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Oscillations and Chaos on the Free Surface of a Heated Fluid.

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Summary. — We report the observation of oscillatory and chaotic instabilities on a fluid layer with a free surface, heated from below. The system is driven in a bidimensional state by a spatial modulation of the heat flux on the free surface. For increasing temperature gradients the system yields oscillations periodic in time, initially at a frequency of 8 mHz, then with a second frequency lower by a ratio 30 and eventually with an aperiodic behaviour corresponding to the onset of turbulent regime. The oscillatory regions are localized in space.

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Introduction.

We report the observation of oscillatory and chaotic instabilities in a cell of fluid with a free surface, heated from below.

A large amount of experimental data and theoretical studies is available on the Rayleigh-Benard instability (RB) (1), that is for a fluid confined between rigid horizontal plates and heated from below, where the successive bifurcations are due to the competition between thermally induced buoyancy and viscous damping.

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(1) CH. NORMAND, Y. POMEAU and M. G. VELARDE: *Rev. Mod. Phys.*, **49**, 581 (1977).

In the free-surface case a second destabilizing mechanism appears, due to thermally induced gradients in the free-surface tension [2-4].

The onset of the instabilities is ruled by the Rayleigh and Marangoni numbers, respectively, which are

$$(1) \qquad R = \frac{\alpha g \Delta T\, d^3}{\nu k} \quad \text{and} \quad M_a = \frac{-(\mathrm{d}\sigma/\mathrm{d}T)\,\Delta T\, d}{\varrho \nu k},$$

where α is the thermal-expansion coefficient, g the gravitational acceleration, ΔT the applied temperature difference between the bottom and the top of the fluid, d the thickness of the fluid layer, ν the kinematic viscosity, k the thermal diffusivity, ϱ the fluid density and σ the surface tension.

To a first approximation for vertical thermal gradients, the relative weights of the two effects are ruled by the equation [2,3]

$$(2) \qquad \frac{M_a}{M_c} + \frac{R_a}{R_c} \simeq 1,$$

where the critical numbers $R_c = R_c(M = 0)$, $M_c = M_c(R = 0)$ depend on the adimensional heat flux L at the free surface ($L = qd/\lambda$, where q is the heat flux at the free surface and λ the thermal conductivity of the fluid). For $L = 0$ and an undeformed free surface it results that $R_c = 669$ and $M_c = 79.6$. Of course, $L = 0$ is experimentally unfeasible and the two numbers R_c, M_c must be evaluated for the appropriate thermal boundary conditions. An experimental investigation of eq. (2) was performed by PANTALONI *et al.* [3], although these authors did not account for any radiant heat transfer at the upper interface.

Replacing eqs. (1) in (2), we see that for a small enough thickness we can have practically only Marangoni effect, that is, we satisfy eq. (2) with $R \to 0$.

In RB it has been established theoretically [5] and experimentally [6] that, for $R > R_c$, the steady flow becomes time dependent and a cascade of bifurcations arises that drives the fluid to a chaotic motion. The best experimental observations of this fact are done in cells of small aspect ratio Γ (that is, the

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(2) D. A. NIELD: *J. Fluid Mech.*, **19**, 341 (1934).

(3) M. G. VELARDE and J. L. CASTILLO: in *Convective Transport and Instability Phenomena*, edited by J. ZIEREP and H. OERTEL jr. (Braun Verlag, Karlsruhe, 1982).

(4) J. PANTALONI, B. BAILLEUX, J. SALAN and M. G. VELARDE: *J. Non-Equilibrium Therm.*, **4**, 201 (1979).

(5) F. H. BUSSE: *J. Fluid Mech.*, **52**, 97 (1972).

(6) P. BERGÉ and M. DUBIOS: *Opt. Commun.*, **19**, 129 (1976); J. P. GOLLUB and S. V. BENSON: *Phys. Rev. Lett.*, **41**, 14 (1978); M. GIGLIO, S. MUSAZZI and U. PERINI: *Phys. Rev. Lett.*, **47**, 4 (1981); A. LIBCHABER, C. LAROCHE and S. FAUVE: *J. Phys. Lett.*, **43**, 211 (1982).

ratio of the horizontal dimension with respect to the vertical one), because in cells with large Γ the fluid motion has a very complicated spatial structure.

In the Marangoni case it is practically impossible to perform an experiment with small aspect ratio. Indeed, according to eq. (2), the Marangoni effect prevails on RB if the fluid layer is a few millimetres deep. This means that for small Γ the surface curvature due to the meniscus on the lateral walls can not be neglected. To overcome this difficulty we work with a large-aspect-ratio cell. In general, the onset of the steady flow is associated with the appearance of a three-dimensional structure (mainly hexagons).

A three-dimensional pattern is ill characterized by a few mode amplitudes, and a detailed description requires a lengthy set of local measurements. Furthermore, it is difficult to compare measurements at different ΔT, since the space pattern changes with ΔT. Thus we have imposed a two-dimensional roll pattern by a suitable thermal boundary condition on the top surface. This method forces the appearance of a two-dimensional pattern which is described by a single time-dependent amplitude, easily measurable.

1. – Experimental set-up.

Our fluid is silicone oil that at 20 °C has the following properties:

$$\nu = 2.88\cdot 10^{-2}\ \text{cm}^2\ \text{s}^{-1}, \qquad k = 1.05\cdot 10^{-3}\ \text{cm}^2\ \text{s}^{-1}, \qquad \alpha = 8.9\cdot 10^{-4}\ {}^\circ\text{C}^{-1}$$

$$\varrho = 0.88\ \text{g cm}^{-3}, \qquad \mathrm{d}\sigma/\mathrm{d}T = -0.2\ \text{dyn cm}^{-1}\ {}^\circ\text{C}^{-1}.$$

The fluid thickness is $d = 2$ mm. With this fluid depth and $L > 1$, eq. (3) yields a surface tension term contribution larger than 80 %, and we can state that the convective instability is mainly ruled by the Marangoni effect.

The cell (fig. 1) has a large aspect ratio with $\Gamma_x = 30$ for the larger dimension (x-axis) and $\Gamma_y = 15$ for the smaller one (y-axis).

The mean temperature of the cell (15 °C) is stabilized with a circulation of water whose temperature is maintained stable within 0.02 °C. The bottom plate, made of copper, is further heated by an electrical resistor with the current controlled in order to maintain the temperature difference between the upper and lower plates stable within 2 m°C. The lateral walls of the cell are made of plexiglass to allow for optical inspection.

The space-periodic thermal perturbation of the free surface is imposed by shaping the lower face of the aluminium cover plate as a grating with groves of pitch 6 mm and depth 3 mm. The minimum distance between the grating and the free surface is about 1 mm. The spatial period of 6 mm corresponds, for a 2 mm deep fluid layer, to the expected critical wave-length when heating from below [2,3]. Because of the periodic thermal horizontal gradient, induced

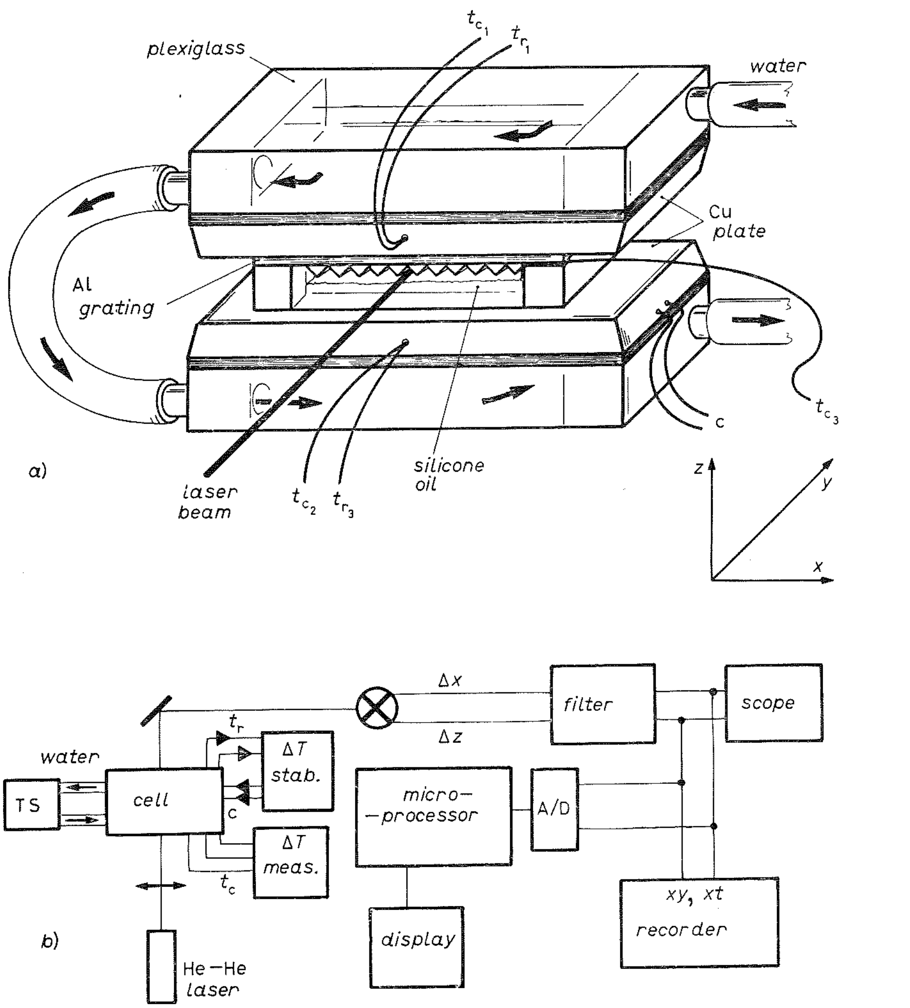

Fig. 1. – *a*) Experimental set-up: t_c are Cu-Constantan thermocouples, t_r are Pt resistor and *c* are the wires for the electrical heater; *b*) block diagram of the control and measuring system.

on the surface, the critical M_c is very low and ill defined, hence the steady-flow regime appears for very small ΔT.

However, since we are interested in the higher-order time-dependent phenomena, we label our results directly in terms of the actual temperature difference ΔT between the two plates.

The detection system consists of a laser beam of diameter 0.5 mm, which crosses the fluid layer parallel or perpendicular to the groves of the grating and is deflected by the thermal gradients inside the fluid. The deflection is measured by a four-quadrant detector that gives the position of the beam in an (x, y)-plane. The output of the detector is sent to a low-pass filter with a suitable cut-off, whose output is connected to the recorder and to the data acquisition system that evaluates the Fourier transform or the autocorrelation of the signal and then sends it to a Videographic system (Tektronix 4051) for visualization.

2. – Experimental results.

The experimental results are shown in fig. 2 to 7. The fluid motion does not show any appreciable time dependence up to $\Delta T = 8.5$ °C at which it is possible to see a low-frequency (LF) oscillation whose period is about 70 min. In every period a higher-frequency (HF) burst of smaller amplitude is present (fig. 2*a*)).

We see in fig. 2*b*) that the HF period is not uniform along the burst, but it shortens down to two minutes. This behaviour is conserved between 8.5 °C and 10.4 °C, but, for increasing ΔT, the length of the bursts becomes longer.

Above 10.4 °C the HF is stable with an amplitude higher than LF. In fig. 3*a*) is shown the X deflection as a function of time (the Z signal has the same behaviour) and in fig. 3*b*) the relative spectrum is given.

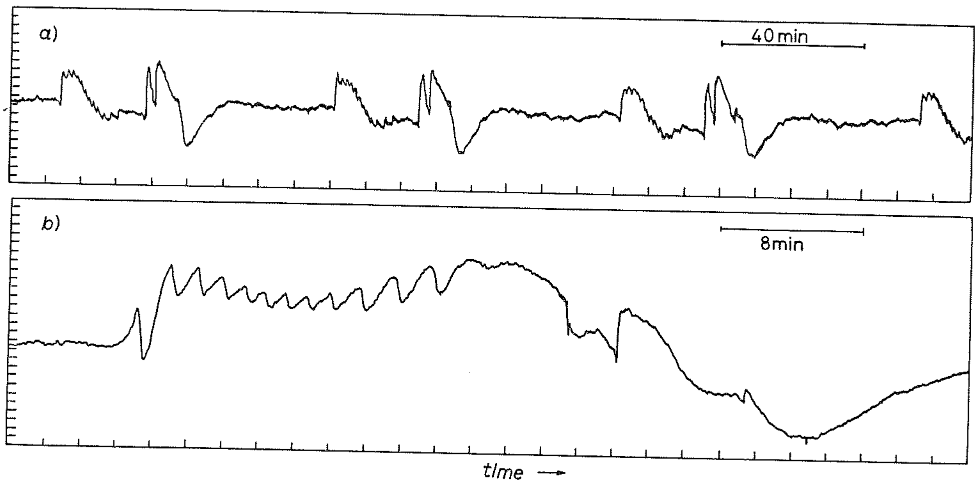

Fig. 2. – *a*) Amplitude of the horizontal beam deflection *vs.* time at $\Delta T = 8.6$ °C; *b*) expanded view of the horizontal deflection at the same ΔT in a different point of the roll.

At $\Delta T = 11.05$ °C the LF amplitude becomes larger than that of the HF, as shown in fig. 4*a*). In this region we observed a practically continuous spectrum (fig. 4*b*)) due to the spiking behaviour of the LF superposed to the small HF amplitude. However, the spectrum does not contain the amount of information shown in the time plot. Conditional spectra (corresponding to a three-point

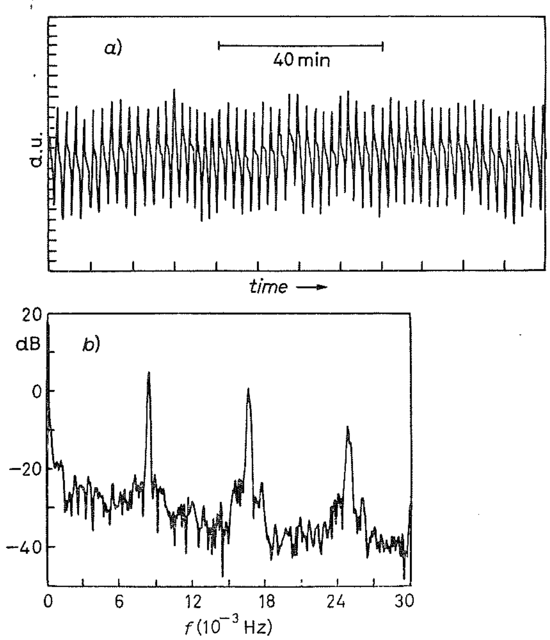

Fig. 3. – *a*) Horizontal deflection in the oscillatory regime ($\Delta T = 10.9$ °C), *b*) corresponding power spectrum, $f_{osc} = 8.3 \cdot 10^{-3}$ Hz.

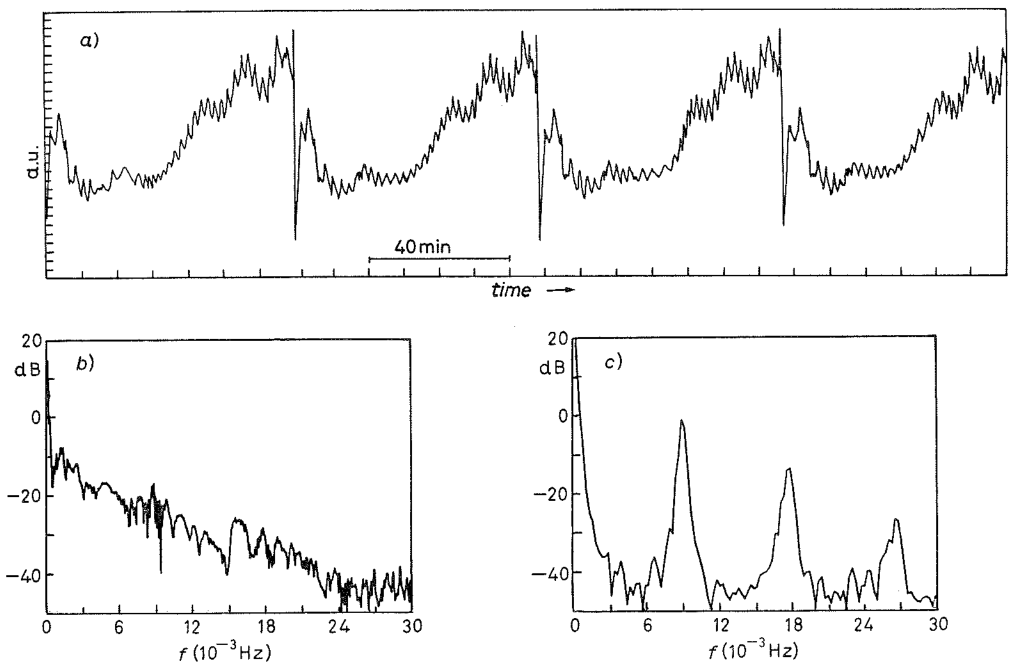

Fig. 4. – *a*) Vertical deflection *vs.* time at $\Delta T = 11.25$ °C, *b*) power spectrum corresponding to the signal of fig. 4*a*), *c*) spectrum of the signal of *a*) taken between two successive spikes.

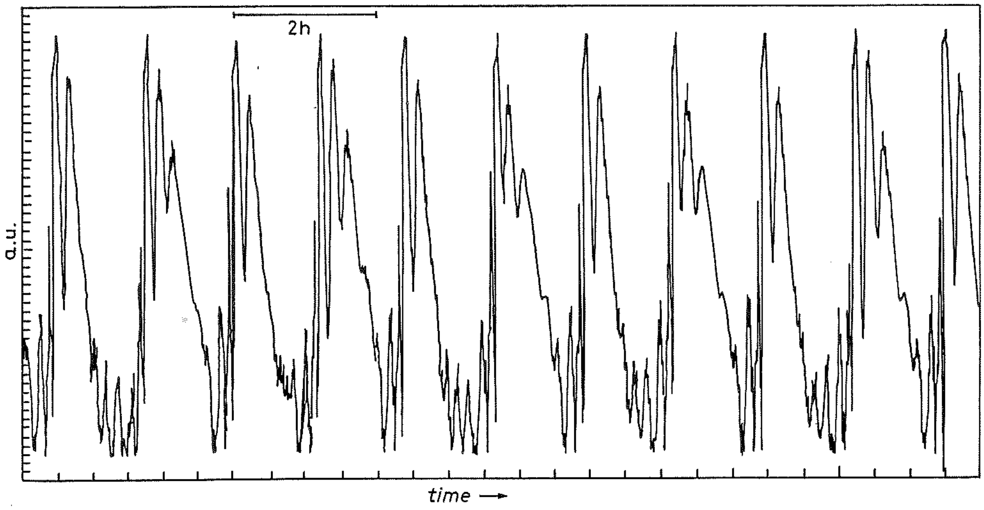

Fig. 5. – Horizontal deflection *vs.* time at $\Delta T = 11.25$ °C, when ΔT is decreased from 11.37 °C, where the fluid has a chaotic behaviour.

correlation function) are more appropriate in such a case and an example is shown in fig. 4*c*).

At $\Delta T = 11.37$ °C the system becomes chaotic, that is, the periodicity of the signal is destroyed. It is difficult to state which is the actual route to chaos, whether intermittency or breaking of the torus formed by LF and HF.

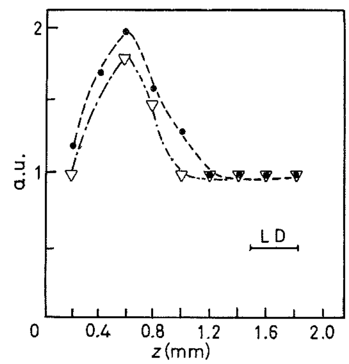

Fig. 6. – Amplitude of the deflections Δx (•) and Δz (▿) as a function of the z coordinate ($\Delta T = 8.6$ °C) with the laser beam parallel to the grooves of the greating. LD is the laser diameter.

By reducing the temperature by small steps a more complicated regime is observed. Figure 5 shows the time behaviour at $\Delta T = 11.25$ °C. In this figure we can easily see that the LF period has changed from 70 min to 80 min and the HF oscillation has a very low amplitude (it can hardly be seen in fig. 5).

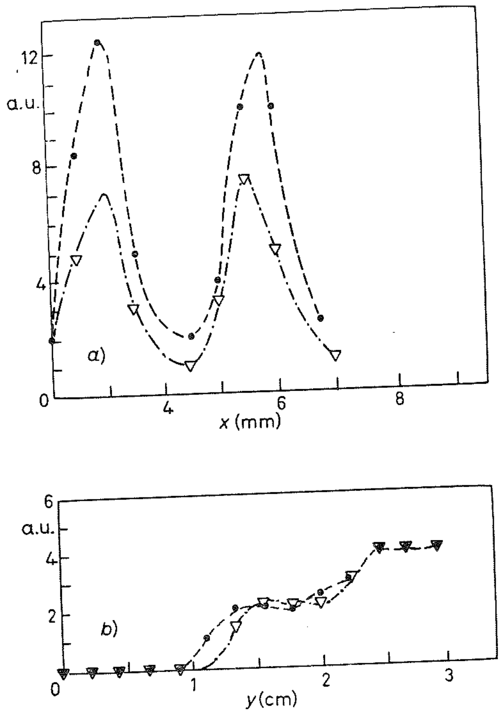

Fig. 7. – *a*) Amplitude of the deflections Δx (•) and Δz (▿) as a function of the x co-ordinate with the laser beam parallel to the grooves, *b*) amplitude of the deflections Δy (▿) and Δz (•) as a function of the y co-ordinate with the laser beam perpendicular to the grooves.

Furthermore, a subharmonic of the LF (period 2) is present and it appears another oscillation with a frequency of about 6 times the LF.

The biperiodic regime ends decreasing ΔT down to 10.56 °C, indeed only the HF oscillation remains and it disappears at $\Delta T = 9.0$ °C.

Obviously the reported data are collected after waiting a suitable time, of order of hours, to be sure that the new dynamic regime be well stabilized.

All the preceding data are obtained with the laser beam parallel to the grooves. In order to show the spatial distribution of the fluid motion in the monoperiodic regime, we have measured the peak-to-peak deflection amplitude with the laser beam incident on the fluid layer in different points and crossing it perpendicular or parallel to the grooves.

In fig. 6 the amplitudes of the deflections ΔX and ΔZ are shown as functions of the vertical co-ordinate Z of the incident point with the beam parallel to the grooves y. The deflection shows that the maximum perturbation inside the fluid is localized near the surface. One should deconvolute the finite size of the laser beam from this diagram to have the real behaviour.

In fig. 7 the horizontal and vertical deflection amplitudes are shown *vs.*

the x–or y–co-ordinate of the incident point with the beam, respectively, parallel or perpendicular to the grooves. The z–co-ordinate of the incidence point corresponds to the position of the maximum in fig. 6.

In the first case, x scan, we see evidence of a spatial periodicity equal to the grating pitch; in the second case, y scan, we have a monotonic behaviour. As expected, the flow is two-dimensional, due to the spatial modulation of the thermal gradient imposed by the grating at the upper aluminium block, and the oscillator appears localized near one of the longer walls of the cell.

3. – Conclusions.

As a conclusion, we have reported preliminary evidence of the successive destabilization of the roll patterns in a Marangoni cell up to chaos, via the appearance of two independent frequencies. While for rigid boundaries and convective instabilities there is already a vast amount of data, no evidence of chaos had been previously reported for a free-surface and tension-driven instability.

The dependence of the dynamics on the past history (hysteresis) shows the simultaneous presence of many basins of attraction for the same parameter or generalized multistability, as already observed in laser instabilities ([7]). Further research is in course to explore the jumps over different attractors induced by noise, as observed in other physical systems.

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([7]) F. T. ARECCHI, R. MEUCCI, G. PUCCIONI and J. TREDICCE: *Phys. Rev. Lett.*, **49**, 17 (1982).

● RIASSUNTO

Si riportano le prove sperimentali di oscillazioni periodiche, e successivamente di aperiodicità (turbolenza) in uno strato di fluido scaldato dal di sotto, e con la superficie libera (effetto Marangoni). Mediante fresatura di canali sulla parte inferiore del coperchio della cella, si forza una modulazione spaziale di flusso termico che vincola i moti a due dimensioni. All'aumentare del gradiente di temperatura, si hanno inizialmente oscillazioni periodiche a 8 mHz, successivamente compare una seconda frequenza, piú bassa di un fattore 30, infine si ha un comportamento aperiodico che segna la nascita del regime turbolento. Le oscillazioni non interessano tutta la superficie del fluido, ma sono localizzate nello spazio.

Осцилляция и хаос на свободной поверхности нагретой жидкости.

Резюме (*). — Рассматриваются осцилляторные и хаотические неустойчивости на свободной поверхности слоя жидкости, нагреваемого снизу. Рассматриваемая система находится в двумерном состоянии за счет пространственной модуляции потока тепла на свободную поверхность. При увеличении градиентов температур в системе возникают осцилляции, периодические во времени, первоначально с частотой 8 мГц, а затем со второй частотой, которая меньше первой в 30 раз, и в итоге возникает апериодическое поведение, соответствующее началу турбулентного режима. Области осцилляций локализованы в пространстве.

(*) *Переведено редакцией.*