions obtained at different LO phases θ by using a "classical" tomographic reconstruction [27, 33] based on the Radon transform, or it can be more precisely calculated from the reconstructed density matrix elements by means of $W(x,y) = \sum_{n,m}^{M} \rho_{nm} W_{nm}(x,y)$, where $W_{nm}(x,y)$ is the Wigner function of the operator $|n\rangle\langle m|$.

An ultra-fast pulsed homodyne detection scheme has been recently developed by Zavatta *et al.* [38] to analyze the quantum states and was the first system capable of operating at the full repetition rate of common mode-locked lasers in the time domain [24].

4. – Direct experimental proof of commutation relations

From the previous description it is possible to observe that both the addition and the subtraction of a single photon from the spatiotemporal mode containing the thermal light state are performed in a conditional way. This means that photon-added or photon-subtracted states are obtained only upon the "click" of a single-photon detector in a separate channel, usually called the "trigger" channel (*i.e.* the PDC idler channel for the photon addition, and the HT-BS reflection channel for the photon subtraction schemes, respectively).

By exploiting this peculiar characteristic of our schemes, we have combined three different modules, each implementing the action of a specific quantum operator on the thermal light state, in order to perform separate sequences of operators as shown in the scheme of fig. 2. By choosing the right combination of clicks coming from the detectors, we can select any desired sequence of quantum operations. A single click simply conditions the generation of a photon-added or photon-subtracted thermal state, whereas a double click can either produce a first-subtracted-then-added thermal state or vice versa, depending on the combination of clicks.

For each sequence the resulting quantum state is analyzed and characterized by homodyne tomography. The results are reported in fig. 3 where the reconstructed photon number distribution (diagonal terms of the density matrix) and the Wigner function are shown for the input thermal state (A), for the added-then-subtracted thermal state (B), and for the subtracted-then-added thermal state (C).

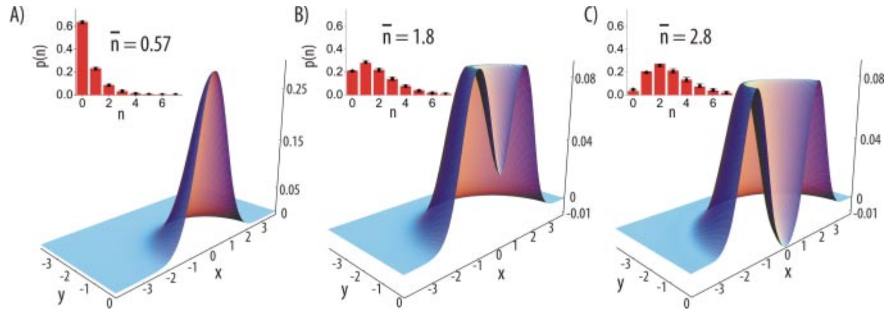

Fig. 3. – Experimentally reconstructed photon number distributions and Wigner functions for (A) initial thermal state $\hat{\rho}_{\rm th}$ with $\bar{n} = 0.57$, (B) added-then-subtracted thermal state $\hat{a}\hat{a}^\dagger\hat{\rho}_{\rm th}\hat{a}\hat{a}^\dagger$, (C) subtracted-then-added thermal state $\hat{a}^\dagger\hat{a}\hat{\rho}_{\rm th}\hat{a}^\dagger\hat{a}$.

A clear negativity of the reconstructed Wigner functions is obtained when photon addition is the last operator acting on the states, thus showing their high degree of nonclassicality. However, the Wigner function of states resulting from the two sequences of addition and subtraction show other interesting features. The two final states are deeply different from each other and from the original thermal state. In both cases, the Wigner function exhibits a clear central dip, which is absent in the Wigner function of the thermal field; such a dip reaches negative values for the subtract-then-add sequence, whereas it stays well in the positive region for the add-then-subtract sequence. This provides a direct experimental verification of the non-commutativity of the quantum bosonic creation and annihilation operators and gives a visually convincing demonstration that a simple view of classical particle addition and subtraction is incorrect in this case.

As recently proposed by Kim *et al.* [10], a full demonstration of the bosonic commutation relation reduces to realizing the superposition of operator sequences $\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}$, and showing that it corresponds to the identity operator 1. While most of the mathematical structure of quantum mechanics is based on the commutation relation, this is the first time that such a fundamental relation is now accessible to a direct experimental test.

A superposition of conditional quantum operators can be achieved by the experimental scheme of fig. 4A. The conditioning (or trigger) photons from the different operations are sent to a 50–50 beam splitter in order to build a single-photon Mach-Zehnder interferometer, and a single-photon detector is placed in one of the beam splitter outputs.

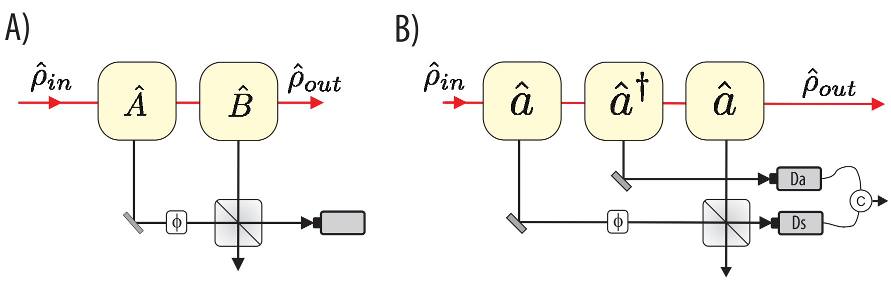

Fig. 4. – (A) Superposition of arbitrary quantum operators $\hat{A} + e^{i\phi}\hat{B}$. (B) Superposition of creation and annihilation operator sequences $\hat{a}\hat{a}^\dagger - e^{i\phi}\hat{a}^\dagger\hat{a}$.

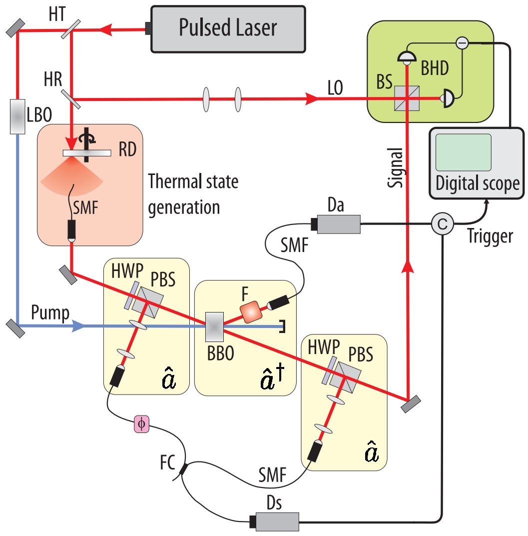

Fig. 5. – Simplified experimental setup for the implementation of arbitrary balanced superpositions of alternated sequences of creation and annihilation operators (see text for details).

Here the interferometer acts as a quantum information eraser: after the detection of a single photon it is not possible, not even in principle, to know which operation was performed on the state, thus the click of the detector heralds the operator superposition $\hat{A}+e^{i\phi}\hat{B}$ where ϕ is the path difference (phase) between the two interferometer arms (see fig. 4A); a similar scheme has been adopted in [39]. In order to realize the superposition $\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}$, the scheme of fig. 4B is used. Here, when a coincidence between the "clicks" of the detectors Da and Ds is observed, it is no longer possible to identify if the single-photon addition happened before or after the single-photon subtraction. This situation thus implies the application of the operator $\hat{a}\hat{a}^\dagger - e^{i\phi}\hat{a}^\dagger\hat{a}$ to the original state (for details see [10]).

The full experimental setup used to implement the scheme of fig. 4B is shown in fig. 5. It consists in a laser source (mode-locked Ti:Sa laser) emitting 1.5 ps pulses at a repetition rate of 82 MHz, with a wavelength of 786 nm. We again use the convenient modular scheme described above, where one single-photon addition stage is placed between two single-photon subtraction ones [7]. The photon addition stage is based on conditional single-photon parametric amplification in a type-I BBO (β-barium borate) nonlinear crystal, where the pump beam for parametric down-conversion is obtained by frequency doubling the main laser output in a LBO (lithium triborate) crystal [40]. Single-photon subtraction stages are based on the conditional removal of a single photon by adjustable low-reflectivity beam splitters, obtained with combinations of half-wave plates (HWP) and polarizing beam-splitters (PBS).

A small portion of the main laser beam is obtained by means of a high-transmission beam splitter (HT in the figure) and is used to provide the local oscillator for homodyne detection and for the generation of pseudo thermal light. A rotating ground glass disk (RD) is inserted in the path of the laser beam and a bare single-mode fiber (SMF) is used to collect a portion of the scattered light to provide the pulsed thermal light states [15] of mean photon numbers $\bar{n} \approx 1$ that have been used as the input states in

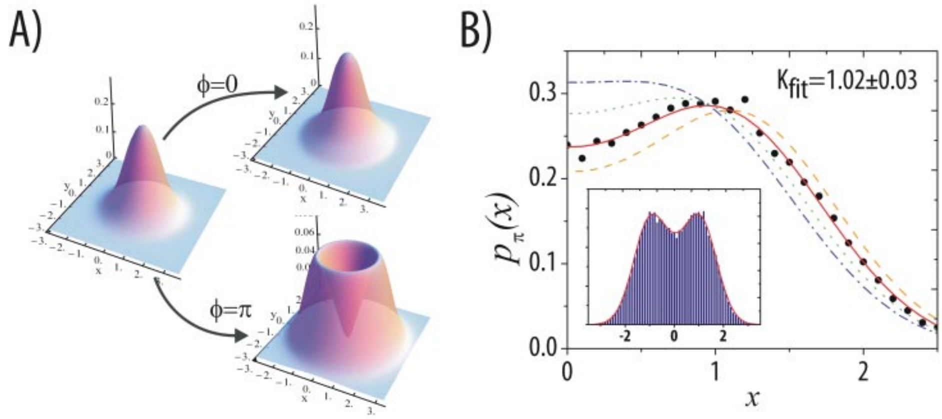

Fig. 6. – (A) Experimentally reconstructed Wigner functions of the original thermal state and of those resulting from the application of the commutator and anti-commutator superpositions. The state is not changed for $\phi = 0$. About 10^4 (10^5) quadrature data points have been acquired in the (anti-)commutator case. A thermal field with a mean photon number $\bar{n} = 0.9(1)$ is used as the initial state. (B) Histogram of raw quadrature data (solid dots) for the anti-commutator setup at $\phi = \pi$. Also shown are theoretical curves calculated for the actual experimental parameters [11] and different values of the commutator ($K = 0$: dashed; $K = 2$: dotted; $K = 3$: dash-dotted). The solid curve is the result of the best fit to the experimental data.

the experiment. The balanced homodyne detection (BHD) signal is acquired and stored by a digital oscilloscope on a pulse-to-pulse basis triggered by a coincidence (C) between clicks from the Da and Ds single-photon detectors, placed after a 3 dB fiber coupler (FC) combining the herald signals from the two subtraction modules. By a click from detector Ds we know that a single photon has been subtracted, but we cannot identify if it was before or after the photon addition. In these conditions, a coincidence event heralds the application of the general *cat operation* $\hat{a}\hat{a}^\dagger - e^{i\phi}\hat{a}^\dagger\hat{a}$, with an adjustable phase ϕ, to any input light field.

The *cat operations* have been introduced in [11] as a general operator superposition; in fact, one can look at Schrödinger's cat paradox [41] as the result of a superposition of two very different operations (*to kill* or *not to kill*) to a living cat. By varying the phase ϕ with a piezo-actuated mirror, any arbitrary balanced superposition of the two operator sequences can be obtained. In particular, by setting $\phi = 0$ or $\phi = \pi$, one can directly implement the commutator or the anti-commutator of the creation and annihilation operators, respectively.

Since both the initial thermal states and those resulting from the above manipulations possess no intrinsic phase, the phase of the local oscillator is not actively scanned, and phase-independent marginal distributions are obtained. However, the final states still clearly depend on the phase ϕ of the superposition. High experimental efficiency is obtained by minimizing all spurious losses and making sure that all the single-photon operations are performed in exactly the same spatiotemporal mode as the one selected by the LO. This requires narrow spatial and spectral filtering (F) in the herald mode of the parametric down-conversion crystal [42-44] and an accurate matching of the fibre-coupled fields reflected from the two subtracting beam splitters to the LO spatial mode.

Figure 6A shows the experimentally reconstructed Wigner functions of the original thermal state and of those resulting from the application of the commutator ($\phi = 0$)

and anti-commutator ($\phi = \pi$) superpositions. The Wigner functions are obtained from 10 diagonal density matrix elements (13 for the anti-commutator case) reconstructed by means of a maximum likelihood algorithm [29, 30]. The remote manipulation of the state by the implementation of different superpositions of creation and annihilation sequences is clearly observed. The variation of the phase ϕ would not have resulted in such different output states if the applied operations had only been statistical mixtures instead of coherent superpositions. The fidelity between the original thermal state and the final one is about $F = 0.992$ for the commutator case ($\phi = 0$). This demonstrates that the implemented operator superposition is essentially equivalent to the identity operator.

Because of the normalization of quantum states, the above results just demonstrate the commutation relation up to a multiplicative constant K, *i.e.* one might still have $[\hat{a}, \hat{a}^\dagger] = K1$. However, in this case, the anti-commutator setup implements the $2\hat{a}^\dagger\hat{a}+K1$ operator, which produces an output state strongly depending on the exact value of the constant K. Figure 6B reports the measured homodyne quadrature distribution for the same initial thermal state after the application of the anti-commutation operator ($\phi = \pi$). Also reported are the theoretical distributions calculated for the same experimental parameters but with a few different values of the constant K. Experimental data are consistent with $K = 1$, whereas different integer values are in evident disagreement. A best fit of the experimental homodyne data gives $K = 1.02(3)$, thus quantitatively demonstrating the bosonic commutation relation for the first time.

5. – Conclusions

In this paper we have shown some of our recent results in the manipulation of the quantum state of light by single-photon operations. The impact of these techniques is demonstrated here by the first direct proof of the bosonic commutation rules. Besides their importance for fundamental studies in quantum physics, these results are also expected to open new exciting opportunities for the development of novel quantum information tasks, as recently proposed [45, 46].

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