FROM OPTICAL BISTABILITY TOWARDS OPTICAL COMPUTING

The European Joint Optical Bistability Project

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CHAPTER 6

ALTERNATIVE OPTICAL BISTABILITY SCHEMES

6.10 BISTABILITY IN QUANTUM OPTICS: CHAOS AND NOISE EFFECTS

Over the past years the Istituto Nazionale di Ottica (I.N.O.) group has explored the simplest laser systems which yield bistable and chaotic behavior, namely the so called class B lasers /1,2/ with homogeneous gainline and in single mode operation.

We have explored some configurations ruled by a number N of degrees of freedom sufficient to yield chaos ($N \geq 3$) but small enough to allow an easy comparison with the corresponding theoretical model.

Precisely, we have shown evidence of multistability and chaotic instabilities in the following configurations:

i) laser with modulated losses;

ii) laser with an injected signal;

iii) laser with feedback;

iv) bidirectional ring laser.

For each of these configurations, we have investigated the routes to chaos by measuring the time signals and the power spectra, and evaluating specific indicators of deterministic chaos, as the fractal dimensions, and the Kolmogorov entropies. Some examples are shown in Figs. 1 to 3.

A new phenomenon, discovered in 1982 both in nonlinear electronic oscillators as well as in lasers, consists in the co-existence of more than one attractor for the same control parameters (generalized multistability). In such cases it was observed /14,15/ that addition of a small amount of random noise, while not modifying substantially the high frequency (short time) behavior, was adding a low frequency (long time) contribution, which in

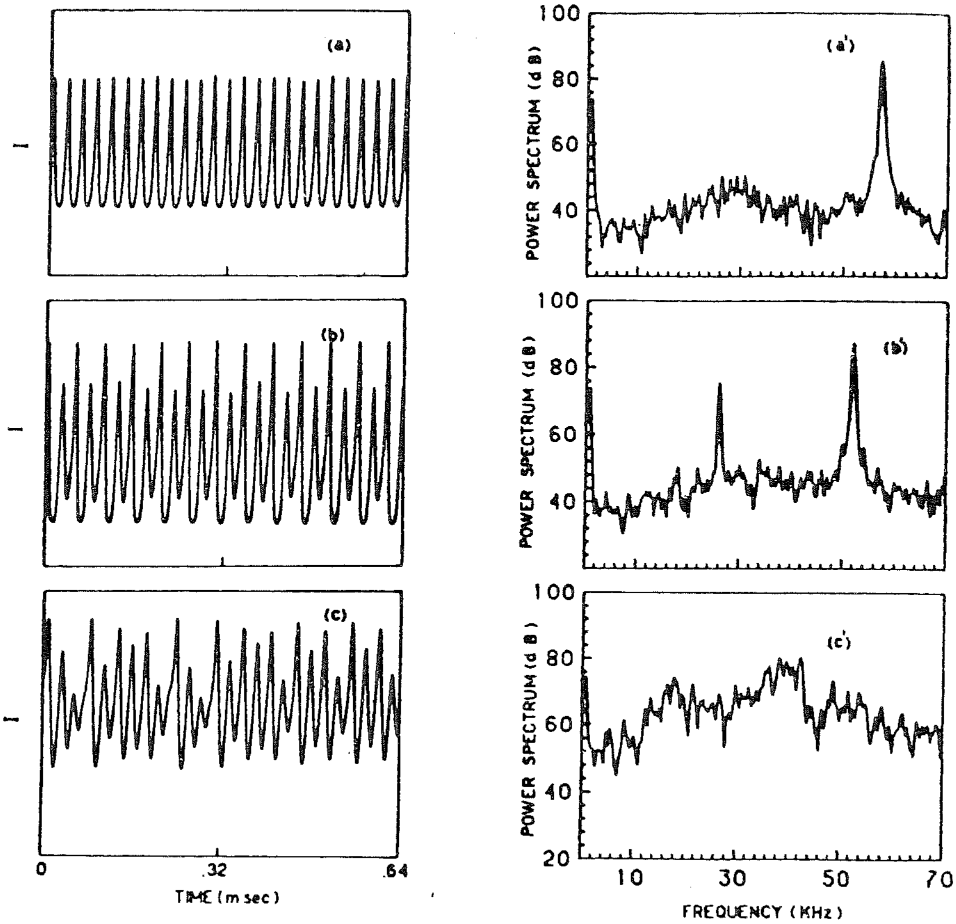

FIGURE 1
Laser with feedback: Digitizer time plots of the experimental laser intensity (left) and the corresponding power spectra (right) for increasing values of the control parameter B. a) corresponds to the onset of the first Hopf bifurcation, at a frequency f=57.3 KHz, B=0.364; b) shows the appearence of a subharmonic bifurcation f/2 where the fundamental frequency is f=52.0 KHz, B=0.378 and c) shows the appearence of chaos, B=0.383.

many instances appeared as a 1/f spectrum. Since the 1/f spectra appear in many areas of science and technology, we have carried a specific theoretical investigation to understand this phenomenon /16,17,13/. Here we show the main results of Ref. 13. The empirical conditions under which the 1/f spectra appear are:

i) coexistence of at least two attractors (so called "generalized mulstistability";

ii) presence of noise;

iii) some "strangeness" in the attractors.

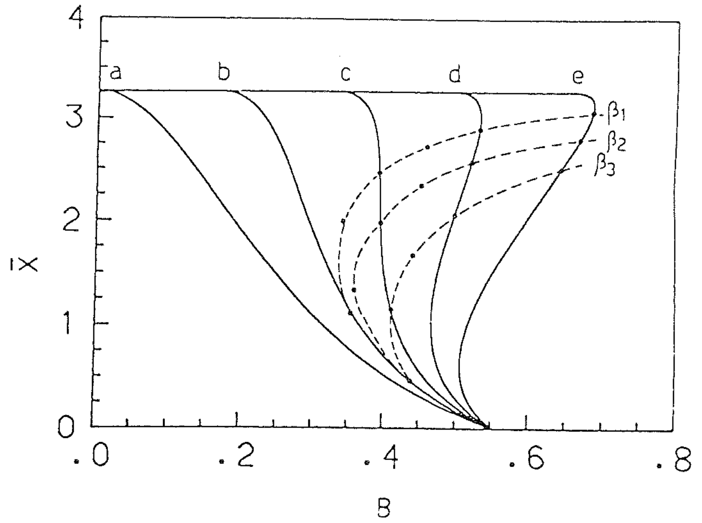

FIGURE 2

Laser with feedback: Plots of normalized stationary intensity x vs bias-voltage B for different values of the feedback coupling constant f. Curves a) to e) refer to f =0.00, 0.052, 0.102, 0.152 and 0.202, respectively. Dashed lines correspond to the loci of the first Hopf bifurcations for three different values of the damping constant β of the feedback loop.

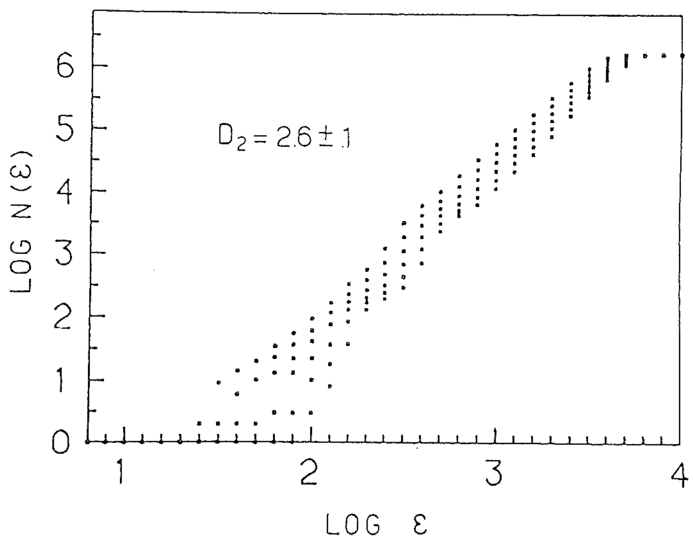

FIGURE 3

Laser with feedback: Plots of log $N_n(\epsilon)$ vs log ϵ for different values of embedding dimension n(n=10-15). Square dots come from experiment. Theoretical plots coincide with the experimental ones within the dot sizes.

As a matter of fact this third condition is rather vague. What in fact we observe in many cases is a fractal structure of the basin boundary between the two attractors of a bistable system. This means the following. As the phase point wanders within one basin of attraction, if we draw a sphere around the point defining its distance from the other basin of attraction, the radii of these spheres are distributed with all scale lengths, according to the self similar structure of the fractal boundary.

Based on the above considerations, we have built an elementary cellular automaton which models the motion of the phase point within a fractal basin boundary under the presence of random noise. We model the boundary region of two basins of attraction A and B as two adjacent one-dimensional lattices of sites. Suppose we start from site i. At each discrete time step, if i belongs to A ($i = i_A$) it moves one step forward on the same lattice and if it belongs to B it goes one step backward. In the absence of noise, once the motion has started on one basin, it will remain on it forever. In the presence of noise, at each time step there is a finite probability of a "cross" jump at the same lattice site, from stripe A to B: $i_A \to i_B$.

We call L the maximum size of the boundary region and $\ell_i \leq L$ any of the possible sizes of the fractal set. At each time step, the probabilities of permanence and jump are respectively

$$P_{AA} = P_{BB} = \ell_i / L$$

$$P_{AB} = P_{BA} = 1 - \ell_i / L \quad .$$

To build a self-similar structure we allow ℓ_{i_K} to scale as $\ell_{i_K} = (1/2)^{V(i_K)}$ where $V(i_K)$ is a natural number sorted randomly for each site i_K ($i = -\infty$ to ∞, $k = A,B$). To deal with a real numerical experiment we consider finite sequences of N sites (e.g. $N = 10^3$) and we truncate the fractality by imposing $0 \leq V(i_K) < F$. Here, F is a finite integer denoting the maximum partitioning $(1/2)^{F-1}$, that is, the ultimate resolution of the measuring device in appreciating the fractal structure of our set. With all this in mind, for each evolution we extract a double sequence of N integers randomly distributed between 0 and F-1, and denote each site i by the corresponding number $V(i_k)$. This means that we have attributed to each site an "area of respect", that is, a specific separation ℓ_{i_k} from the other attractor. We start, e.g. on the basin A from $i_A = N/2$.

At each step, to account for noise, we generate a random number y uniformly distributed between 0 and 1. If $y \leq (1/2)^{V(i_A)}$, then at the next time the point goes to $i_A + 1$ on attractor A; if $y > (1/2)^{V(i_A)}$, then the point jumps instanteneously to site i_B and at the next time it goes to $i_B - 1$ on attractor B.

By measuring the position coordinate, taking the Fourier transform and squaring it, we can build the power spectra, that is, the transforms of the position correlation functions.

In Fig. 4 we show two power spectra for F=4, and 14 respectively. In fact, we have measured spectra for all fractalities between F=4 and F=14, but we report two samples over slightly more than three frequency decades. The sequence shows that, as the fractality increases, the slope of the log-log plot goes from about 2 (single Lorentzian) to about 1 (1/f spectrum). This appears better in Fig. 5, where the slope α of the $f^{-\alpha}$ spectral law is plotted versus the fractality F. The Lorentzian ($\alpha = 2$) of the random telegraph model is easily recovered for F=0, thus showing that noise induced jumps across a regular line boundary fulfill the intuitive expectation of a single decay rate.

The results of this paper may strongly affect our current understanding of Optical Bistability (OB) phenomena. OB is described in terms of two fixed point attractors, which however are the result of a collective dynamics implying many degrees of freedom. There are no exhaustive analyses of the structure of the basin boundary, thus possible fractal structures may appear if the dynamics is evaluated in detail. On the other hand, in order to reduce the signal power necessary to drive the OB device from one state to the other, the system is usually set very near to the boundary. Thus, unavoidable random noise might induce low frequency spectra of the type above described.

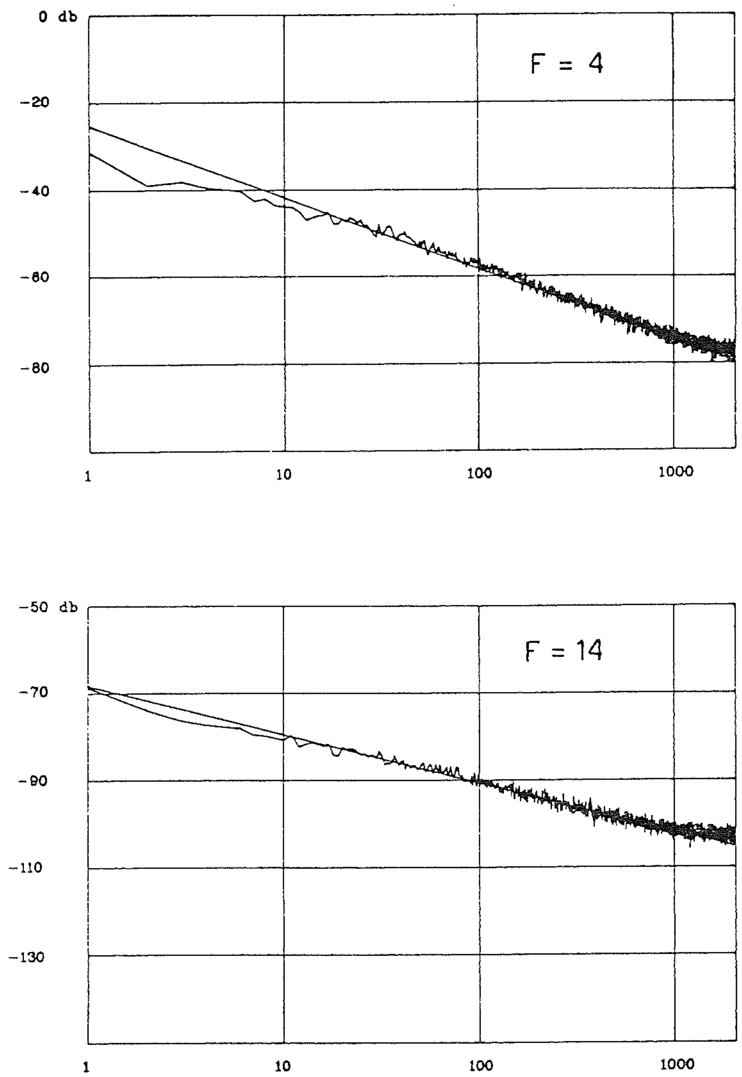

FIGURE 4

Power spectra (vertical) versus frequency (horizontal) in log-log scale. Wavy lines: measured spectra, straight lines: best fits, whose slopes α are reported in the next figure. The two samples shown refer to F = 4, and 14, respectively.

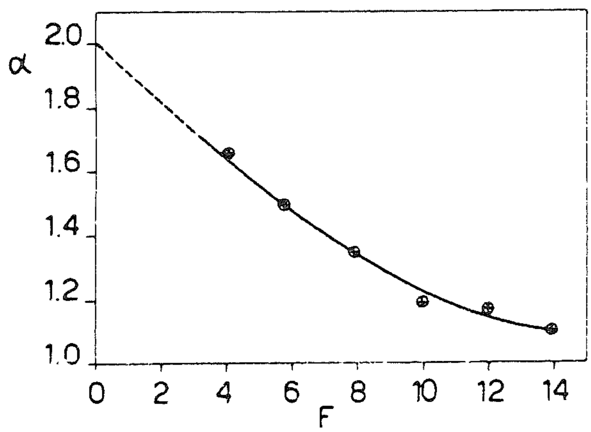

FIGURE 5

Exponents α of the power law $f^{-\alpha}$ versus fractality F.

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/2/ Arecchi, F.T., "Instabilities and Chaos in Single Mode Homogeneous Line Lasers" in: Instabilities and Chaos in Quantum Optics (F.T. Arecchi & R.G. Harrison eds.) Springer 1986 and references therein. In particular here below we report a selected list of 11 papers (from Ref. 3 to 13) done under the partial support of the European Economic Community, Committee for Scientific and Technological Research (CODEST), EJOB Project (Optical Bistability):

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