DISSIPATIVITY OF AN OPTICAL CHAOTIC SYSTEM CHARACTERIZED VIA GENERALIZED MULTISTABILITY

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We present experimental evidence of three different kinds of collisions in a laser with modulated losses and we show that the existence of them are related to the relative overlap of coexisting attractors in the parameter space. The generalized multistability is a quantitative indicator of the dissipativity of a dynamical system and we use it to determine the behavior of the dissipation as a function of the parameters.

Recently, there has been a large interest in the chaotic dynamics of lasers with a modulated parameter [1-5]. Experimental evidence of period doubling, chaotic behaviour and generalized multistability was first given in ref. [1] which includes a theoretical model in good agreement with the experiment. Generalized multistability was defined as the coexistence of many attractors for the same parameter values.

On the other hand, in nonlinear dynamics it is generally possible to have collisions between unstable orbits and chaotic attractors, leading to interior, boundary and external crises [6,7]. The former one preserves the chaotic attractor enlarging monotonically its basin, the second one destroys the attractor by sweeping off its basin of attraction, while the latter one enlarges discontinuously its basin of attraction. The presence of those different crises in the system depend on the amount of dissipation [6]. Boundary crises in the modulated laser have been recently observed [8].

Here we report experimental evidence of the three types of crises which are due to collisions among strange attractors and unstable periodic orbits created in saddle node bifurcations. These collisions are also responsible for the existence of isles that can be reached only by hard mode excitation and for periodic windows that separate different regions [9].

Furthermore, from the shape and size of the multistable region as a function of the modulation amplitude, m, we draw a connection between the amount of attractor overlap in parameter space and the volume contraction rate in phase space, that is, the dissipativety of a dynamical system, thus showing that the current definition of dissipativity can lead to a wrong expectation. We then introduce a more appropriate definition of dissipation which agrees with the experiment.

The experimental set-up has been extensively described in refs. [1-3]. It consists of a single mode CO_2 laser with an intracavity electrooptic modulator yielding time dependent cavity losses. Keeping constant excitation current and modulation frequency ($f = 100$ kHz), we sweep the modulation amplitude, m. Increasing m, we observe a complete period doubling cascade up to $f/16$, demonstrating the high stability of our system. In fig. 1a we show an $f/8$ solution (time behavior, power spectra and phase space portrait). A further increase in m pushes the system into

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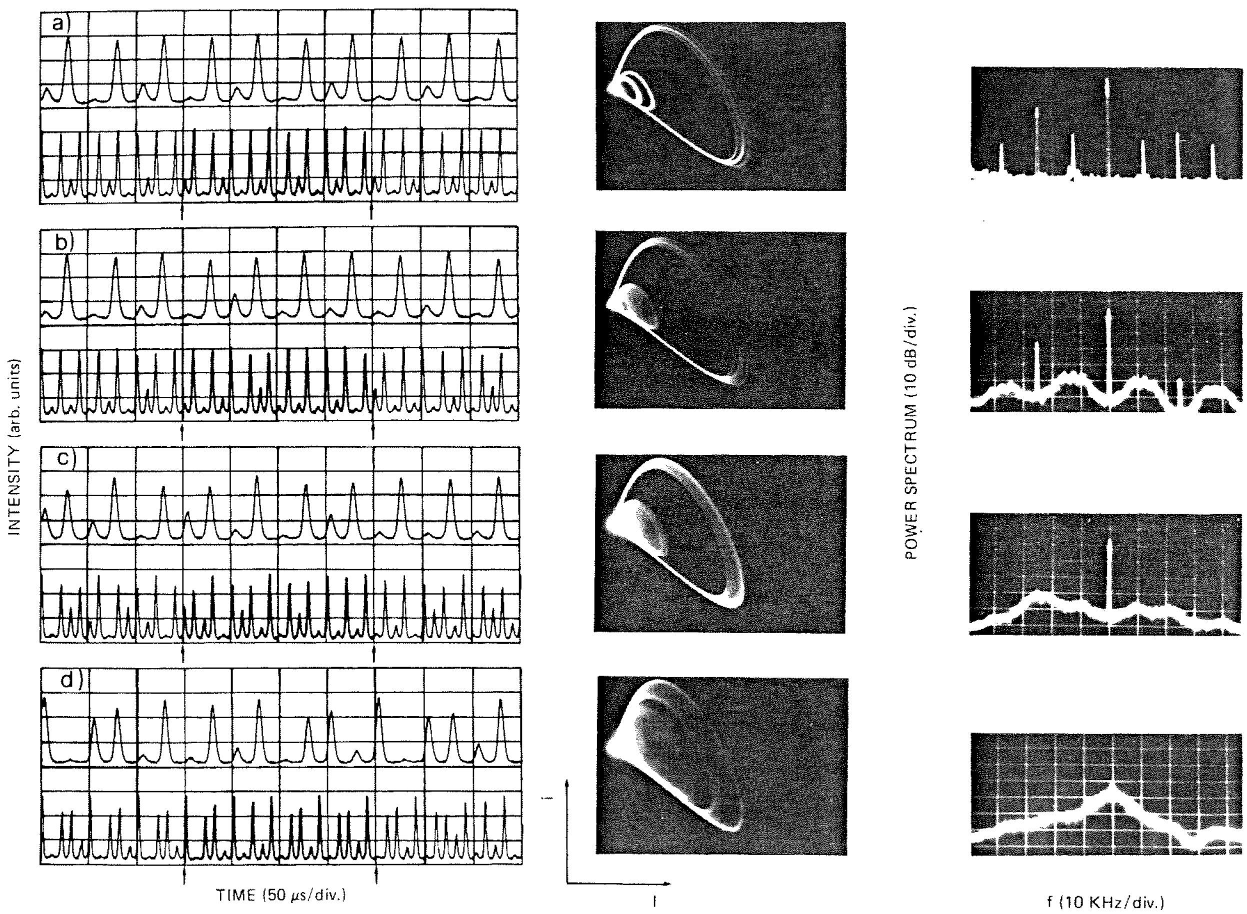

Fig. 1. From left to right: intensity (I) versus time (t), phase space portrait ($\dot{I}$ versus I) and power spectrum for (a) $m=55$ V ($f/8$ orbit), (b) $m=60$ V, (c) $m=66$ V, (d) $m=72$ V. (b), (c), and (d) are experimental evidence of the inverse cascade.

a chaotic region, where the time periodicity is lost and the power spectrum broadens around the $f/8$ component (fig. 1b). For higher values of m we observe the successive broadening of the $f/4$ and $f/2$ peaks which is evidence of an inverse Lorenz cascade [10]. This effect is reflected also on the time signal, that presents a pseudoperiodic behavior, and in the phase space portrait, where we observe bands progressively widened. The inverse Lorenz cascade is the proof of interior crises that widen the strange attractor in phase space. It is worth to notice that only one period five window was observed inside this branch, whereas the period three predicted in a normal Feigenbaum sequence was never detected. Up to the m values considered above, the bifurcation diagram does not depend on the pump rate. From here on we will distinguish two regimes corresponding to a low pump rate (discharge current 4 mA) and high pump rate (8 mA current). For comparison, the maximum output intensity in our laser corresponds to 10 mA.

At high pump rate, after the inverse cascade has taken place, the strange attractor expands abruptly as shown in the phase space portrait of fig. 2a. The laser intensity as a function of time displays bursts of peaks higher than in the previous chaotic region and with a pseudoperiodicity three or four. The concept of "pseudoperiodicity" is clarified with respect

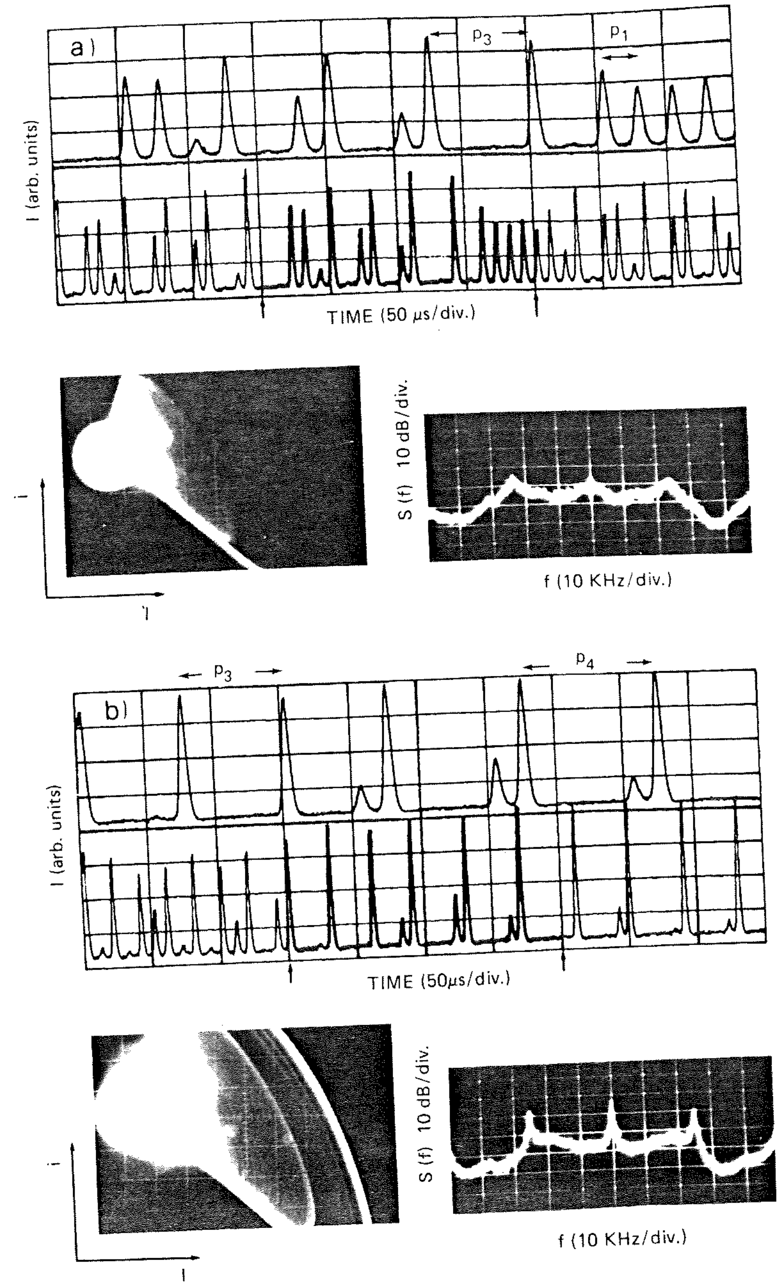

Fig. 2. Intensity versus time, phase-space portrait and power spectrum for (a) $m=80$ V and (b) $m=85$ V. Pseudoperiodicities p_n are assigned in the time signal. Phase-space portraits have the same scale of fig. 1.

to the time plots of fig. 2, where we have made some assignments of pseudoperiod p_n ($n=1$ to 4). The classification is based also on the spectral shapes, which shows the dominance of some frequency bands, and hence the average rate of occurence of their corresponding pseudoperiodicities. The appearance of pseudoperiodicities p_n is the experimental indicator of the existence of an external crisis, because these pseudoperiodicities in the strange attractor could not appear if the period n stable or unstable orbits were not present.

A further increase in m makes the occurence of those bursts more likely as evidenced in fig. 2b by a thicker line in the phase space portrait and by the large subharmonics peaks above the broad background in the power spectrum. Decreasing m, the system goes through the regimes reported above without hysteresis, hence no bistability between different chaotic regions is observed as it is expected in an external crisis.

The phenomena reported in fig. 2 are the direct consequence of a collision between the original strange attractor and an unstable $f/3$ orbit and then an $f/4$ orbit coming from completely different branches. If our interpretations is correct a period three (P3) branch should be present for lower m. We returned to the stable $f/2$ and swept the modulation frequency back and forth in a discontinuous way, stopping always at the same frequency value (note that the frequency synthesizer here used has an accuracy of 10 mHz, while the used modulation frequency is 100 kHz). We then found two different stable solutions, namely the $f/2$ and $f/3$ characterized by only one intensity peak for each three periods of modulation (fig. 3).

Once we land on the P3 branch, we increase m. The system undergoes to period doubling bifurcations ($f/6, f/12$) and then becomes chaotic. The chaotic region is very small. In fact a small increase in m leads to a jump into the $f/4$ stable orbit of the first subharmonic sequence which is the characteristic behavior of a boundary crisis.

The bifurcation diagram corresponding to the process described above is schematically shown in fig. 4a. The P3 branch is an "isle" that can be reached only by hard mode excitation (discontinuous jump in the initial conditions) at variance with the lower P2 branch. It remains confined between a saddle node bifurcation and a boundary crisis (collision of the strange attractor with the $f/3$ unstable periodic orbit originated in the same saddle node).

At the low pump rate, the relative position of the P3 branch with respect to the lower branch moves toward higher m (fig. 4b). For this low pump the chaos originated from the first bifurcation sequence does not widen as m is increased, but it is destroyed by boundary crisis and we observe a jump to the $f/3$ stable orbit. At that point we break the isle. We do not need a hard mode excitation to arrive to the P3 branch which is distinguished from a normal window inside the chaotic region of a typical logistic map, because it shows bistability with the chaotic attractor (i.e. hysteretic behavior as m goes up and down). By a further increase in m we pass through a new chaotic region, and we jump to a P4 branch which appears as an unexpected window. In fact, it is not a window, but a different branch. A similar process puts the system into a P5. A more extended sequence of regimes was presented in fig. 3 of ref. [3], where however the experimental events were displayed without any interpretation.

The succession of events described here is a series of boundary crises destroying succesive strange attractors and pushing the system into stable periodic orbits of higher periodicity. The external crises have dissappeared and have been replaced by boundary crises and we need a larger m to make possible the appearance of different periodic branches. This trend corresponds to the well known passage from the coexistence of many independent orbits in a conservative system to the succesive appearance of one attractor at a time (no multistability) on a logistic map which may be considered as a case for dissipation where the flow has been confined to a one dimensional return map.

The amount of dissipativity, d, is given by the divergence of the flow $\dot{x}=F(x)$ [11]. Using the usual rate equations for a laser [1], we easily get

$$d \equiv \operatorname{div} F = -\gamma A \,, \qquad (1)$$

that is, the product of the population decay rate (γ) times the pump parameter (A). Such a criterion shows a linear increase of dissipativity from the minimum threshold value $A=1$. This would be at variance with our experimental results summarized in figs. 4a and b, which show that dissipation decreases as our pump rate is increased.

We take a different approach to dissipativity. In a damped linear oscillator with damping rate Γ and frequency Ω, even though div $F=-\Gamma$, we would rather take a self consistent dissipativity criterion, attributing the amount of dissipation to the number Q of orbits allowed within a lifetime that is, to $d=1/Q=\Gamma/\Omega$. By applying such a concept, which re-

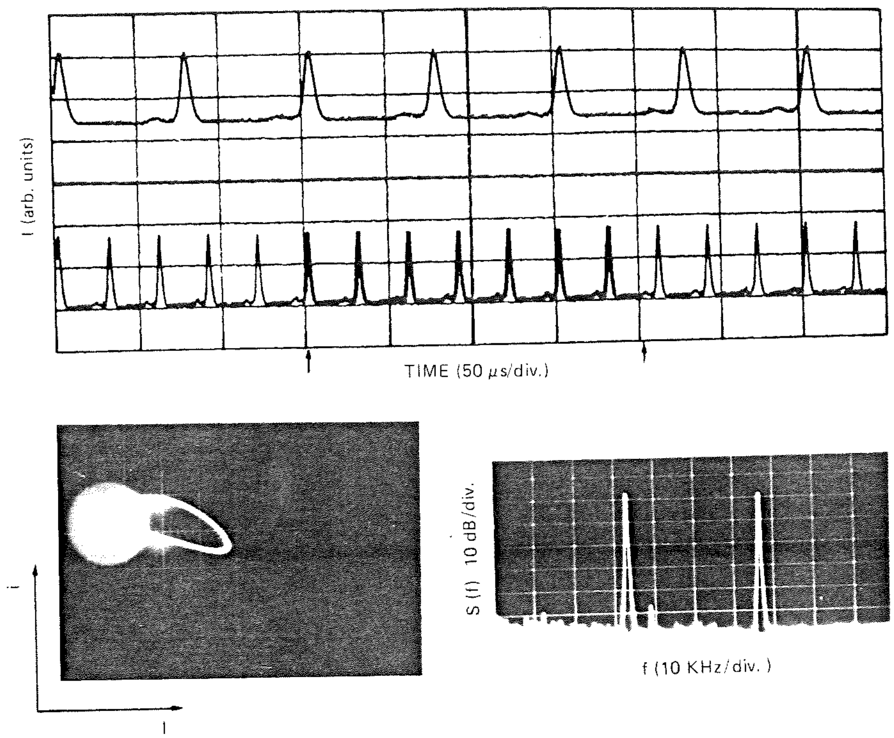

Fig. 3. Idem as fig. 2 for $m = 40$ V. The $f/3$ attractor of the P3 branch is shown.

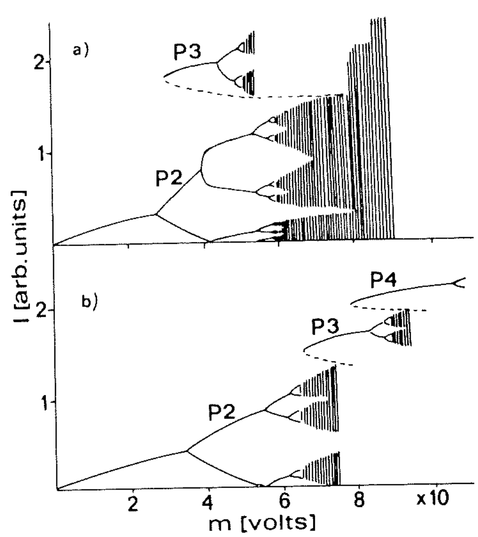

Fig. 4. Schematic bifurcation diagrams, peak intensity versus modulation amplitude, m, for (a) high and (b) low pumping rates. The dashed zones indicate the chaotic regions, and the broken lines are the assumed location of unstable orbits.

fers to a time scale intrinsic to the system, we recall that the eigenvalues of the stability analysis of our system are [12]

$$\lambda = -\gamma A \pm \mathrm{i}[k\gamma(A-1) - (\gamma A)^2]^{1/2} , \qquad (12)$$

where k is the cavity loss rate. Then dissipativity is related to

$$d = (\gamma/k)^{1/2} \, [A/(A-1)^{1/2}] , \qquad (3)$$

when the relation $k \gg \gamma$ is satisfied as it is in any Class B laser, such as the CO_2 laser that we are investigating. Eq. (3) shows that in the range $A=1$ to $A=2$, within which our two reported experiments are confirmed, the dissipativity *decreases* for increasing A rather than increasing as suggested by eq. (1) and it agrees with the experimental result.

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