ON CHAOS IN LASERS WITH MODULATED PARAMETERS: A COMPARATIVE ANALYSIS

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Modulation of the laser excitation rate and modulation of the cavity losses are shown to result in the same dynamical phenomena for lasers in which the polarization can be adiabatically eliminated. However, to obtain the same results, the degree of modulation of the excitation must be larger than the degree of modulation of the losses.

1. Introduction

Over the past years several authors [1–9] have studied different kinds of laser systems with modulated parameters. Some of the theoretical analyses [1–3,6–8] have also been tested experimentally [4–6,8,9]. In such systems they have found oscillations at the modulation frequency, sub-harmonic bifurcations and chaotic behavior. In general these previous studies have studied as separate phenomena the effects of the gain or loss.

Here, we compare loss and pump modulation of lasers for which the polarization can be adiabatically eliminated in modelling their dynamical behavior. We find the expected similarities but also present the first clear evidence that the effects of the two types of modulation can differ dramatically. The behavior of lasers of this type can be described by two equations as first shown by Statz and deMars [10] for ruby lasers and discussed later in detail by others [11–13]. Adiabatic elimination was proposed as a systematic technique for the analysis of laser dynamics in ref. [14] and various detailed procedures for its careful application have since been presented [15,16]. The model discussed here is valid for CO_2 lasers [6], for solid state lasers [3–5,7,9–13,17] and, with small changes, also for semiconductor lasers [18,19]. One may expect the phenomena and relationships, that are discussed in the following sections, to be observed for all lasers in this class [20].

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2. Model

The Maxwell–Bloch equations describing a collection of two-level atoms in a single-mode, resonantly-tuned, laser cavity, after adiabatic elimination of the polarization, are [13]:

$$\dot{E} = -kE + (g^2/\gamma_\perp)EN, \tag{1a}$$

$$\dot{N} = -\gamma_\parallel(N - \bar{N}) - (4g^2/\gamma_\perp)|E|^2 N, \tag{1b}$$

where E is the slowly varying amplitude of the electric field; N is the population inversion with $\gamma_\parallel N$ the excitation rate; k, $\gamma_\perp$ and $\gamma_\parallel$ are the loss rates for the field, polarization and population inversion, respectively; g is the coupling constant given by

$$g^2 = (\omega\mu^2/\hbar\epsilon)V, \tag{2}$$

where μ is the dipole matrix element, V is the volume and ω is the transition frequency.

The non-trivial steady-state solution for N_s and E_s

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of eqs. (1) is given by

$$N_s = (k\gamma_\perp)/g^2, \qquad 1 + (4g^2/\gamma_\perp\gamma_\parallel)|E_s|^2 = g^2\bar{N}/k\gamma_\perp. \qquad (3)$$

Thus the normalized laser excitation rate is:

$$A = g^2\bar{N}/k\gamma_\perp, \qquad (4)$$

where $A = 1$ signifies the threshold for laser action.

A stability analysis of the steady-state solution given by eqs. (3) [11–13,21] finds that it is stable and that the system responds after a perturbation by relaxing back to the steady-state solution with damped oscillations of frequency

$$\Omega = [2\gamma_\parallel k(A-1)]^{1/2}. \qquad (5)$$

3. Modulation of one parameter

In order to achieve a third degree of freedom which is absolutely necessary if chaotic behavior is to be observed [22], we introduce a sinusoidal modulation of the loss rate for the field or of the pump rate, in the form:

$$k = k(t) = k_0(1 + m\cos\omega t) \qquad (6a)$$

or

$$A = A(t) = A_0(1 + l\cos\omega t). \qquad (6b)$$

Introducing the dimensionless variables

$$n \equiv (4g^2/\gamma_\perp\gamma_\parallel)|E|^2, \quad \Delta \equiv (g^2/k_0\gamma_\perp)N \qquad (7)$$

into eqs. (1), we have:

$$\dot{n} = -2kn + 2k_0 n\Delta, \qquad (8a)$$

$$\dot{\Delta} = -\gamma_\parallel(\Delta - A) - \gamma_\parallel n\Delta. \qquad (8b)$$

This system of two first order equations is non-autonomous, since either k or A is time dependent. The system can be transformed into a non-autonomous second order equation by taking the time derivative of eq. (8a) as follows:

$$\ddot{n} = -2\dot{k}n - 2k\dot{n} + 2k_0\dot{n}\Delta + 2k_0 n\dot{\Delta}. \qquad (9)$$

In eq. (9) we replace $\dot{\Delta}$ using eq. (8b) and we replace Δ using

$$2k_0\Delta = \dot{n}/n + 2k, \qquad (10)$$

which follows from eq. (8a), thus obtaining:

$$\ddot{n} - \dot{n}^2/n + \gamma_\parallel[1+n]\dot{n} + 2n[\dot{k} + k\gamma_\parallel(1+n) - k_0\gamma_\parallel A] = 0. \qquad (11)$$

Using eqs. (6a) or (6b), eq. (11) can be written explicitly as

$$\ddot{n} - \dot{n}^2/n + \gamma_\parallel[1+n]\dot{n} - n[\Omega_k^2 - 2k_0\gamma_\parallel n] = [-2k_0 m\omega\sin\omega t + 2k_0 m(1+n)\cos\omega t]n, \qquad (12a)$$

or

$$\ddot{n} - \dot{n}^2/n + \gamma_\parallel[1+n]\dot{n} - n[\Omega_A^2 - 2k_0\gamma_\parallel n] = -A_0 l\gamma_\parallel k_0\cos\omega t, \qquad (12b)$$

respectively, where Ω_k and Ω_a are defined as Ω using k_0 and A_0, respectively.

4. Comparison between loss and pump modulation

From eqs. (12), by inspection, we conclude that both types of modulation yield systems with the common features of forced nonlinear oscillators. We thus expect to observe similar behavior with either kind of modulation.

Time-dependent behavior can be found by solving eq. (12). To see the response of the steady-state solutions to a small perturbing modulation we linearize these equations near their steady-state values and find that the perturbations obey the following equations:

$$\ddot{\epsilon} + \gamma_\parallel A_0\dot{\epsilon} + \Omega_k^2\epsilon = -k_0 m[A_0\gamma_\parallel\cos\omega t - \omega\sin\omega t](A_0 - 1) \qquad (13)$$

for loss modulation, and

$$\ddot{\epsilon} + \gamma_\parallel A_0\dot{\epsilon} + \Omega_A^2\epsilon = [-A_0 l\gamma_\parallel k_0\cos\omega t](A_0 - 1) \qquad (14)$$

for pump modulation.

These are obviously similar results for forced damped harmonic oscillators with resonant frequencies Ω and damping rates $A_0\gamma_\parallel$, as was observed for loss modulation in ref. [3].

Close examination shows that the effective forcing terms have different dependencies on the modulation depth (m or l). In fact, the ratio m/l that gives the same effective forcing amplitude for the two cases is given by

$$m/l \sim [A_0/(A_0 - 1)^{1/2}](\gamma_\parallel/2k_0)^{1/2}, \tag{15}$$

when ω is chosen to be around Ω. For CO_2 lasers or solid state lasers and $A_0 \sim 2$, this ratio of $m/l \sim 10^{-2}$.

It was shown experimentally in ref. [6] that subharmonic bifurcations, generalized multistability, and chaos could be obtained in a CO_2 laser by a loss modulation depth in the range of 1% to 5% with frequencies around the relaxation oscillation frequency Ω. This corresponds to a resonant destabilization of the system as eqs. (13) and (14) explain. In related CO_2 experiments with a pump modulation depth between 1% and 20% we were not able to observe instabilities similar to those obtained with modulated cavity losses. From eq. (15) it is clear that we would need a pump modulation depth, l, around 100% to obtain results similar to those obtained in our experiments with loss modulation.

For such large modulation amplitudes it is difficult to offer a simple interpretation of the dynamical process. If we look at eq. (5) for the frequency of the relaxation oscillations, we see immediately that a modulation of the losses or of the pump rate also modulates the relaxation oscillation frequency. Description of the pulsations as resulting from a resonant destabilization of the nonlinear system by driving at the "resonant" frequency becomes very tenuous when the modulation depth is large. However, while lacking an intuitive description, the system remains a forced nonlinear oscillator of the type that has been studied previously [23,24].

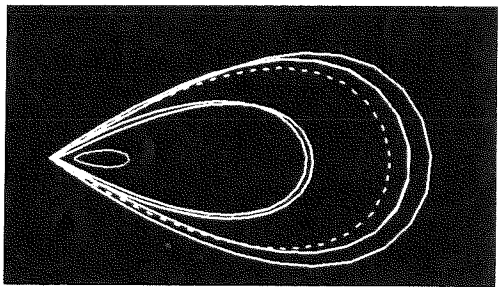

Fig. 1. Generalized multistability in a laser with modulated losses. The time dependent solutions are plotted in the phase plane of $\dot{n}$ (vertical) and n (horizontal) with parameter values $\gamma_\parallel = 10^3$, $\gamma_\perp = 10^8$, $k_0 = 1.5 \times 10^7$, $A_0 = 2$, $m = 0.2857 \times 10^{-2}$, and $\omega = 91.03663$ kHz. The origin $(\dot{n}, n) = (0, 0)$ is found at the cusp. We show three different possible solutions, for different sets of initial conditions. They are the small closed loop, the single dotted curve, and the four-loop solid curve.

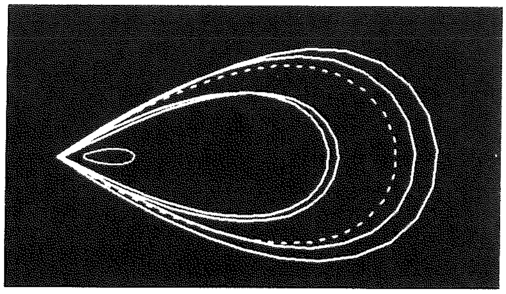

Fig. 2. Similar results obtained with pump modulation. We use a larger modulation amplitude as suggested by eq. (15), $l = 0.536$. The other parameters are the same as in fig. 1.

We also observe that both systems show generalized multistability, that is, we can find different solutions for the same parameter values, namely m or l and ω, depending upon the initial conditions. As an example we show in fig. 1 three different numerical solutions of eq. (12) (loss modulation) corresponding to the same parameter values. This effect makes it very difficult to construct an unique phase diagram and to predict experimental results quantitatively.

As an example of the similarity between loss and pump modulation, fig. 2 shows the solutions of eq. (12b) for a suitably larger pump modulation depth chosen as suggested by our formula (eq. (15) which indicates that $l/m \approx 200$) to give the same results as those found in fig. 1.

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References

[1] T. Yamada and R. Graham, Phys. Rev. Lett. 45 (1980) 1322.

[2] H. Scholz, T. Yamada, H. Brand and R. Graham, Phys. Lett. 82A (1981) 321.

[3] D.V. Ivanov, Ya.I. Khanin, I.I. Matorin and A.S. Pikovsky, Phys. Lett. 89A (1982) 229.

[4] T. Kimura and K. Otsuka, IEEE J. Quantun Electron. QE-6 (1980) 764.

[5] K. Otsuka, IEEE J. Quantum Electron. QE-15 (1979) 655.
[6] F.T. Arecchi, R. Meucci, G.P. Puccioni and J.R. Tredicce, Phys. Rev. Lett. 49 (1982) 1217.
[7] N.I. Polushkin, P.A. Khandokhin and Ya.I. Khanin, Sov. J. Quantum Electron. 13 (1983) 950.
[8] E. Brun, B. Derighetti, R. Holzner and D. Meier, Helv. Phys. Acta 56 (1983) 825.
[9] W. Klische, H.R. Telle and C.O. Weiss, Optics Lett. 9 (1984) 561.
[10] H. Statz and G.A. deMars, in: Quantum electronics, ed. C.H. Townes (Columbia U. Press, NY, 1960) p. 530.
[11] H. Statz, C. Luck, C. Shater and M. Ciftan, in: Advances in quantum electronics, ed. J.R. Singer (Columbia U. Press, NY, 1961) p. 342.
[12] C.L. Tang, J. Appl. Phys. 34 (1963) 2935.
[13] H. Haken and H. Sauermann, Z. Physik 176 (1963) 47.
[14] H. Haken, Synergetics: an introduction (Springer-Verlag, Berlin, 1983).
[15] L.A. Lugiato, P. Mandel and L.M. Narducci, Phys. Rev. A29 (1984) 1438.
[16] C.W. Gardiner, Phys. Rev. A29 (1984) 2814.
[17] G.V. Perevedentseva, P.A. Khandokhin and Ya.I. Khanin, Sov. J. Quantun Electron. 10 (1980) 71.
[18] F. Mogensen, G. Jacobsen and A. Olesen, Opt. Quantum Elec. 16 (1984) 183.
[19] R. Lang, IEEE J. Quantum Electron. QE-18 (1982) 979.
[20] F.T. Arecchi, G.L. Lippi, G.P. Puccioni and J.R. Tredicce, Optics Comm. 51 (1984) 308.
[21] J.R. Tredicce, F.T. Arecchi, G.L. Lippi and G.P. Puccioni, J. Opt. Soc. Am. B2 (1985) 173.
[22] E.N. Lorenz, J. Atmos. Sci. 20 (1963) 130.
[23] P. Holmes, Philos. Trans. Royal Soc. London A292 (1979) 419.
[24] B.A. Huberman and J.P. Crutchfield, Phys. Rev. Lett. 43 (1979) 1743.