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Determination of a Small Photon Number by Statistical Amplifying Transients.

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Abstract. – We introduce a «statistical microscope» which allows the accurate determination of a small number of infrared photons inside a laser cavity by optical (rather than electron) multiplication. For this purpose, we induce a loss switch in a CO_2 laser and then measure the statistical distribution of passage times through a threshold value within the linear regime. Our approach is model independent, since the measured cumulants provide both average values and associated error bars. Furthermore, our method discriminates the initial photon number from noise contributions along the amplifying path, thus it does not depend on the statistical features of the amplifying process.

Since the beginning of quantum physics many discussions have been devoted to macroscopic measuring devices which shed information on microscopic events. Some of these devices, such as the Geiger-Muller counter, may register a single event, however they are nonlinear, in the sense that the output amplitude is saturated, independently of the number of «source» events. Some others are linear, like the proportional counter, however the same amplification mechanism which yields the high sensitivity is affected by intrinsic noise and this gives rise to a large, macroscopic error [1]. Furthermore, in order to produce a ionizing event, both nonlinear and linear detectors need an initial quantum of energy above a suitable ionizing threshold.

Here we introduce a «statistical microscope», based on the combination of a quantum optical transient amplifier and a statistical observation technique, which has the following features:

i) It can detect a few initial photons in a laser cavity by linear optical amplification. The reported amplification factors are of the order of 10^8, but in principle they could be larger.

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ii) The energy of the source photons is around 0.1 eV, that is within the linewidth of a CO_2 laser. In principle, the method can be extended to the far IR, thus detecting photons of less than 0.01 eV.

iii) The linearity of the amplification process is preserved up to the saturation photon number, that is over 11 decades in our case.

iv) The method is self-calibrating, in the sense that the second moment of the observed statistics provides the amplification gain without any previous calibration, and the higher-order cumulants provide the error bars of the experimental points.

v) Even for a noise temperature equal to zero, any amplifier is affected by its intrinsic noise due to spontaneous emission. However, our method allows to discriminate the fluctuations along the amplification process from the initial «source», thus sorting out the initial photon contribution from the amplification noise.

The first observation of a statistical spread in the leading edge of a transient He-Ne laser pulse was associated with the appearance of a large peak in the variance of the transient photon number distribution [2]. This fact was explained in terms of an approximately deterministic decay out of a macroscopic unstable state, to be averaged over the statistical distribution of the initial states [2, 3]. The assumption of deterministic evolution neglected the role of fluctuations *along* the build-up with respect to the *initial* ones.

A quest for a discrimination between fluctuations on the initial condition and those along the path led to a new observation method, based on the statistics of passage times at a given threshold [4, 5]. This method provided an important difference between gas and dye lasers, since in the latter case it allowed to detect the role of pump fluctuations as «noise along the path» [6, 7]. A useful generating function algorithm has been introduced for these problems [8].

Both the He-Ne and the dye laser have in common a population decay rate large with respect to the photon decay rate (so-called class-A lasers [9]). Hence the population adiabatically follows the intensity changes, with a consequent reduction of inversion as the cavity losses are lowered. This adiabatic following forbids any overshoot in the laser intensity. On the contrary, when the population decay is slower than the photon decay (class-B lasers [9]), the initially large population storage provides a large intensity pulse by stimulated emission, and only later the population feels the slower depletion channels. This explains why, after a sudden loss reduction, class-B lasers release giant intensity pulses well above the asymptotic value, whereas class-A lasers do not.

We generate a transient dynamics in a single-mode CO_2 (class B) laser by switching an intracavity modulator (EOM) from absorption to transparency in a time shorter than the build-up time of the giant pulse. Details on the experiment are reported elsewhere [10]. As we switch the EOM voltage from V_0 to V_1, the cavity loss rate k decreases from k_0 to k_1 with a time constant of 0.6 μs as shown in fig. 1. A laser pulse builds from a low photon number n_0 (laser below threshold for loss k_0) up to a peak value, at $t_2 \simeq 3$ μs. The pulse develops over a time much shorter than the molecular decay time, thus, the population change during the pulse is mainly due to the radiative interaction. We notice from fig. 1 that the time jitter already built in the first part of the leading edge is preserved along the whole pulse. This shows that noise in the initial condition is more important than noise along the trajectory. Indeed, ref. [4] proves that a constant time variance is the indicator of no relevant noise along the trajectory. In order to remain in the linear regime we consider the evolution up to t_1 when the photon number n_1 is still smaller than the saturation value. Up to t_1, we can take the population inversion as fixed at its pre-pulse value.

As appears in fig. 1, superposition of many successive pulses gives a spread in the build-

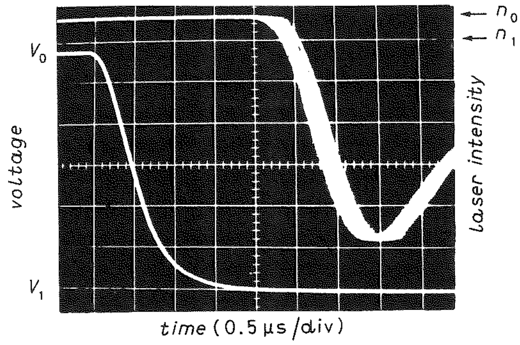

Fig. 1. – Oscilloscope plots of the EOM voltage driven from V_0 to V_1 and of a set of transient intensity traces starting from the off-state (laser intensity increasing downwards from a very low initial value n_0, practically coincident with the baseline). The exposure corresponds to 10^4 successive transients. The time jitter is uniform along the evolution. To avoid nonlinearities, the threshold n_1 for the time distribution $p(t_1)$ (see fig. 2) is adjusted below the saturation photon number, that is rather close to the off-state (at just 10% above the baseline, that is exactly at the crossing with the first horizontal screen division).

up time. By setting a threshold circuit on the photo-detector output at the photon number $n = n_1$ and using a time-to-amplitude converter, we measure the statistical distribution $p(t_1)$ of arrival times to n_1. An example is shown in fig. 2. From this we can evaluate accurately $\bar{t}_1 \equiv \langle t_1 \rangle$ and $\overline{\delta t} \equiv \langle \Delta t^2 \rangle^{1/2}$ and higher-order cumulants of $p(t_1)$, as skewness and kurtosis.

A simple argument gives the connection between the time statistics $p(t_1)$, and the statistical spread $p(n_0)$ of the initial photon numbers n_0. Since very below threshold the field is Gaussian, $p(n_0)$ is a simple exponential

$$p(n_0) = (1/\langle n_0 \rangle) \exp[-n_0/\langle n_0 \rangle]. \tag{1}$$

A noise-free linear amplification with constant gain a would be ruled by the equation

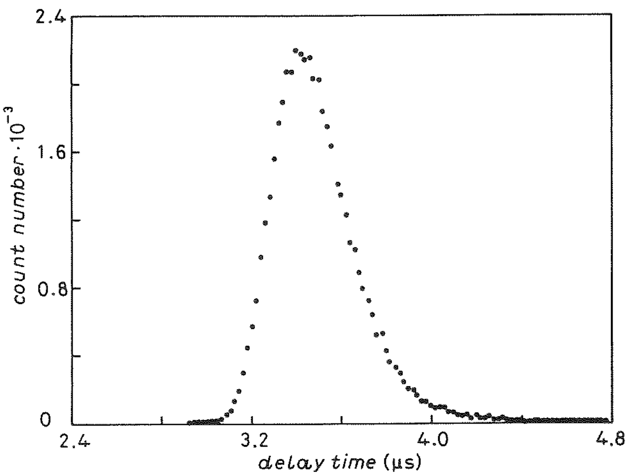

Fig. 2. – Statistical distribution $p(t_1)$ of the passage times through the threshold photon number n_1. This distribution corresponds to experiment 3 of the two successive figures.

$\dot{n} = a \cdot n$ (n = photon number). Stopping the evolution at time t_1, we would measure

$$n_1 = n_0 \exp[at_1]. \tag{2}$$

A spread in n_0 will induce a corresponding spread in t_1. Deriving $p(t_1)$ via $p(n_0)$, eqs. (1) and (2) provide a relation between the first moments $\langle n_0 \rangle$ and $\bar{t}_1$, of $p(n_0)$ and $p(t_1)$, respectively,

$$\langle n_0 \rangle = n_1 \exp[-a\bar{t}_1]. \tag{3}$$

This is the core of the «statistical microscope» method, that is an accurate calibration of n_1, a and $\bar{t}_1$ (all reliable laboratory operations as we shall see) permits to retrieve the small initial number $\langle n_0 \rangle$, otherwise unaccessible.

At the fixed threshold n_1, t_1 is spread around $\bar{t}_1$ with an r.m.s. spread $\overline{\delta t}$. Via eqs. (1) and (2) we evaluate

$$\overline{\delta t} = \frac{\sqrt{\psi'(1)}}{a}, \tag{4}$$

where $\psi'(x)$ is the trigamma function [11]. Thus a is inferred from the experiment without having to rely on a model for coupling coefficients or population inversion. Inclusion, however, of noise along the amplification process is equivalent to an additive contribution to initial noise as pointed out in ref. [12] and dealt with in detail in ref. [10]. This equivalence means that the time spread $p(t_1)$ can still be seen as the result of a noise-free amplification process as eq. (2), starting, however, from a modified initial distribution $p(n_0)$, where $\langle n_0 \rangle$ of eq. (1) is replaced by the sum N_0 of the initial photon number $\langle n_0 \rangle$ plus the contribution n_p from noise along the path ($N_0 = \langle n_0 \rangle + n_p$). Thus the statistical microscope equation becomes

$$N_0 = n_1 \exp[-a\bar{t}_1]. \tag{5}$$

In the experiment the switch takes a finite time t_s. This has to be accounted for in the last equation by a time translation. The complete expression is then [10, 13]

$$N_0 = n_1 \exp[-a(\bar{t}_1 - t_s) - \psi(1)], \tag{5'}$$

where $\psi(1) = -\gamma = -0.577\ldots$ is the digamma function [11] $\psi(x)$ at $x = 1$.

This way, we have found a method to retrieve a small photon number N_0 (of the order of 10^3 or 10^2) by measuring a large photon number $n_1 = 1.7 \cdot 10^{11}$ and the statistics of the passage times across the threshold n_1.

Two problems remain: i) how to separate in N_0 the physical quantity $\langle n_0 \rangle$ from the virtual n_p which is an extra photon number attributed to the initial state to account for noise along the amplification path; ii) how to evaluate the error bars.

Now, it is a well-known result that the equilibrium variance for a statistical process can be expressed as the ratio of the noise correlation amplitude D to the dissipation rate a. Applying this to our case we find [10]

$$N_0 = \langle n_0 \rangle + n_p = \frac{D}{|a_0|} + \frac{D}{a}. \tag{6}$$

Here, $D=\gamma_{sp}N_2\eta$ is a diffusion coefficient due to spontaneous noise (1) ($\gamma_{sp}=0.3\,s^{-1}$ is the spontaneous emission rate, N_2 the total population of the upper laser level and $\eta \simeq 10^{-6}$ an angular coupling efficiency); and a and a_0 are the net gain rates, that is the difference between gain and cavity loss rate k. k is related to the EOM voltage via a relation as $k=\bar{k}(1+f(V))$, where $\bar{k}=2.0\cdot 10^7\,s^{-1}$ and $f(V)$ is the modulator characteristic. As the voltage V is switched from V_0 (corresponding to a high loss k_0) to V_1 (low loss k_1), the gain rate is switched from a negative value a_0 to a positive one a.

In table I and fig. 3 and 4 we give data for three experimental situations, corresponding to the same initial photon number $\langle n_0\rangle = D/|a_0|$, that is (see inset in fig. 3), same discharge current and initial EOM voltage $V_0=778$ V.

TABLE I. – *Numerical values for the three experiments of fig. 3-4.*

Experiment	1	2	3
V_1 (V)	420	520	595
$\bar{t}_1$ (μs)	2.39	2.79	3.48
$\overline{\delta t}$ (μs)	0.115	0.140	0.185
$a\cdot 10^{-6}$ (s^{-1})	11.13	9.14	6.92
N_0	1533	1504	1735
n_p	194	225	327
$D\cdot 10^{-9}$ (s^{-1})	2.15	2.05	2.26

Values of V_0, V_{th} and $|a_0|$ are kept constant for the three experiments and equal to 778 V, 746 V and $1.6\cdot 10^6\,s^{-1}$, respectively.

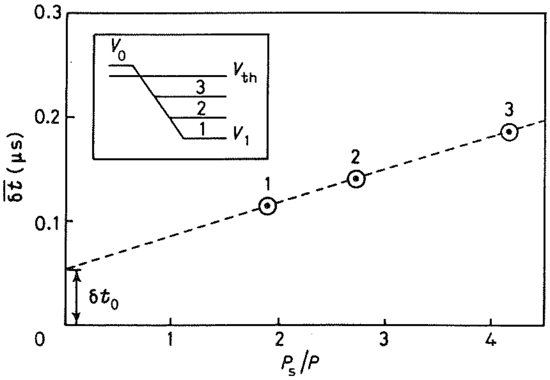

Fig. 3.

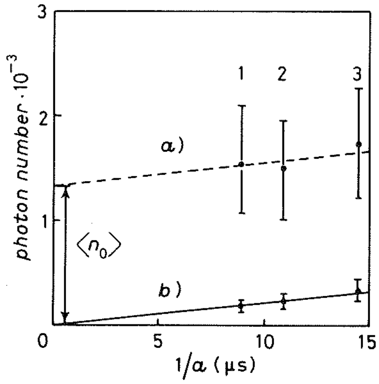

Fig. 4

Fig. 3. – Plot of the average time spread $\overline{\delta t}$ *vs.* the reciprocal of the output power P. P is evaluated for long times, after the transient has died out, the normalization P_s is the saturation power. The error bars are smaller than the black dots (the circles around the dots are just indicators). The intercept $\overline{\delta t}_0$ is the asymptotic uncertainty for large powers. The inset shows the relative EOM voltage settings in the three experiments (V_0 and V_1 initial and final voltages, respectively, V_{th} threshold voltage).

Fig. 4. – *a*) Total photon number N_0, and *b*) equivalent photon number n_p corresponding to noise along the path, *vs.* the reciprocal of the gain rate a. The vertical intercept of N_0 for $1/a=0$ is the initial photon number $\langle n_0\rangle$.

(1) In the far IR we should also account for a thermal (blackbody) contribution to D. However, for $\lambda=10\,\mu m$ this becomes important only for an initial photon number $n_0<1/20$.

The steps by which table I is built are the following. By measuring the first two cumulants of $p(t_1)$ we retrieve $\bar{t}_1$ and $\overline{\delta t}$ (and hence a). Thus by eq. (5) we evaluate N_0. By knowledge of a_0 and hence of n_0 we isolate the noise along the path $n_p = N_0 - \langle n_0 \rangle$ as given in part *b*) of fig. 4. The diffusion coefficient D is eventually calculated from eq. (6). The finite sweep time t_s provides corrections to eq. (6) given in ref. [13], which, however, in our case affect D by less than 10%. This was checked experimentally by repeating the measurement after an increase by a factor 3 in the sweep time.

In fig. 3, the average spreads $\overline{\delta t}$ are reported *vs.* the reciprocal of the output power P. Since eq. (4) yielded the simple relation $\overline{\delta t} = 1.28/a$ and in the linear regime P grows as a, $\overline{\delta t}$ *vs.* $1/P$ is a straight line, as shown by the data. For $1/P \rightarrow 0$, that is $a \rightarrow \infty$, the noise along the path disappears (see eq. (6)), hence the intercept with the vertical axis gives the residual width $\overline{\delta t_0}$ due only to the initial photon number (which is equal in all these cases).

The statistical time distributions have been collected over 300 s at a rate of 150 counts/s for a total of $M_0 = 4.5 \cdot 10^4$ counts. Each channel is affected by a relative counting error of the order of the reciprocal square root of the count number. This error will affect each moment by an amount evaluated in ref. [14]. Of course, increasing the measurement time would reduce the counting error, however it would also introduce long-time drifts in some experimental parameters. Hence the chosen counting time is a compromise between the two conflicting requirements. The counting error for the k-th moment M_k requires knowledge of the moments up to M_{2k}. With this in mind, the error bar in $\overline{\delta t} \equiv \langle M_2 - M_1^2 \rangle^{1/2}$ is evaluated in terms of the first four cumulants of $p(t_1)$. For the first of the three points in fig. 3 we have evaluated $\delta(\overline{\delta t}) = 1.55 \cdot 10^{-3}\,\mu\text{s}$ which corresponds to a relative error $\delta(\overline{\delta t})/\overline{\delta t} = 1.4 \cdot 10^{-2}$. Thus, in all the reported measurements the relative error bars are of the order of $(1 \div 2)\%$, that is smaller than the black denotation marks in fig. 3.

From eq. (5) we can evaluate the relative error bar in N_0, that is

$$\frac{\delta N_0}{N_0} = \bar{t}\,\delta a + a\,\overline{\delta t} + \frac{1}{\sqrt{n_1}}, \tag{7}$$

where $\delta a = 1.28 \cdot \delta(\overline{\delta t})/\overline{\delta t}^2$ and the last term is the standard measurement error $\delta n_1/n_1$ of the macroscopic quantity n_1. By replacing the numerical values, the second and third term contribute only as 10^{-3} and 10^{-5}, respectively, whereas the first one yields a 30% error bar. Since $\delta N_0/N_0$ varies as $1/\sqrt{M_0}$, to reduce this error to 3% we should increase the total number of counts by a factor 100. Once the relative error in N_0 has been calculated from the experimental data, use of eq. (6) with the values of a and a_0 inferred from the EOM voltages allows to establish the error in the diffusion coefficient and hence in the noise along the path n_p. It is easily proved that $\delta n_p/n_p = \delta D/D \simeq \delta N_0/N_0$. The corresponding data are collected in table I and fig. 3.

In conclusion, our «statistical microscope» permits to retrieve a small photon number via a macroscopic measurement separating the initial photon number from the fluctuations along the amplification process. Furthermore, knowledge of the first four cumulants of the passage time distribution permits a calibration of residual errors, and these ones can be reduced by increasing the count number. In the limit of a large number of counts, the procedure here outlined seems to imply a precise determination of $\langle n_0 \rangle$, if $\langle n_0 \rangle \gg 1$. In fact, in order to arrive at the crucial assumption eq. (6), we have considered [10] an amplification factor $\exp[a\bar{t}_1] \gg 1$. We recall that in our case $a\bar{t}_1 \simeq \bar{t}_1/\overline{\delta t} \simeq 20$. For $a\bar{t}_1$ less than 1, that is in the very early part of the growth process, our procedure fails. However, that region has no interest for the method here proposed, which instead is based on a large amplification factor.

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