ROUTE TO CHAOS IN A CO_2 LASER WITH INJECTED SIGNAL

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It is well known that the equations for a single-mode laser can be reduced to the Lorenz equations[1] and a chaotic behaviour should be seen. However, this type of chaos has never been observed because of the actual values of the parameters in physical systems. Hence, it is necessary to introduce an external modulation[2] or to have a multimode laser where the degrees of freedom are larger than three. In these systems, experimental results have been reported [3-4]. In this communication a transition to chaos by intermittency is numerically shown in a single-mode laser with an injected signal without any modulated parameter. We refer to a CO_2 laser medium where the pressure broadening provides a homogeneous gain line. A CO_2 laser system has a relaxation time ($1/\gamma_{\parallel} = 10^{-3}$s) much larger than the dipole decay time ($1/\gamma_{\perp} = 10^{-8}$s); hence, the single mode dynamics is described by the coupling between two degrees of freedom (rate equations). Introducing an external field whose frequency ω_1 is detuned with respect to the cavity resonance ω_0 and the atomic transition frequency ω_c, we have the three degrees of freedom necessary for the onset of chaos.

The coupled field-molecules equations are:

$$\left.\begin{aligned}
\dot{x} &= \theta y + \frac{1}{1+\delta^2}\,\Delta x + \frac{\delta}{1+\delta^2}\,\Delta y - x + A \\
\dot{y} &= -\theta x + \frac{1}{1+\delta^2}\,\Delta y - \frac{\delta}{1+\delta^2}\,\Delta x - y \\
\frac{k}{\gamma_{\parallel}}\dot{\Delta} &= \frac{-1}{1+\delta^2}\,\Delta(x^2+y^2) - \Delta + \Delta_0
\end{aligned}\right\rbrace \qquad (1)$$

where x,y are the real and imaginary parts of the field, Δ is the population inversion normalized to its threshold, A is the amplitude of the external field, considered as real (all the fields are normalized to the Rabi frequency$/(\gamma_\perp\gamma_\parallel)^{1/2}$), $\theta = (\omega_c-\omega_1)/k$ and $\delta = (\omega_0-\omega_1)/\gamma_\perp$ are the detunings of the cavity frequency ω_c and the atomic frequency ω_0 with respect to the external field frequency ω_1 (Fig. 1), k is the cavity loss (3×10^7 s^{-1} for a CO_2 laser), $\dot{\Delta}_0$ is the pump rate, and the dot denotes the derivative with respect to the adimensional time $\tau=kt$. Fixing the values for the parameters θ and δ, we have resolved numerically Eq. (1) for different values of the external field amplitude A.

For very small values of A, the intensity $n = x^2+y^2$ oscillates with a frequency which is nearly the eigenvalue frequency resulting from the linearization of the Eq. (1). In this case the instantaneous frequency of the laser field sets at the value assigned by linear pulling and the influence of the external field is practically negligible. In other words, the frequency

$$\omega_{inst} = \omega_1 - \dot{\phi} \quad , \tag{2}$$

where $\dot{\phi} = d\phi/d(kt)$, and $\phi = \arg(x+iy)$ is such that

$$(\omega_0 - \omega_{inst})/\gamma_\perp = (\omega_{inst} - \omega_c)/k \quad . \tag{3}$$

Increasing A, two frequencies appear on the intensity oscillations, as it is shown in Fig. 2a (intensity as a function of time) and in Fig. 2b (Poincare map for $\dot{n}=0$). A 2-torus of two incommensurate frequencies is shown in the Poincare map. The higher and lower frequencies have an approximate ratio around 20. For increasing values of the external field, the system does not go to the chaotic behaviour via quasi-periodicity, but the two frequencies lock at a commensurate value. In the Poincare map we observe a discrete number of

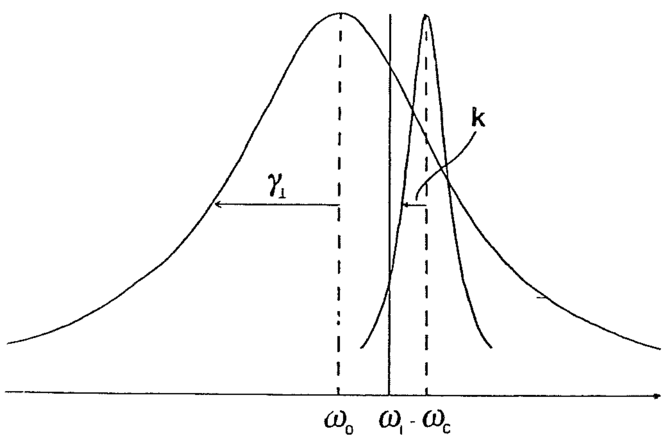


Fig. 1 Qualitative plot of the frequency relations among atomic line (homogeneous width γ) centered at ω_0, cavity line (width k) centered at ω_c, and external field centered at ω_1. The choice of the frequency position is such that the injected signal acts as a perturbation around the position of the internal laser frequency, imposed by the pulling conditions (see Eq. (3)).

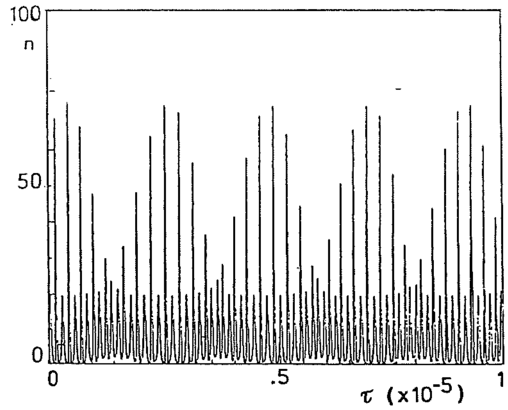


Fig. 2a Intensity $n=x^2+y^2$ of the laser radiation versus the normalized time $\tau=kt$ for the following set of control parameters: $\Delta_0=10$, $\delta=-0.01$, $\theta=0.025$, $A=0.021$. The time behaviour appears as the "crossing" of two oscillations, one at high frequency with peaks around $n=20$, the other at lower frequency with peaks up to $n\simeq 75$, the ratio of the two frequencies is around 20. The crossing picture is misleading as one can see by observing the crossing points which do not repeat regularly.

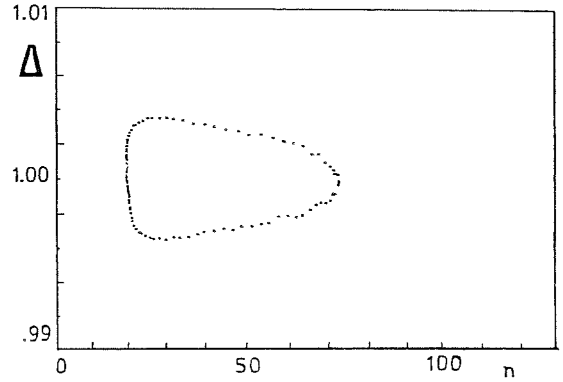
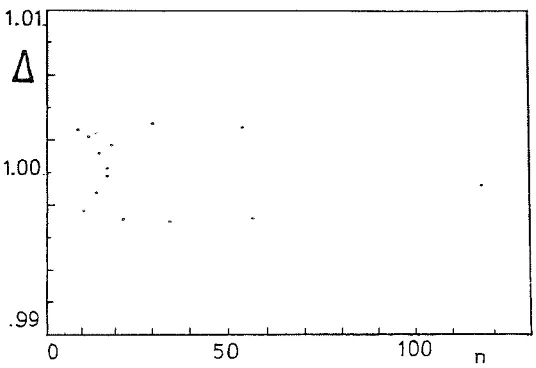
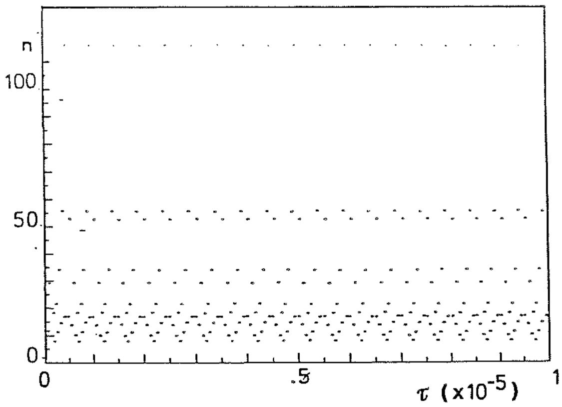


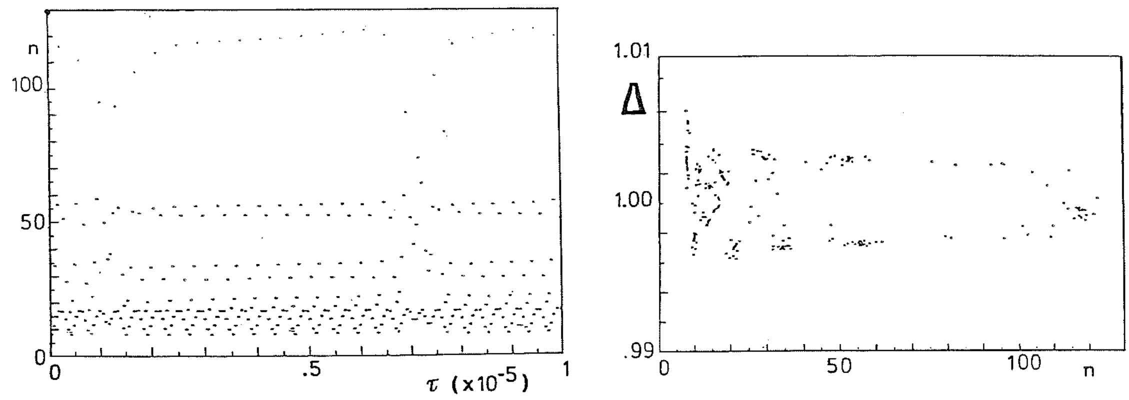
Fig. 2b The Poincare section in the plane Δ-n shows a continuous line, corresponding to the intersection of a torus with a plane. The continuity of the section is evidence that the two frequencies are incommensurate.





b)

Fig. 3a-b For $A=0.02295$ (all the other parameters as in Fig. 1) the two frequencies lock at a ratio n_1/n_2 (n_1,n_2 integers) where $n_2=14$. This is shown by the discrete number of 14 points in the Poincare section (Fig. 2a). In Fig. 2b we give the time plot n vs τ, limited to the peaks in order to display evidence of the regularity.



Figs.4a-b For A=0.022955 the system unlocks and goes into an intermittency region. This is shown clearly in the time domain (again limited to the sequence of peaks). The corresponding Poincare section shows a broadening around the discrete points.

points corresponding to the 2-torus (Fig. 3). The number of points is equal to the denominator of the ratio $n_1/n_2(n_1,n_2$ integers) of the two frequencies. A further increase in the parameter A destabilizes the lower frequency by intermittency as is shown in Fig. 4, and the system goes eventually to a chaotic regime (Fig. 5).

Chaos in a laser with injected signal has also been described in a model atomic system without adiabatic elimination of the polarization[5]. There, the extra-degree of freedom introduced by the new variable is compensated for by "freezing" one of our parameters, namely imposing the constraint $\omega_0=\omega_c$.

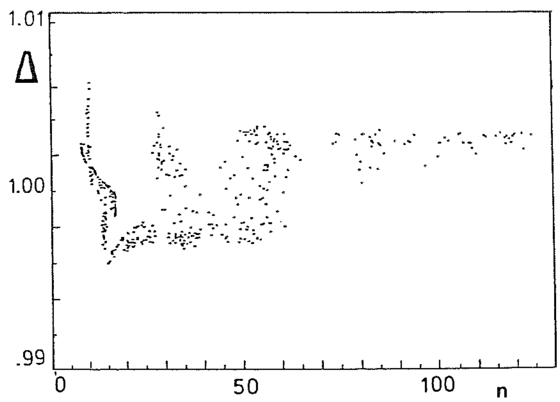


Fig. 5 For A=0.02297 the system has become chaotic. Since the time plot would be irrelevant, we show the Poincare section, which is organized in "sheets", each one broadened.

We have preferred to refer our calculation to a real molecular system, since we are carrying an experiment of this kind. It is worth noticing that the manifold of the initial conditions can be split into sub-manifold leading to two (or more) different asymptotic trajectories (attractors) in phase space. This is the phenomenon of "generalized multistability" already described in reference 3.

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