Modulation response, mixed-mode oscillations and chaotic spiking in quantum dot light emitting diode


Abstract
In this work quantum dot light emitting diode (QD-LED) was modeled in a dimensionless rate equations system where it is not done earlier. The model was examined first under bias current without any external perturbation where it exhibits chaotic phenomena since the model has multi-degrees of freedom. Then, it is perturbed by both small signal and direct current modulations (DCM), separately. The system exhibits mixed-mode oscillations (MMOs) under DCM. This behavior was reasoned to continuous states of two-dimensional wetting layer (WL) which works as a reservoir to quantum dot (QD) states. QD capture was the dominant rate controlling the dynamic behavior in QD-LED. The nonlinear dynamic behavior of our model is compared very well to the experimental observations in the QD-LED.

Keywords:
Nonlinear and chaos
QD-LED
DCM
MMOs

1. Introduction

Nanocrystals are classified by their size (in the range of 1–100 nm) and properties. They are described as those having at least one dimension, less than or equal to, nano-size. Quantum dots (QDs), also known as nanocrystals, are a nontraditional type of semiconductor crystals with limitless applications in enabling material industries [1]. The QDs, which can also be called artificial atoms, are nanometer-scale structures that selectively hold or release electrons. They consist of a few hundred to a few millions of atoms. QDs bridge the gap between the single atoms and the solid state and, because of this they exhibit a combination of atomic and solid-state properties. The emission wavelength, or the color emitted, of QDs depends on the size, and using simple chemistry with semiconductor nanocrystals, the color can be precisely controlled. Quantum dots (QDs) are grown by the self-assembling process in a wetting layer (WL), a two-dimensional quantum-well layer. Thus, QD structure consists of completely quantized subbands in the QDs attached with a two-dimensional WL [2].

QD-LEDs are widely used as incoherent light sources in applications such as lighting and short-distance optical fiber communications [3]. An important performance characteristic of a QD-LED is the output efficiency, i.e., the amount of light extracted from the structure at a given input current. Since higher modulation speed implies a larger information capacity, a high modulation speed is important for short-distance communication applications [4].

Some light sources exhibit intensity and phase fluctuations. These fluctuations are of great importance since they induce errors in optical measurements. Their origin lies in the quantum nature of transition process itself. In fact, every spontaneous emission event in the oscillating mode varying the phase of the electromagnetic field (quantum noise) is responsible for the carrier density variation [5].

Semiconductor nitrides such as aluminum-nitride (AlN), gallium-nitride (GaN), and indium-nitride (InN)
are commonly utilized as materials for their potential use in high-power and high temperature optoelectronic devices [6].

The properties of III-N QDs are closely related to those of bulk materials that have been reviewed in many articles [7]. Although the bulk III-N LEDs can be crystallize in wurtzite, zinc blende, and rock salt structures in ambient conditions, the thermodynamically stable structure is the wurtzite phase [8].

Here, we elucidate the role that photon reabsorption and nonradiative recombination processes play an important role in QD-LED. We present a rate-equation analysis which incorporates the essential aspects of electronic dynamics between states. Actually, we will show in the next section that a fully deterministic model explains such a distribution. Furthermore, we discuss the influence of various parameters upon the dynamic QD-LED output in detail, small-signal modulation response, and we present simulations of the turn-on dynamics with DCM of QD-LED.

This paper is organized as follows. After introducing the theoretical model details in Section 2 and its results in Section 3, we discuss the dependence of the small-signal response upon the dynamic parameters in Section 4. In Section 5, we focus on the population dynamics induced by the interaction of the QD states with the DCM reservoir of wetting-layer state, and we conclude in Section 6.

2. QD-LED model

In the QD-LED system the electrons are first injected into WL before they are captured by the QDs. The system equations describe the dynamics of the carriers in the QD ground state, $n_{QD}$, the number of carriers in the WL, $n_{WL}$, and the number of photons in the optical mode, $S$, are as follows:

$$
S^* = Wn_{QD}^2 - Wn_{QD}S - \gamma_S S,
$$

$$
n_{QD}^* = \gamma_{rwl}n_{WL}(1 - n_{QD}/2N_d) - \gamma_{rwl}n_{QD} - (Wn_{QD}^2 - Wn_{QD}S).
$$

$$
n_{WL}^* = \frac{I}{e} - \gamma_{rwl}n_{WL} - \gamma_{rwl}(1 - n_{QD}/2N_d).
$$

(1)

Here, the induced processes of spontaneous emission and re-absorption in the QDs are modeled by the first and second terms of the first equation of the system (1), the Einstein coefficient $W$ is given by $W = [(|\mu|^2 \sqrt{\varepsilon_{bg}})/(3\pi \varepsilon_0 a h)]/(w/e)^2$, where $\varepsilon_{bg}$ is the static relative permittivity of the background medium, $\varepsilon_0$ is the vacuum permittivity, $c$ is the speed of light in vacuum, and $\mu$ is the dipole moment of the QDs. $\gamma_{rwl}$ and $\gamma_{rwl}$ are the nonradiative decay rates of the number of carriers in the QD and WL respectively; $N_d$ is the total number of QDs; and $I$ is the injection current, $e$ is elementary charge, $\gamma_S$ is the capture rate from WL into the dot, $\gamma_{rwl}$ is the output coupling rate of photons in the optical mode, respectively. The energy diagram of the QD-LED under study is shown in Fig. 1.

The main goal of this work is to provide a model reproducing qualitatively the experimental results and showing the chaotic spiking of QD-LED. Applying DCM to QD-LED induces a dramatic change in the photon statistics wherein strong, frequency modulated current bunching is indicative of random intensity fluctuations associated with the spike emission of light, as will be reported in this work. To do so, we rescale the system (1) to a set of dimensionless equations. Defining new variables and dimensionless parameters by the following:

$$
x = \frac{S}{S^*}, \quad y = \frac{n_{QD}}{n_{QD}^*}, \quad z = \frac{n_{WL}}{n_{WL}^*},
$$

$$
\gamma = \frac{\gamma_{rwl}}{\gamma_S}, \quad \gamma_1 = \frac{W}{\gamma_S}, \quad \gamma_2 = \frac{W}{\gamma_{rwl}},
$$

$$
\gamma_3 = \frac{\gamma}{\gamma_S}, \quad \gamma_4 = \frac{\gamma_{rwl}}{\gamma_S}, \quad N_d = a, \quad \delta = \frac{1}{\gamma_S}.
$$

and the time scale $\tau' = \gamma_{rwl}t$. The system Eqs. (1) can be rewritten in the following form:

$$
\gamma_1x^* = \gamma(y^2 - yx(x + 1))
$$

$$
y^* = \gamma_2z(\gamma_1 - \gamma_2/2a) - y(\gamma_3 + \gamma y) + \gamma_2xy
$$

$$
z^* = \gamma_4(\delta - z + yz/2\gamma a) - z
$$

(2)

Here, the upper subscript “dot” (‘) refers to differentiation with respect to (‘). The bias current is represented by ($\delta$).

3. Results of QD-LED model

In this section, the dynamics of chaos in QD-LED without external effect was discussed. The rate Eqs. (2) are solved numerically using the fourth-order Runge–Kutta method by Matlab system. The parameters used in the simulation are listed in Table 1. The initial values are obtained by solving the system (2) at steady state.

Fig. 2 shows a numerical example of chaotic oscillations in QD-LED without feedback. The injection current, and another parameters have been chosen carefully. These parameter values are critical; above and below them the system is at steady state. Fig. 2 shows the time series with their corresponding attractors. The dynamics in QD-LED are inverted; they began...
Fig. 2. Numerical simulations of the QD-LED model equations. Left panels: time series for the light intensity. Right panels: reconstructed phase space. For the chaotic spiking regime for selected bias current $\delta_0$ (a)-(d) correspond to $\delta_0 = 0.002–0.5$ as indicated in the first bifurcation.

Fig. 3 shows the inter-spike interval (ISI) probability distribution of Fig. 2. In Fig. 3(a) an exponential decay is shown which corresponds to chaotic state. This is due to unstable periodic orbit shown in the right panel of Fig. 2(a). This behavior is also shown in semiconductor laser with optoelectronic feedback [9]. In Fig. 3(b) ISI shows several sharp peaks. In Fig. 3(c) ISI shows double sharp peaks. The single sharp peak shown in Fig. 3(d) corresponded to periodic behavior shown in Fig. 2(d). It is shown that the system attains a new state by increasing the bias current by one order of magnitude. The left panel shows the corresponding attractor in 3-dimensional (3D) phase space which depicts the time series behavior in all of these states.

Fig. 5(a) shows the chaotic behavior of QD-LED which depicts the behavior shown in Fig. 2. It also shows that an inverted behavior begins from chaos and ends at regular state at high current. In order to provide further understanding of the chaotic behavior of our dynamical model we extracted the Lyapunov exponent (LE) "which is referred to as $\lambda$ in the figure" where it is frequently used to help the understanding of the dynamic behavior of nonlinear systems. The LE has a positive value in chaotic regions while it has a zero or a negative value in periodic and stable oscillations. The output power has fixed states in the bifurcation diagram at the point where the LE has a value less than zero. Indeed, LE is positive for the chaotic region in Fig. 5(b) while it is negative for stable states.

To explain the above QD-LED behavior the time series of QD ground state and that of WL are plotted in Fig. 6. For WL, since it has a large number of states (it is considered as a continuum state) its carrier number is always higher value as shown in the right panel of Fig. 6. This behavior is shown
Fig. 5. (a) The bifurcation diagram for the model equations. (b) Lyapunov exponent of QD-LED for the same dynamic states as in (a) where the parameter values are set as in Fig. 2.

4. Small signal response

For the mode under consideration, the total spontaneous emission rate not its spontaneous emission rate is used to determine its modulation characteristics. Therefore, different optical modes approximately will have the same modulation characteristics despite the difference in photon decay time [11].

To further understand the dynamical behavior of the QD-LED, we examine the frequency modulation response of our dynamical system Eqs. (1). When our system is affected by modulation, then it is expected to respond to external perturbation.

Thus, we analyze the small signal response in terms of its poles (photon and bias current) dependence on $S(\omega_m)/\|\dot{\omega}_m\) .

Fig. 4. Numerical simulations of the QD-LED model equations. (a) Time series for the light intensity. (b) Reconstructed phase space, for bias current $\delta_o < 3.51$ as indicated in the first bifurcation.
Assuming a harmonic modulation of the pumping rate $I(t) = I_o + I_e^{\omega_m t}$, with $\omega_m$ being the angular modulation frequency. In order to obtain the modulation response, the rate equations are linearized by a modified small-signal analysis. Considering a sinusoidal current modulation $I_e^{\omega_m t}$ around the injection current $I_o$, the following system equations for the optical mode varying around their steady-state solutions are obtained:

$$S(t) = S_0 + S e^{i\omega_m t},$$
$$n_{QD}(t) = n_{QD0} + n_{QD} e^{i\omega_m t},$$
$$n_{wl}(t) = n_{wl0} + n_{wl} e^{i\omega_m t}$$

(3)
Inserting (3) into the rate Eqs. (1) and it should be noted here that all the three time-independent terms \( X_0 \) will be neglected in the system (3). It is, then, solved for steady state case using their standard Fourier analysis\[12,13\] to yield the modulation response \( \frac{S(\omega_m)}{I(\omega_m)} \) of the QD-LED. The final form is shown in Eq. (5), at the bottom of the page. The modulation transfer function can be expressed as

\[
R_M(\omega_m) = \frac{S(\omega_m)}{I(\omega_m)}
\]

In the free-running case, the small signal photon number for the QD-LED is obtained, after some long mathematical manipulations, as follows:

\[
\frac{S(\omega_m)}{I(\omega_m)} = \frac{W\gamma(1-2n_{QD})(1-n_{QD})/q\left[\gamma_{ref} + \gamma(1-n_{QD})\right]}{\gamma_{QD}(W+\gamma_4)-W^2(1-2n_{QD}) + i\omega \left[\gamma_{QD} + W + \gamma_4\right] + \omega^2 \left[\gamma_{QD} + W + \gamma_4\right] + \omega^2 \frac{\gamma_{ref} + W + \gamma_4}{i\gamma(1-n_{QD})} + \frac{\gamma_{ref} + W + \gamma_4}{(i\omega)^2} + \frac{\gamma_{ref} + W + \gamma_4}{(i\omega)^3}}
\]

The following parameters are used in the calculations: \( I_0 = 0.9, \gamma_s = 0.1 \text{ ns}^{-1}, \gamma_{QD} = 0.77 \text{ ns}^{-1}, \gamma_{ref} = 0.14 \text{ ns}^{-1}, \) \( N_d = 6 \times 10^{12} \) and the spontaneous emission rate is 0.02. In Fig. 8, the bandwidth of the spectrum is essentially shown the same as QD lasers \[5,14\]. Fig. 8 illustrates that the modulation frequency is increased when the capture rate from WL into QD is shortened. This result coincides with that obtained from Fig. 7, where the system becomes more dynamical at shorter capture rate. This may be reasoned to the chaotic behavior of carriers at small value of bias current as shown in Fig. 2(a).

One should first note that, like QD lasers \[14\], the modulation response at high value of the capture rate is highly damped, with no observable resonance peak. More importantly, however, Fig. 8 shows that as the \( \gamma_c \) is decreased, the 3-dB modulation bandwidth decreases and observable the highest resonance peak. In a QD laser \[5\], the highest bandwidth appears with decreasing the capture rate. In fact, the highest bandwidth in this example (53 GHz) and
Fig. 11. Time series of the variable $x$ as obtained by numerical solution of Eqs. (2): (a) $f_m = 0.005$, (b) $f_m = 0.012$, (c) $f_m = 0.017$, and (d) $f_m = 0.026$. The fixed parameters are $I_{dc} = 0.16$ and $I_{ac} = 0.2$.

the highest resonance frequency is (20 GHz) which occurs at the capture rate $0.019 \text{ ps}^{-1}$.

5. DCM

The unique differentiating feature of QD-LED is its response to the direct modulation for the pump. Namely the QD-LED characteristics are directly modulated by an injection current. Thus, it is a good examination of our QD-LED characteristics where it is examined here to external perturbation. It differs from the first examinations (frequency modulation – examinations to the entity of the system without perturbation) in that it is an examination of our system to perturbation.

Apart from small and compact light sources, QD-LEDs are suitable light sources of optical communications and information devices because of their direct modulation property [14]. Directly modulation of the QD-LED through the injection current represents an introduction of extra degree of freedom to QD-LED and it induces instability and chaos in the output. This means that it examine the controllability of chaotic behavior in QD-LEDs by direct injection current modulation.

We investigate the transition between periodic and chaotic states and analyze the effects of injection current modulation on the chaotic attractors by using the same theoretical model and adding an additional perturbation to allow the occurrence of non-periodic output and chaos. There are several possibilities to accomplish this. The easiest one is to modulate the injection current periodically with a modulation frequency $\omega_m$ [13]. The injection current $I(t)$ in Eqs. (1) and (2) has to be replaced by a source of injection current [15]

$$I(t) = I_{dc} + I_{ac} \sin(2\pi f_m t)$$

where $I_{dc}$ is the dc part of the injection current and $I_{ac}$ is the amplitude of the ac part of the injection current. The injection parameters: the dc bias current (the dc bias strength), $I_{dc}$, the modulation current (modulation depth), $I_{ac}$, and modulation frequency, $f_m$, have been chosen according to Table 1.

The system Eqs. (2) are solved numerically after introducing Eq. (6) into the third equation of it. The $dc$ bias strength value used was 0.1 while modulation frequency $f_m = 5 \times 10^{-2}$. In Fig. 9(a) we plot bifurcation scenario of QD-LED under direct modulation which shows an evidence of complex periodic and chaotic mixed-mode oscillations. Here the system began from double periodic behavior at $I_0(I_{dc}) = 0.1$ and zero $I_{ac}$. This point can also be shown in behavior of QD-LED in Fig. 5 above. With the increase of DCM the system becomes in MMOs phenomena. Finally, at high current modulation the system returns to chaotic behavior which begins from it originally (see the beginning of Fig. 5). Similar results are also obtained experimentally for bulk LED in [8]. To unveil the link between the transient dynamics of unforced systems $I_{dc}$-bifurcation and the forcing-induced stochastic $I_{ac}$-bifurcation, we plot $I_{ac}$ versus LE in Fig. 9(b). A comparison between Figs. 5(b) and 9(b) shows that the spectrum is chaotic when LE is characterized by “at least” one positive value with increasing DIC, while in Fig. 5(b) most of LE is less or equal to zero (less refers to the gradient, and equal to conservative systems). Besides, bifurcation scenario of
QD-LED under frequency modulation is also studied in Fig. 10; it shows transition periodically of system dynamic to the chaos state through the following cases: quasi-periodic and MMO. The power drops with increasing frequency modulation until a steady state is attained.

In Fig. 11 the bias current, $I_{dc} = 0.16$, and the modulation current $I_{ac} = 0.2$. By increasing the frequency modulation we observe the dynamical sequence. Fig. 11(a) corresponds to the lowest frequency modulation ($f_m = 0.005$), where a single periodic oscillations was observed. For modulation frequency lower than this value, semi-periodic oscillations are appeared (not plotted here). By increasing the frequency modulation ($f_m = 0.012$, 0.017 and 0.026) periodical phenomena are clearly observed. The expansion of inter-spike interval was observed. Note that a somewhat large increment in frequency modulation moves the system dynamics initially to another periodic state, finally the system reaches chaotic state by a smaller change of frequency modulation, from one state to another state (periodic to chaos). In all cases of Fig. 11(a) MMO was detected. MMO represents a chaotic spiking regime where large intensity pulses are separated by irregular time intervals from small-amplitude chaotic oscillations as shown in Fig. 12 [16].

6. Conclusion

Here, we modeled QD-LED in a dimensionless rate equations system where it is not done earlier. By no means do we argue that our model is “universal” or something of the kind. Then, we analyzed the modulation response for QD-LED, taking into account the effects of both nonradiative recombination and the photon reabsorption processes. We demonstrate theoretically the existence of slow chaotic spiking sequences in the dynamics of QD-LEDs without external perturbation. The timescale of these dynamics is fully determined by increasing the injection current and their erratic nature is evidenced by means of the ISI probability distribution. We examine the frequency modulation response, where it is shown that the response for modulation such us that of QD lasers is strongly dependent upon the capture rate from WL into a dot. Other important feature of the model, which appears essential for explaining the MMOs, is the incorporation of a frequency modulation which is described in a DCM. By modulating the injection QD-LED via the variation of $I_{ac}$ part of the injection current, the modulation frequency was investigated. Bifurcation route indicated that QD-LEDs can move the system dynamics to reach chaos state. Windows such as MMO and intermittency seem to occur in this output too. Our results are compared very well to the experimental observations in the QD-LED.

References