DETERMINISTIC MODE ALTERNATION, GIANT PULSES AND CHAOS IN A BIDIRECTIONAL CO_2 RING LASER

G.L. LIPPI, J.R. TREDICCE, N.B. ABRAHAM [1,2] and F.T. ARECCHI [2]
Istituto Nazionale di Ottica, Largo Enrico Fermi 6, 50125 Firenze, Italy

Received 17 December 1984

A CO_2 ring laser with a single longitudinal mode propagating in each direction shows a variety of stable, periodic, and aperiodic phenomena depending on gas pressure, cavity detuning and relative excitation. Three distinct low frequency time scales for dynamical behavior are observed and are explained by numerical solutions of an appropriate model.

Experimental investigations of the onset of deterministic chaos have been recently reported in many areas of physics [1]. The essential elements for such chaos are nonlinear interactions and a sufficiently large phase space (i.e., at least three coupled degrees of freedom), a classic example being the three equations of the Lorenz model [2]. In quantum optics, single-mode lasers with a homogeneously-broadened response of the medium can have the same kind of dynamical solutions found in the Lorenz equations [3]. However, the parameter values required to see chaos in homogeneously-broadened systems have not yet been achieved in any experimental system. In particular, for many of the most common lasers (CO_2, YAG and semiconductors) the number of key dynamical variables is reduced because of rapid relaxation of the polarization. This means that the polarization plays no essential role in the dynamical evolution of the system, justifying its adiabatic elimination in the corresponding models. In order to see chaos in such systems, the phase space must be enlarged by external driving which may be modulation of the gain or loss [4] or an injected optical field [5]. Chaos has also been seen experimentally in laser systems where the phase space has been modified by the inclusion of many modes or of many different material resonant frequencies as in a laser with an inhomogeneously-broadened gain profile [6], though simple modes in these cases have usually not been found.

Here we show that the bidirectional CO_2 ring laser system, is one of the simplest cases of mode coupling (two counter-propagating modes) that gives an intrinsic chaos and it can be described by a seven-equation model. Specifically, for large pumping we find that the steady states in which one mode quenches the other are destabilized by small amounts of cavity detuning, resulting in a slow alternation of the forward and backward modes almost as if the system were switching between the two steady states. Results that look remarkably similar have recently been reported in *noise-induced* switching in a bidirectional ring dye laser [7] fully described by only two coupled field equations. Here by contrast the slowly-varying response of the population inversion enlarges the phase space of the system to give *dynamical* switching with chaotic durations of each phase. For small pump values we observe a periodic spontaneous generation of giant spikes (as if there were spontaneous Q-switching) occurring simultaneously in both modes. Between the two phases we find a gradual change from one to the other with emission of chaotic spikes.

Overall, the system displays three different characteristic time scales: a very low frequency mode alternation, a pulsing related to the population recovery time, and the normal rate-equation relaxation oscillation of energy exchange between the field and the

[1] Permanent Address: Dept. of Physics, Bryn Mawr College, Bryn Mawr, PA 19010 USA.
[2] Also with Dipartimento di Fisica, Università di Firenze.

0 030-4018/85/$03.30 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

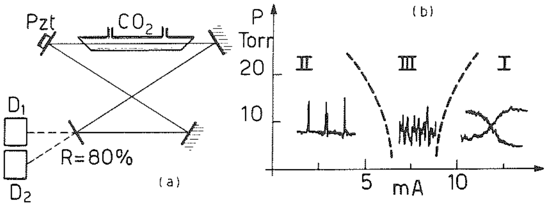

Fig. 1. (a) Experimental set-up: Gain cell with partial pressures: CO_2 1 Torr, N_2 1.5 Torr, He variable. Cavity length 4.2 m; PZT piezoelectric mirror translator; D_1 and D_2 detectors for forward and backward intensities. (b) Phase diagram for total pressure (P) and discharge current (i). Regions are: (I) mode alternation, (II) self Q-switch, (III) irregular pulsation.

population inversion.

The experimental set-up is shown in fig. 1a. The two detected signals are analyzed with a real-time spectrum analyzer and a transient digitizer. Fig. 1b shows a phase diagram with different operating regimes: I. mode switching, II. Q-switching, and III. aperiodic behavior. For different values of the detuning, the boundaries do not change qualitatively.

In region I, bistability between the two states of one mode quenching the other is observed for small detunings $\delta < \delta_c \sim 0.2$. ($\delta$ represents the detuning of the laser cavity resonance frequency from the center of the material resonance measured in units of the polarization relaxation rate $\gamma_\perp$.) Mode alternation, observed for $\delta > \delta_c$, is shown in fig. 2. The maxima for the steady state output of the two modes differ, indicating clearly that the field loss rates are not exactly the same in the two directions. The frequency of the switches depends on the amount of detuning. The output switches approximately from zero intensity to the steady state nonzero solution.

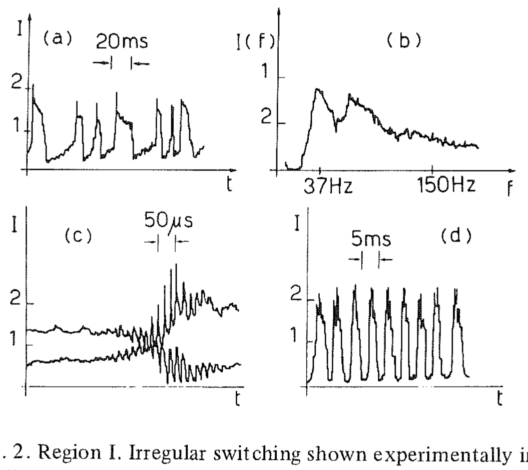

Fig. 2. Region I. Irregular switching shown experimentally in (a) (P = 15.0 Torr, i = 8 mA) gives a broadened peak in power spectrum at 30 Hz (b). Switch induced high frequency ringing shown in expanded scale (c) similar to relaxation oscillations in approach to steady state values. Numerical result (d) for $\gamma_\parallel = 2.5 \times 10^3$, $\gamma_\perp = 10^8$, $K_1 = 1.4 \times 10^7$, $K_2 = 1.4001 \times 10^7$, $z_0 = 3$. All vertical scales in arbitrary units.

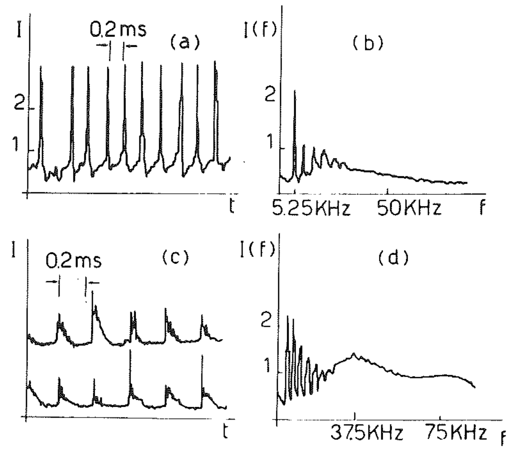

Fig. 3. Region II experimental results. High intensity sharp peaks (a) appear simultaneously in both modes, with the spectrum (b) (P = 13.0 Torr, i = 4.5 mA). Damped relaxation oscillations (c) contribute to higher frequencies in the power spectrum (d) (P = 11.0 Torr, i = 3.5 mA). The frequency of Q-switch is proportional to the current and to the total gas pressure with values close to $\gamma_\parallel$. Fig. 3c shows both modes offset for clarity. Vertical scales in (a), (b) and (d) in arbitrary units.

In region II (fig. 3) we find giant pulse generation in which both modes pulse simultaneously with a very regular repetition rate of the order of the population recovery rate. Higher frequency relaxation oscillations, when observed, are exactly out of phase in the two modes as shown in fig. 3c.

Between regions I and II chaotic pulsing was observed (fig. 4) and calculations of the characteristics of the underlying attractor using the algorithm proposed by Grassberger and Procaccia [8] indicates the existence of a low dimensionality attractor for the time series of fig. 4c and a larger yet finite dimensionality for the more complex time series of fig. 4e.

The key feature necessary to properly describe the

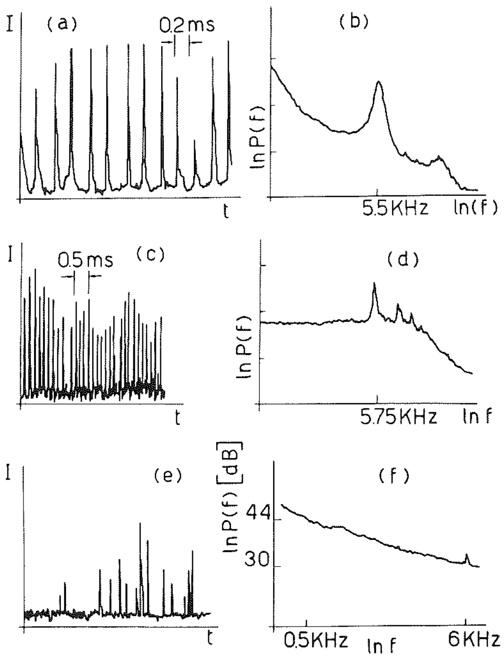

Fig. 4. Region III. Experimental time series (a, c, e) and corresponding power spectra (b, d, f). Peak intensities of the pulses become more irregular with increasing current 5 mA, 5.5 mA, 6 mA, (a, c, e, respectively) at pressure 13.5 Torr. Spectra change from sharp peaks as in fig. 3b to broadened peaks (b, d) with increasing background and low frequency divergence, $f^{-0.6}$ (f).

two mode laser is the inclusion of the grating formed in the population and the resulting Bragg-like scattering of one mode into the other which can double the coupling between the two modes [9]. The model equations suitable for description of a CO_2 system (where the polarization has been adiabatically eliminated and the population grating is described by only the lowest order harmonic component) are:

$$\dot{x} = (1 + i\delta)^{-1}[zx + w^*y] - x, \quad (1)$$

$$\dot{y} = (1 + i\delta)^{-1}[zy + wx] - (K_2/K_1)y, \quad (2)$$

$$(K_1/\gamma_\parallel)\dot{z} = -(z - z_0) - (1 + \delta^2)^{-1} \times [z(|x|^2 + |y|^2) + w^*x^*y + wxy^*], \quad (3)$$

$$(K_1/\gamma_\parallel)\dot{w} = -w - (1 + \delta^2)^{-1} \times \{zx^*y + w[|x|^2 + |y|^2 + i\delta(|y|^2 - |x|^2)]\}. \quad (4)$$

Here x and y are the two complex field amplitudes, z is the spatially uniform component of the population inversion, and w is the amplitude of the population grating formed at the second spatial harmonic of the wavelength of the field. K_1, K_2 and $\gamma_\parallel$ are the relaxation rates of the two fields and the population inversion, respectively; δ is the cavity detuning defined above. z_0 is the excitation parameter, values greater than one initiating laser action. Time is renormalized to $\tau = K_1 t$ and the dot signifies derivative with respect to τ.

Numerically generated time series showing agreement with the experimental results are given in fig. 2d and fig. 5. The giant pulse generation is obtained only with differences between K_1 and K_2. Physically this means that the frequencies of the steady state solutions for the two counterpropagating modes (taken separately) will differ because of different amounts of mode pulling. With the population grating scattering one field into the other we have a mechanism for Q-switching much like that found recently in the study of a single-mode CO_2 laser with a detuned injected signal [5,6].

Pulsing in bidirectional ring YAG lasers is a well studied problem where a model such as ours has been used [9]. However the linewidth of such lasers is so large that significant detuning is incompatible with single-mode operation. Thus there has been no physical basis for using detuning in the models to explain the experimental results. Other factors such as backscattering, external noise, isotope splittings and excitation modulation have been cited as the causes of pulsing and switching. By contrast we find that the complex dynamics observed in our CO_2 laser is simply explained by inclusion of modest detunings.

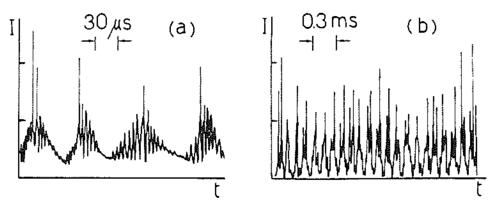

Fig. 5. Numerical results (with parameters: $z_0 = 5$, $K_1 = 1.4 \times 10^7$, $K_2 = 1.425 \times 10^7$ for (a) and $K_2 = 1.4001 \times 10^7$ for (b) show good agreement with experimental results in figs. 3c and 4c, respectively.

We wish to thank G.P. Puccioni for helpful discussions and L. Albavetti and P. Poggi for technical assistance. The work of N.B.A. was supported in part by NSF Grant INT-83000142 and an Alfred P. Sloan Research Fellowship. Work partly supported by the European Joint Project on Optical Bistability.

References

[1] N.B. Abraham, J.P. Gollub and H.L. Swinney, Physica 11D (1984) 252; Universality in chaos, ed. P. Cvitanović, (A. Hilger, London, 1984).
[2] E.N. Lorenz, J. Atmos. Scie. 20 (1963) 130.
[3] H. Haken, Phys. Lett. 53A (1975) 77.
[4] F.T. Arecchi, R. Meucci, G.P. Puccioni and J.R. Tredicce, Phys. Rev. Lett. 49 (1982) 1217;
W. Klische, H.R. Telle and C.O. Weiss, private communication.
[5] J.R. Tredicce, F.T. Arecchi, G.L. Lippi and G.P. Puccioni, Optics Comm. 51 (1984) 308.
[6] For a review of many chaotic laser systems see articles and references therein in a special issue of J. Opt. Soc. Am. B to appear in January 1985.
[7] S. Singh and L. Mandel, Phys. Rev. A20 (1979) 2459;
R. Roy and L. Mandel, Optics Comm. 34 (1980) 133.
[8] P. Grassberger and I. Procaccia, Phys. Rev. A28 (1983) 2591; Phys. Rev. Lett. 50 (1983) 346.
[9] See for example: A.S. Agabekyan, A.Z. Grasiuk, I.G. Zubarev, V.I. Svergun and A.N. Oraevskii, Radiotekh. Elektron. 9 (1964) 2156;
B.L. Zhelnov, V.S. Smirnov, A.P. Fadeev, Opt. Spek. 28 (1970) 744;
G.V. Perevedentseva, P.A. Khandokhin and Ya.I. Khanin, Kvan. Elektron. (Moscow) 7 (1980) 128;
N.I. Polushkin, P.A. Khandokhin and Ya.I. Khanin, Kvan. Elektron. (Moscow) 10 (1983) 1461;
P. Mandel and G.P. Agrawal, Optics Comm. 42 (1982) 269; and references therein.