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Evidence that transverse effects cause an instability in a single-mode CO_2 laser

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We show experimental evidence of spontaneous oscillations in a single-transverse- and single-longitudinal-mode CO_2 laser under conditions for which the plane-wave Maxwell-Bloch model predicts stable laser operation. These instabilities seem to result from transverse structure in the mode, as a model which accounts for the transverse profile of the field in the cavity predicts behavior in close agreement with our experimental results.

As early as the first years of laser studies, instabilities and spontaneous pulsations were observed under many different conditions, for example, in cw pumped ruby lasers,[1] and in multimode systems.[2] However, these early observations came in experiments which were not designed to study laser instabilities, and little more than reports of the existence of such a phenomenon was given. More recently, since the growth of the study of nonlinear dynamical systems and the demonstration of analogies between the dynamics of lasers and convecting fluids,[3] experiments have been designed to study the dynamical behavior of lasers in detail.[4,5] Two major experimental goals have been the achievement of a variety of different behaviors and the realization of a system described by the Lorenz-Haken[3] theory.

We report a different type of instability; spontaneous intensity oscillations are observed in a single-mode CO_2 laser in the absence of any kind of external modulation. The laser parameters are highly stabilized, under conditions which are normally assumed to be described by one of the most common models used to describe laser dynamics—the field-population rate equations. However, this model (and all other plane-wave laser models) fails to predict the observed results. Nevertheless, this laser is one of the simplest used so far in studies of laser instabilities, and the existence of spontaneous pulsations suggests the need for revisions of some of the basic assumptions of these models.

The experimental system is described in detail in Ref. 6. It consists of a CO_2 laser pumped by means of a stabilized dc current discharge. It is mounted on a shock-absorbing optical table and enclosed in a Plexiglas box to prevent external mechanical and thermal disturbances. A single CO_2 laser line is selected by a grating in the optical cavity, and single-mode operation is verified by observing the single beat note in the mixture of the output signal and the output of a stabilized single-mode CO_2 laser operating on the same transition. Transversal single-mode operation is achieved by inserting in the cavity appropriate apertures, while longitudinal single-mode operation is assured by the ratio between the cavity-mode spacing and the transition linewidth (~ 1) and the use of low values in the laser pump (≤ 2). The ratio of the homogeneous to inhomogeneous linewidths is about 2.4:1, which provides rather satisfactorily a homogeneously broadened laser system.

An operating point in the parameter space for the laser was selected by setting the cavity losses at the desired level by applying the appropriate dc voltage to an intracavity electro-optic crystal, by choosing the value of the dc excitation current, and by selecting the desired cavity length by applying a dc voltage to the piezoelectric crystal which controlled a cavity mirror. After the laser relaxed to its steady state (in fractions of a second) the single-mode operation was checked by heterodyning[6] and then the output intensity was observed and recorded directly.

The experimental results are summarized in Fig. 1. Just above threshold the laser displayed a stable cw output intensity (solid line) which increased with increasing excitation current (proportional to the pump parameter). At some pump value, which can be estimated to be of order 1.3 (normalized to the value at threshold), the laser intensity oscillated spontaneously, and continued to do so for

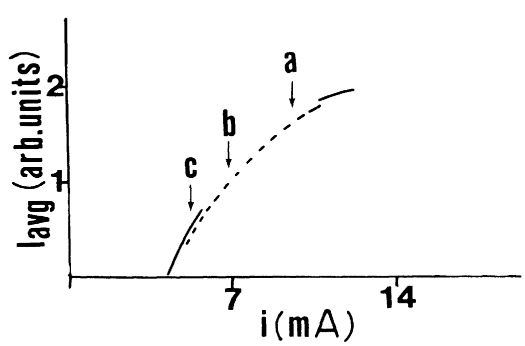

FIG. 1. Average output intensity as a function of the excitation current. The continuous line represents stable laser output, while the broken line corresponds to the region of intensity oscillations.

increased pumping until a cw output was restored for pump levels above some higher value, typically ~1.8.

This instability occurs for very low pump values, unlike those reported in other experiments with single-mode homogeneously broadened lasers for which the required pump values are fairly high.[7] Moreover, the fact that the polarization decay rate for a CO_2 laser exceeds the cavity decay rate (the so-called "good cavity limit") means that the pulsations exist in a region where the Lorenz-Haken model predicts only a stable steady state. The small amount of inhomogeneous broadening cannot be used to explain the observed effects.[8]

Different values of the cavity losses (changed by the electro-optic modulator) did not lead to special behavior of the output intensity of the laser. The only influence that this parameter had on the dynamics was to change the effective pump value. This sometimes prevented us from reaching the upper stable region or even the unstable region, when the losses were high.

For different cavity lengths, and consequently different detuning between optical transition frequency and cavity frequency, we did not observe any difference in the laser behavior. The instability appears for the same pump values and the output intensity oscillates at the same frequency. This suggests that this instability has an origin as an amplitude instability rather than a phase one.

The laser instability appears with a subcritical Hopf bifurcation at the lower pump threshold; it begins immediately with spiking, as shown in Fig. 2(c), with peak intensities about 30 times higher than the cw laser output.[9] There is a narrow region below the instability threshold of coexistence between very strong spiking solution and a stable steady-state solution. The pulsing changes with a slight increase in the pump into the pattern shown in Fig. 2(b), where an almost sinusoidal oscillation at a frequency very close to the preceding one is observed (~3.5 kHz). Here the amplitude of the oscillation is about 60% of the cw laser intensity. The pulsation pattern is very sensitive to changing pump and the instability is reduced in amplitude to about 10% of the cw output level at point *a* in Fig. 1 [time dependence is shown in Fig. 2(a)]. The pulsation frequency also is very sensitive (increasing by more than an order of magnitude with a slight increase in excitation), so fast that the transition appears, experimentally, almost abrupt. Beyond this point the laser output oscillates at a frequency which decreases with increasing excitation and the instability disappears with an inverse supercritical Hopf bifurcation at an oscillation frequency of about 30 kHz.

Particular care has been paid to eliminate various destabilizing phenomena such as jumps between longitudinal modes which have been observed to cause relaxation oscillations,[10] noise in the excitation current which might induce Q switching[11] (the importance of dealing with a cw pumped laser is evident here), jumps in the lasing optical transition among the various molecular lines of the CO_2, and fluctuations in the laser temperature. External perturbations of mechanical, thermal, and electrical origin have been kept at the lowest possible levels.[12] This great care in performing the experiment resulted in a signal-to-noise ratio as high as 70 dB.[6]

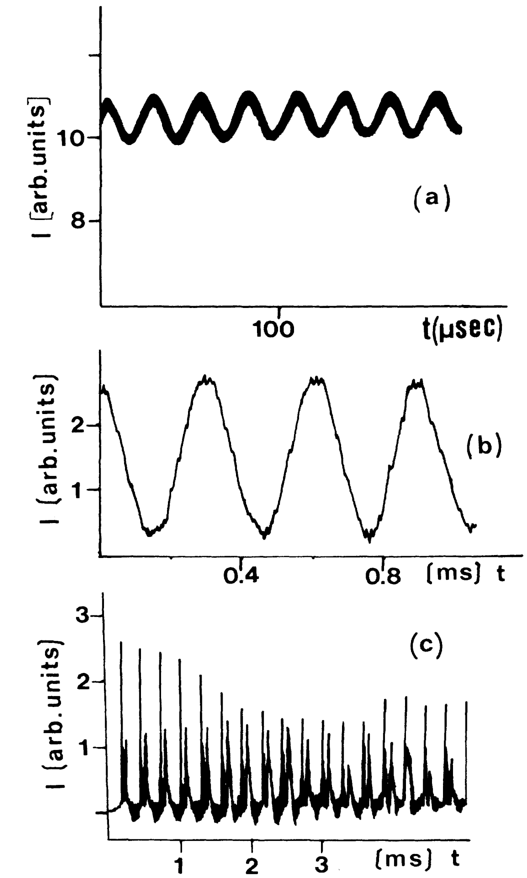

FIG. 2. Laser output intensity as a function of time at points in Fig. 1 marked (a), (b), and (c). (a) Oscillation amplitude about 10% of the cw level, frequency about 40 kHz. (b) Oscillation amplitude about 60% of the cw level, frequency about 3.5 kHz. (c) Spiking solution, peak intensity about 30 times the cw level, frequency about 3 kHz.

With these precautions there is no doubt that a disagreement between theoretical predictions and our results has to be attributed to an insufficient description of the real system by the usual model rather than in experimental problems.

The usual Maxwell-Bloch equations, in the plane-wave approximation,[13] with adiabatic elimination of the polarization lead to the well-known rate equations

$$(\partial E/\partial t) = -K[(1+i\delta\Omega)E - 2\alpha L\,|\ln\mathcal{R}|^{-1}EN/(1+i\Delta)]\ ,$$

$$(\partial N/\partial t) = -\gamma_{\parallel}[N-1+|E|^2N/(1+\Delta^2)]\ ,$$

where E and N are the field amplitude and population inversion, respectively, with decay rates K and $\gamma_{\parallel}$; αL is the gain; Δ is the detuning between the frequency of operation of the laser and the atomic resonance frequency normalized to the gain bandwidth; $\delta\Omega$ is proportional to the detuning of the laser frequency from the cavity frequency; and $\mathcal{R}$ is the reflectivity of the output mirror (assuming that the losses for the field are due to the output mirror only). The stationary solution for the intensity is of the form

$$1+\Delta^2+|E_{st}|^2=2\alpha L|\ln\mathcal{R}|^{-1} . \quad (1)$$

This model predicts stable cw operation of a CO_2 laser for all the acceptable parameter values and the transient behavior includes damped relaxation oscillations.[14]

As an alternative, we consider a transverse profile in the field as it has been shown previously theoretically,[15] and experimentally,[16] that it can play a major role in determining the dynamics of passive systems. Although lasers are active systems and present different characteristics, it is possible to also improve laser models by including transverse effects.[17,18]

Assuming the field to have a Gaussian transverse profile of the form

$$E(r,z)=I(z)^{-1/2}\exp\{i\theta(z)-r^2[Q_R(z)+iQ_I(z)]\} ,$$

we have in effect enlarged the phase space of the problem by adding the beam waist and radius of curvature of the wave as dynamical variables. The steady-state equations become[17,18]

$$(1/I)(\partial I/\partial z)=2Q_I+2\alpha L(1+\Delta^2+I)^{-1} , \quad (2a)$$

$$(\partial\theta/\partial z)=-Q_R+\delta\Omega-\alpha L\Delta(1+\Delta^2+I)^{-1} , \quad (2b)$$

$$(1/Q_R)(\partial Q_R/\partial z)=2Q_I-2\alpha L(1+\Delta^2+I)^{-1}\times[I/(1+\Delta^2+I)-\beta/Q_R] , \quad (2c)$$

$$(\partial Q_I/\partial z)=(Q_I^2-Q_R^2)+2\alpha L\Delta Q_R(1+\Delta^2+I)^{-1}\times[I/(1+\Delta^2+I)-\beta/Q_R] , \quad (2d)$$

with the boundary condition

$$E(0)=E(L)\mathcal{R}^{-1/2}\exp(-i2r^2R_0^{-1}) ,$$

where z is the position along the axis, β is the curvature of the pump profile $\chi(r)=1-\beta r^2$, and R_0 is the radius of curvature of the curved mirror (assuming that all other mirrors are plane[19] and that the length of the medium is equal to the total cavity length[20]). The physical meaning of the variables is clear: I is the intensity on the axis ($r=0$), and Q_R and Q_I are inversely proportional to the beam width squared and the radius of curvature of the field, respectively.[21]

Assuming that the intensity on the axis and the beam width are constant throughout the medium we can integrate the equation which results from taking the difference between Figs. 2(a) and 2(c) and obtain

$$1+\Delta^2+I=2\alpha L|\ln\mathcal{R}|^{-1}[1+I/(1+\Delta^2+I)-\beta/Q_R] . \quad (3)$$

This equation is equivalent to Eq. (1) if we consider the losses to be intensity-dependent, $K(I)$, of the form

$$K(I)=(c\gamma_\perp/2L)|\ln\mathcal{R}|[1+I/(1+\Delta^2+I)-\beta/Q_R]^{-1} . \quad (4)$$

This suggested[22] that the dynamics of a transverse field might be modeled with the usual rate equations modified by replacing the constant cavity-damping rate K by an intensity-dependent $K(I)$.

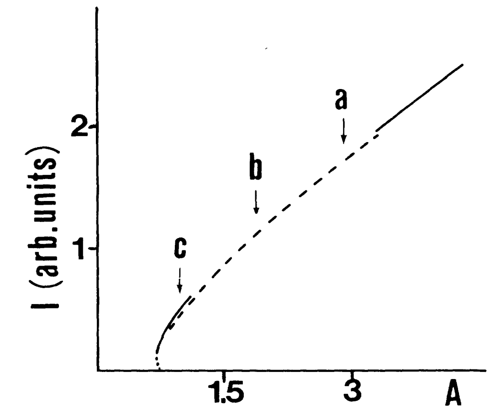

FIG. 3. Steady-state output intensity vs pump as predicted by the improved model. The solid line represents stable output, the broken one, intensity oscillations (cf. Fig. 1).

By using a four-level model valid for CO_2 lasers[23] with $K(I)$ given by Eq. (4), the steady-state intensity and its stability (as obtained in Ref. 22) are shown in Fig. 3. The origin of the instability might be viewed heuristically as a competition between the radial field profile that the pump tries to create and the one sustained by the cavity. While the Gaussian beam supported by the geometry of the cavi-

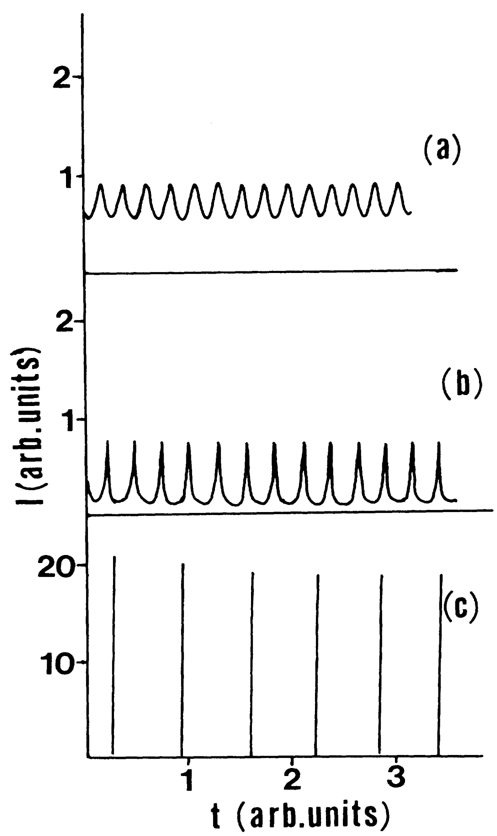

FIG. 4. Laser intensity oscillations as predicted by the model in points (a), (b), and (c) of Fig. 3. Compare to Fig. 2.

ty has a fixed beam width (because Q_R is fixed), the medium can be initially inverted in a large volume (β small) which then tries to expand the beam. In fact, the region of instability becomes wider as $\beta \rightarrow 0$, while it narrows down when β is increased.[24]

Typical numerical solutions of the dynamical equations are shown in Fig. 4. The similarity between Figs. 1 and 3 is striking, not only because the low-threshold instability is reproduced in a manner no other model has given, but also because it shows the existence of a stable region for higher pump values. Figure 4, very similar to Fig. 2, shows the kind of temporal output intensity signal of the laser as predicted by this model. The agreement between these theoretical predictions and the experimental results is striking and suggests that transverse effects have a primary role in the instability in this case.

In conclusion, we have shown that a careful experiment reveals the existence of spontaneous low-threshold oscillations in the good cavity limit for a single mode CO_2 laser with no external modulation. The agreement between the experiment and the revised theory suggests that transverse effects may cause single-mode instabilities in many other lasers.

Note added in proof. Recent reports of optogalvanic chaos in a CO_2 laser[25] have led us to check again the stability of our discharge during the observed optical instabilities. While we can induce instabilities in the power-supply plasma-tube circuit at 4 kHz for very low currents, we believe such instabilities were not present during the experimental measurements reported here. The purely optical pulsations do however require cooling by the water bath surrounding the laser tube below ambient room temperature, which suggests that we are achieving a more dramatic variation in the transverse gain profile.

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