SWEPT DYNAMICS OF A CO_2 LASER NEAR THRESHOLD: TWO- VERSUS FOUR-LEVEL MODEL

F.T. ARECCHI [1], W. GADOMSKI [2], R. MEUCCI and J.A. ROVERSI [3]
Istituto Nazionale di Ottica, 50125 Firenze, Italy

Received 2 September 1987

The swept behavior of a single mode homogeneously broadened CO_2 laser has been characterized by applying a triangular modulation to an electro optic modulator inside the optical cavity. We observe a dynamical hysteresis effect depending on the modulation depth and on the sweep rate. Our experimental results can be explained in the framework of a four level molecular model which takes into account the coupling between the two resonant levels and the rotational manifold.

1. Introduction

The dynamical behavior of a single mode homogeneously broadened CO_2 laser is usually described by two coupled rate equations, for the laser intensity and the population inversion [1]. This model comes from a semiclassical laser theory considering two-level atoms. This framework provides fairly good results to explain a large class of phenomena such as the chaotic behavior when an external sinusoidal modulation is introduced [2]. However, in the above mentioned case, we have found some quantitative disagreement between theoretical and experimental values of the modulation depth for the same bifurcation. To get rid of such a discrepancy within the two-level model one has to introduce a damping rate of the relaxation oscillations larger than that experimentally observed. Such a fast relaxation is typical of a four-level system, where the two resonant levels are strongly coupled with the rotational manifold. Furthermore, some dynamical aspects predicted by a two-level laser model, such as dynamical hysteresis near threshold [3], have not been observed in our experiment on a CO_2 laser.

In this paper we report a comparison between experimental results and the theoretical ones for a two- and four-level laser model.

[1] Also Dept. of Physics, University of Firenze, Italy.
[2] Permanent address: University of Warsaw, Warsaw, Poland.
[3] Permanent address: Universidade Estadual de Campinas, Campinas, S.P., Brasil.

2. Experimental set-up and results

Our experimental set-up consists of a CO_2 laser tube inside an optical cavity defined by a grating in order to select the P(20) line and a total reflecting mirror mounted on a Piezo Translator to adjust the frequency of the cavity mode to the center of the molecular line. The cavity length is 2 meters long. The radiation is coupled out by means of a low reflectivity ZnSe beam splitter and detected by a HgCdTe detector.

A triangular high voltage sweep is applied to an electro-optic modulator (EOM) inside the cavity. The voltage applied to the EOM drives the laser from its maximal intensity value corresponding to $V=0$, to the zero-intensity, which means below threshold, when $V > V_{th}$; V_{th} corresponds to the laser threshold.

In fig. 1 we report the time evolution of the laser output intensity and the corresponding signal applied to the EOM. For each temporal behavior the associated x–y representation (laser intensity versus modulation voltage) is shown. In fig. 1a the sweep rate occurs at a frequency of 11 Hz. At this low sweep rate the laser intensity clearly shows a purely symmetric behaviour which corresponds to a x–y plot without hysteresis, fig. 1b. On increasing the sweep

0 030-4018/88/$03.50 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

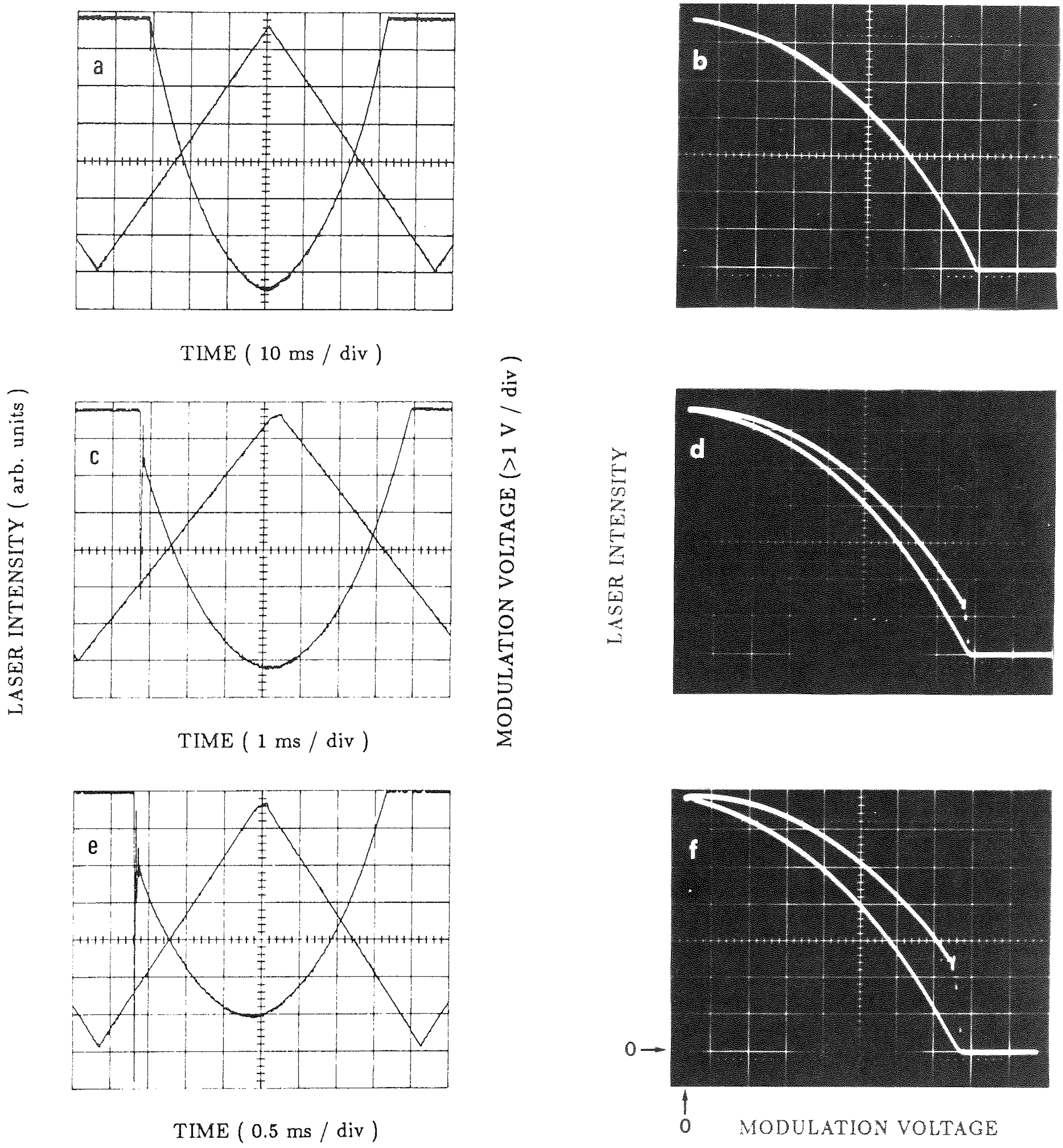

Fig. 1. Time evolution of the laser output intensity and triangular modulation applied to the EOM (left side). Corresponding x–y plots (laser intensity versus modulation voltage) are reported in the right hand side. (a), (c) and (e) refer to a modulation frequency $f=11$ Hz, 96 Hz and 232 Hz respectively. The laser threshold is about 700 V (0.52 in normalized units).

rate new features appear, fig. 1c. First, we observe the growth of the initial spike associated with the switch-on of the laser and a very fast damping with only few relaxation oscillations. Second, we observe the appearance of "negative" hysteresis as clearly shown in the x–y representations, figs. 1d and 1f.

3. The two- and four-level models

First let us refer to the numerical solution of a two-level system. Such a system is described by two rate equations for the intensity $x(t)$ and population inversion $y(t)$:

$$\dot{x} = -K_0 x(1+f(u)-y) , \quad (1a)$$

$$\dot{y} = -\gamma_{\parallel}(y+xy-y_0) , \quad (1b)$$

where $K_0=(c/L)(T+D_f)$ is the nonmodulated cavity parameter, L is cavity length, T and D_f are the effective transmission and diffractive loss of the cavity respectively, $\gamma_{\parallel}$ is the population decay rate. The intensity $x(t)$ is normalized to the saturation intensity $I_s=\gamma_{\parallel}/2G$, with G being the field–matter coupling constant. The population inversion $y(t)$ is normalized to the threshold inversion K_0/G. $f(u)=\alpha\sin^2(u)$ is the modulation function where $\alpha=(1-T)/(T+D_f)$ and u is the voltage applied to EOM normalized to $\pi/V_{\lambda/2}$, $V_{\lambda/2}=4240$ V is the $\lambda/2$ modulator voltage and y_0 is the normalized pump parameter.

For our numerical calculation we have used $K_0=3.0\times10^7\ \mathrm{s}^{-1}$, $\gamma_{\parallel}=0.5\times10^4\ \mathrm{s}^{-1}$, $y_0=2.17$, $T=0.05$ and $D_f=0.16$.

In fig. 2a we refer to a sweep rate close to the experimental value of fig. 1e. It shows the same kind of hysteresis but the damping rate does not match the experimental observations. In order to reach a much faster damping we need $\gamma_{\parallel}$ two orders of magnitude larger. The corresponding plot is given in fig. 2b for $\gamma_{\parallel}=0.5\times10^6\ \mathrm{s}^{-1}$. An initial spike much lower than the experimental one and the lack of hysteresis are evident. In order to obtain a higher spike and the same damping rate as that experimentally observed it is necessary to increase simultaneously $\gamma_{\parallel}$ and the sweep rate about two orders of magnitude. This situation is shown in the fig. 2c.

Even if we disregard the unrealistic parameter values used in the last case, we are not able to find this behavior in the experiment. The computed hysteresis of fig. 2c corresponds rather to a theoretical model described by Mandel et al. [3] on two-level laser equations when the pump is slowly varied in time.

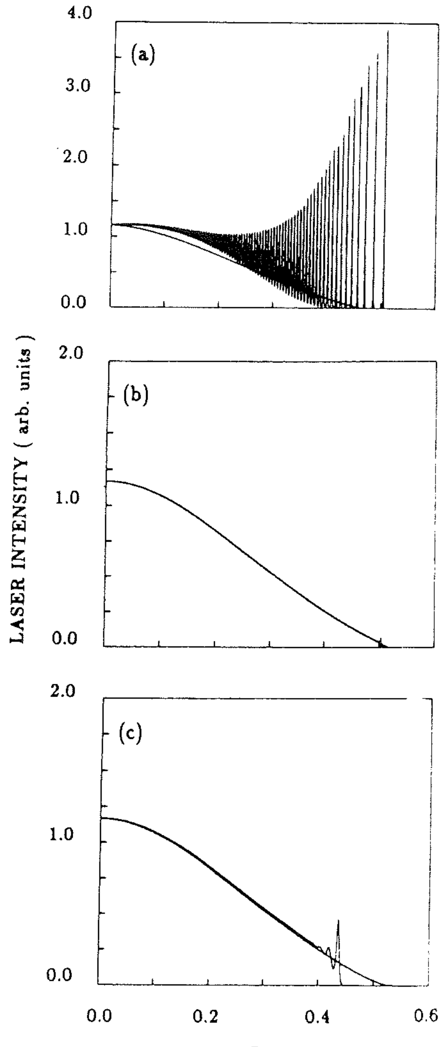

Fig. 2. Numerical results for the laser output intensity versus B (the triangular modulation voltage applied to the EOM) for a two-level laser model. The laser threshold was set at $B=0.52$. (a), (b) and (c) correspond to $\gamma_{\parallel}=0.5\times10^4\ \mathrm{s}^{-1}$ and $f=250$ Hz, $\gamma_{\parallel}=5\times10^5\ \mathrm{s}^{-1}$ and $f=250$ Hz, and $\gamma_{\parallel}=2\times10^6\ \mathrm{s}^{-1}$ and $f=10$ kHz respectively.

Let us consider now a four-level molecular model [4,5] in which the two resonant energy levels are

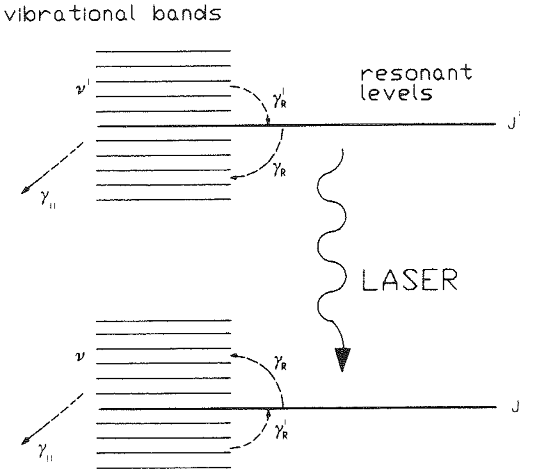

Fig. 3. Schematic diagram of energy levels for a four-level system, with each of the two resonant levels coupled to the rotational manifold of the corresponding vibrational band.

coupled to all other rotational levels of the same vibrational v band. According to the idea presented in refs. [4,5] we rewrite, with a small but essential change, the rate equations for intensity (x), population inversion of the resonant levels (y) and population inversion of the vibrational band (z) as follows:

$$\dot{x} = -K_0 x(1 + f(u) - y) , \quad (2a)$$

$$\dot{y} = -(\gamma_R + \gamma_{\parallel})y + \gamma'_R z - \mu xy + \eta\gamma_{\parallel} y_0 , \quad (2b)$$

$$\dot{z} = -(\gamma'_R + \gamma_{\parallel})z + \gamma_R y - (1-\eta)\gamma_{\parallel} y_0 , \quad (2c)$$

where $\mu = \gamma_{\parallel}(\gamma_R + \gamma'_R + \gamma_{\parallel})/(\gamma'_R + \gamma_{\parallel})$ and $\eta = (\gamma_R + \gamma'_R + \gamma_{\parallel})/(\gamma_R + \gamma_{\parallel})$, $\gamma_{\parallel}$ is the decay rate from the band (v,J) to (v',J') for $v' \neq v$, γ_R and γ'_R are the decay rates from resonant level (v,J) to the vibrational band (v,J') (for all $J' \neq J$) and for the reverse process, respectively. The intensity $x(t)$ is normalized to the saturation intensity $I_s = \mu/2G$. Both $y(t)$ and $z(t)$ are normalized to the threshold inversion K_0/G, where K_0 and G are the same as above.

In fig. 3 we present a schematic diagram of the energy levels and main decay processes that take place in the four-level model. As expected the two resonant level limit can be obtained by letting γ_R and $\gamma'_R \to 0$.

In our numerical calculations dealing with a four-level model we have used [5] $\gamma_R = 10^7$ s^{-1} and $\gamma'_R = \gamma_R/Z$, with $Z = 16$, while the other parameter values ($\gamma_{\parallel}, K_0, T, D_f, y_0$) are the same as for two-level model.

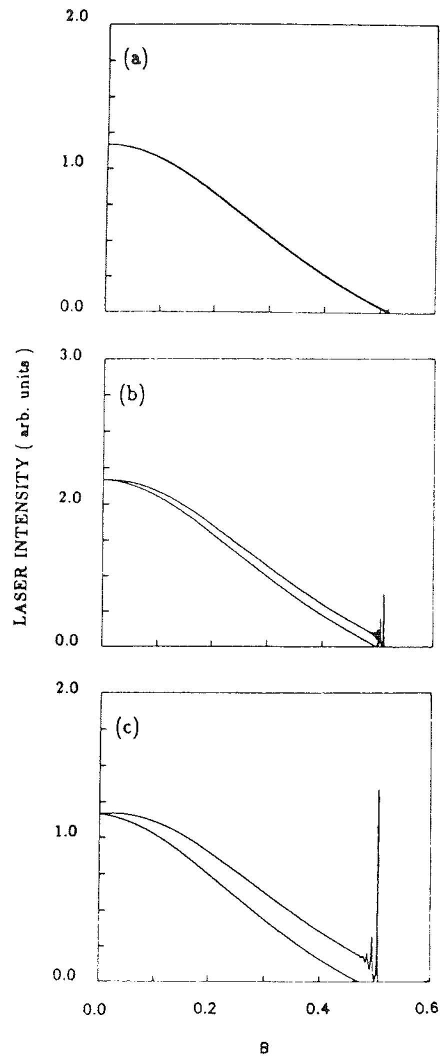

Fig. 4. Numerical results for the four-level model. (a), (b) and (c) correspond to a modulation frequency $f = 10$ Hz, 100 Hz and 250 Hz respectively, $\gamma_R = 10^7$ s^{-1} with $\gamma_{\parallel} = 0.5 \times 10^4$ s^{-1}. The other parameters assume the same values as in fig. 2.

In figs. 4a, 4b and 4c we report the x–y plots for the three increasing values of the sweep rate corresponding to the experimental conditions of fig. 1. It is evident that our experimental observations are fitted by the numerical results of the four-level model.

4. Conclusion

As a conclusion we observe that a two-level model appears inadequate to describe the swept behavior of a CO_2 laser. A more complete model taking into account the coupling between the two resonant levels and the rotational manifold leads to a good agreement between experimental and numerical results.

Acknowledgement

The authors are very grateful to A. Poggi for useful discussions. One of us (J.A.R.) has a fellowship of CNPq (National Research Council of Brazil). This work was partly supported by the European Economic Community.

W.G. was supported in part by the Polish Ministry of Science and Higher Education under project CPBP 01.06.

References

[1] F.T. Arecchi, in: Instabilities and chaos in quantum optics, eds. F.T. Arecchi and R.G. Harrison (Springer Verlag, 1987) p. 9.
[2] F.T. Arecchi, R. Meucci, G.P. Puccioni and J.R. Tredicce, Phys. Rev. Lett. 49 (1982) 1217;
G.P. Puccioni, A. Poggi, J.R. Tredicce and F.T. Arecchi, Phys. Rev. Lett. 55 (1985) 339.
[3] P. Mandel and T. Erneux, Phys. Rev. Lett. 53 (1984) 1818.
[4] J. Dupré, F. Meyer and C. Meyer, Phys. Rev. Appl. (Paris) 19 (1975) 285.
[5] E. Arimondo, F. Casagrande, L.A. Lugiato and P. Glorieux, Appl. Phys. B30 (1983) 57.