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INSTABILITIES IN A SEMICONDUCTOR LASER WITH DELAYED OPTOELECTRONIC FEEDBACK

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A single mode CW semiconductor laser with an optoelectronic feedback gives a modulated output, with a sharp frequency peak depending on the feedback delay. A theoretical model, with few parameters assigned from the experiment, yields quantitative agreement with observations.

In molecular lasers, introduction of a feedback resulted in the onset of deterministic chaos [1–3]. In the case of semiconductor lasers, external feedback has been provided either by optoelectronic [4–7] or by purely optical means [8–10]. Only in this latter case chaos was observed. On the contrary, an optoelectronic feedback has always led to a stable single frequency oscillation. Even though a correlation was established between oscillatory frequency and the delay time [6], no detailed theory has been given so far to relate the observed behavior to a model.

The role of delay requires a preliminary consideration. As well known [11] a delay differential equation of first order requires an infinity of initial conditions (all those included in a time interval of length equal to the delay τ). However, if the dynamics includes a filter with a limited bandwidth $\Delta\nu$, then the relevant number of degrees of freedom will be [12]

$$N=2\,\Delta\nu\,\tau\,. \tag{1}$$

Thus delay will play an effective role only when $N\gtrsim 1$. Therefore in molecular lasers or in the experiment of ref. [4] delay is not included since, e.g. in ref. [4] $\Delta\nu=15$ MHz against a delay in the feedback loop which in ordinary laboratory condition did not exceed 20 ns.

In order to explore the role of a delayed optoelectronic feedback, we have performed an accurate experiment with a single mode cw laser (Hitachi HLP 1400). Here we report the observations and present the model equations, whose solutions are in quantitative agreement with the experiments. Precisely, we detect the laser intensity with an avalanche photodiode (APD) (see fig. 1). The detector photocurrent is fed back to the laser junction, being summed up through a broad band "bias-tee" to the dc pump current. The bias-tee low frequency decoupling (200 kHz cut-off) assures that the dc behavior is not per-

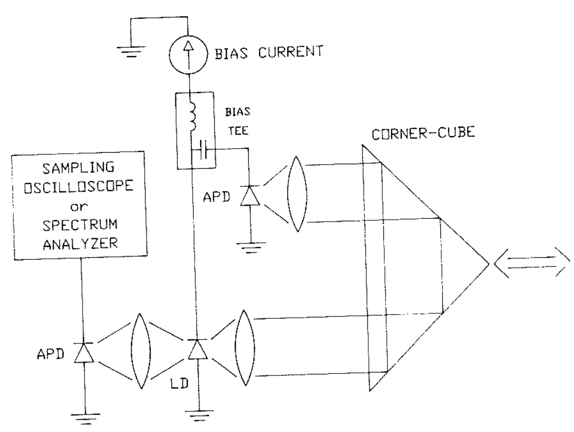

Fig. 1. Experimental set-up. LD: laser diode; APD: avalanche photodiode; the pump current is decoupled from the feedback loop through a bias-tee (here represented as an LC filter). The laser rear facet output is used for monitoring.

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turbed. In the absence of an amplifier in the feedback loop the high frequency cutoff is due to the stray capacitance of the laser diode and it is estimated to be above 2 GHz. The frequency response of the APD is much higher (above 10 GHz). By use of eq. (1) we see that we have relevant delay effects down to τ as small as half a nanosecond.

The three control parameters of our experiment are: pump (controlled by the injection current), feedback gain (controlled by the gain of the APD) and delay (adjusted continuously from 2.5 to 6 ns by insertion of an optical delay (translatable corner cube) between laser and APD). Feedback gain and delay are measured at open loop by a network analyzer.

For a typical setting of the control parameters the cw output is not qualitatively perturbed. However the spectral distribution (fig. 3) shows the onset of small peaks corresponding to strongly damped excitation by broadband noise (spontaneous emission).

For some selected values, a new behavior appears, that is, the output displays a strong modulation with a narrow (≈ 100 kHz) spectral distribution (figs. 2 and 3c). Once an oscillation has been found it is "robust" against parameter variations. For a fixed pump, these resonance conditions correspond to a feedback gain and delay such that the open loop measurement of the resonance frequency yields a gain of 0 dB and a phase of 0°. The resonance is always at the low frequency side of the natural relaxation peak. As we change the delay τ, the resonance disappears, and it appears again for an extra delay $\Delta\tau$ such that the phase is again 0°.

This phenomenology is satisfactorily reproduced by the following equations, based on a class B laser [13] with a fast feedback with a low frequency cutoff,

$$\frac{1}{k}\frac{\mathrm{d}x}{\mathrm{d}t}+X=XY,$$

$$\frac{1}{\gamma}\frac{\mathrm{d}Y}{\mathrm{d}t}+Y=1+\alpha(a-XY+BZ_{\mathrm{d}})\,,$$

$$\frac{1}{\beta}\frac{\mathrm{d}Z}{\mathrm{d}t}=Z=\frac{1}{\beta}\frac{\mathrm{d}X}{\mathrm{d}t}\,. \qquad (2)$$

Here X is the laser intensity normalized to the saturation value, Y is the difference between carrier population n and its value n_0 at transparency, normalized to the same difference $n_{\mathrm{th}}-n_0$ at threshold; k^{-1} and γ^{-1} are the X and Y lifetime, respectively. The coupling constants have been included in the rescaling parameters (saturation intensity and thresh-

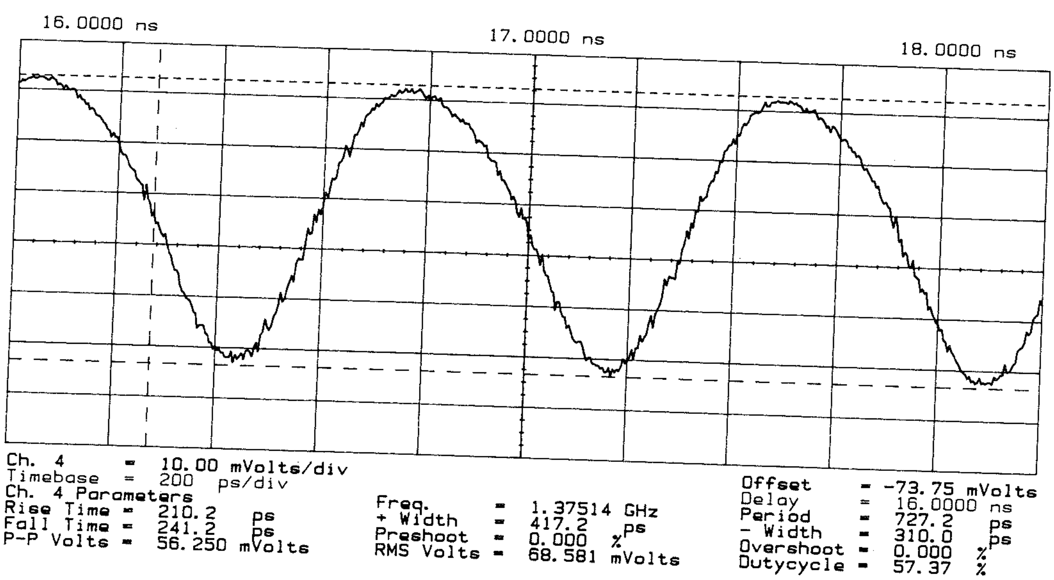

Fig. 2. Time dependent laser output at resonance observed with a digitizing sampling oscilloscope (HP54120, timebase 200 ps/div); the modulation depth is ≈ 0.6.

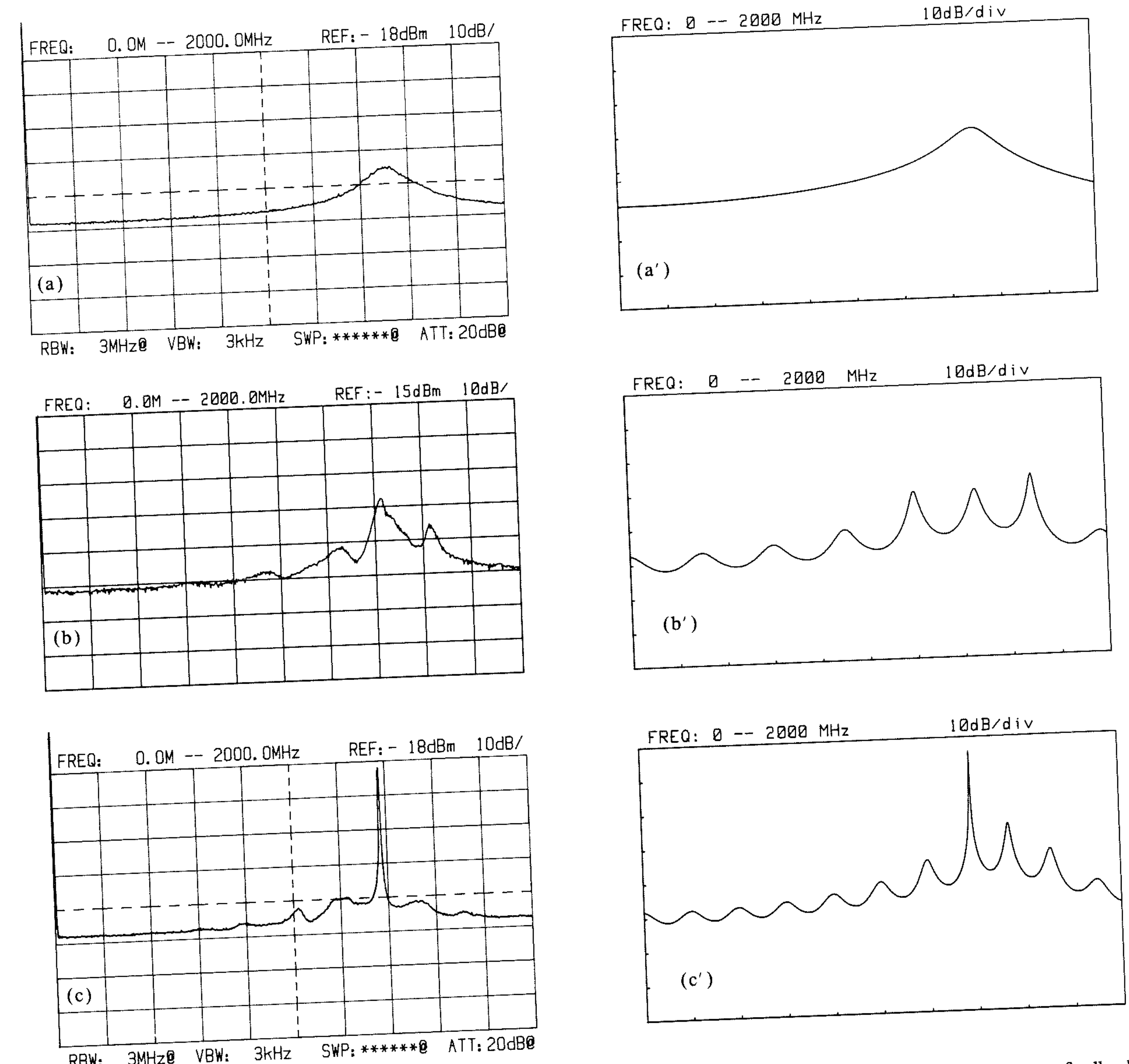

Fig. 3. Experimental power spectrum of the laser output versus theoretical spectrum computed from eq. (5): (a), (a′) without feedback, the broad peak corresponds to the natural relaxation oscillation; (b), (b′) with feedback but away from resonance, the small peaks correspond to resonances excited by noise but rapidly decaying; (c), (c′) with feedback at resonance, the large narrow peak (30 dB above (a) and (b)) corresponds to a resonance in the delay equations.

old population). In order to write the semiconductor laser equations as in eqs. (2), that is, in a form similar to that used in two level systems [13], it was necessary to introduce in the model equations the parameter

$$\alpha = \frac{n_{th}}{n_{th} - n_0} \approx 3 \qquad (3)$$

(in a two level laser it would be $\alpha = 1$).

The pump, normalized to the threshold value is

$a+1$. BZ_d is the feedback addition to the pump, where B is the low frequency loop gain,

$$Z_d(t)=Z(t-\tau) \tag{4}$$

is the delayed detector photocurrent, and Z is coupled to the intensity X (detector efficiency included in B) through the third equation in (2) which acts as a high pass filter with a low frequency cutoff.

Eqs. (2) may look oversimplified as compared with more elaborated versions of semiconductor laser equations [14]. In fact we have not included a detuning. Detuning plays a main role in inducing a phase dynamics. Phase variations are crucial in the case of optical feedback, however they play no role for optoelectronic feedback which is intensity sensitive. Thus, inclusion of a detuning term in eqs. (2) would not alter significantly the main results. For the same reasons, eqs. (2), which assume a single longitudinal mode, are still adequate to describe the behavior of the laser in the oscillatory regime in which, due to the dependence of the cavity refractive index on the carrier density, the emission spectrum becomes multi-frequency. The fixed points of eqs. (2) coincide with the two fixed points (OFF and ON) of the laser without feedback. A linear analysis around the ON point yields for $\beta\to 0$ the following equation

$$\frac{d^2X}{dt^2}+2\Gamma\frac{dX}{dt}+\Omega^2X=\Omega^2BX_d\,, \tag{5}$$

where $X_d(t)=x(t-\tau)$ is the delayed intensity.

The resonances of eq. (5) correspond to

$$\tan(\omega\tau)=\frac{2\Gamma\omega}{\omega^2-\Omega^2}\,,\quad B\sin(\omega\tau)<0 \tag{6a}$$

$$\Omega^4B^2=(\Omega^2-\omega^2)^2+(2\Gamma\omega)^2\,, \tag{6b}$$

where

$$\Omega^2=(k\gamma\alpha a)\,,\quad 2\Gamma=\gamma(1+\alpha a)\,.$$

Eqs. (5), (6) depend only on four measurable parameters, that is the laser relaxation frequency Ω and its half width Γ, the loop gain B and the delay τ. Introducing the values form the experiment of fig. 3c (see table 1, column 1 to 4) we obtain the theoretical spectrum of fig. 3d′, which displays a quantitative agreement with fig. 3c, with the exception of the high frequency side of the natural relaxation peak. In fact, in the real system there are stray capacitances due to the laser mounting which depress the high frequency part of the spectrum, and these capacitances were not taken into account in the model. In table 1, the 7th column shows the calculated frequency as compared to the measured one, for different delays (see 4th column) which provided the same resonance. Notice that the resonance condition eq. (6) is equivalent to the zero phase condition observed in the experiment. Since eq. (6a) is periodic in $\omega\tau$ the same resonance frequency can be obtained with an infinite number of τ-values, separated by a constant interval $\Delta\tau=2\pi/\omega$. In column 5 the values of $1/\Delta\tau$ are reported, in good agreement with f-values from column 6.

Our model is at variance with the heuristic proposal of refs. [5–7] that the resonances would be an integer multiple of $1/\tau$. Indeed the last column of table 1 shows that this latter conjecture yields a bad agreement as compared with that (6th column) coming from our model.

Such an optoelectronic system shows comparatively pure power spectra, and good intensity and tunability, thus, it may be conveniently used as a high

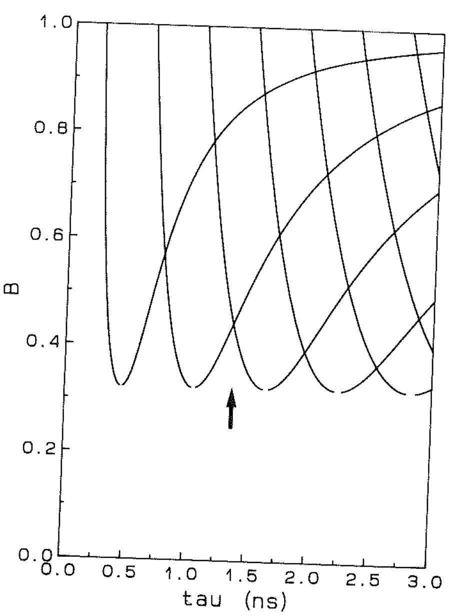

Fig. 4. (τ, B) parameter space. Curves are the loci of Hopf bifurcations of eq. (5). For a fixed gain B we have a periodicity in the delay τ (see eq. (6a)). The overlapping of instability regions suggests the possibility of observing coupling between different resonant modes.

Table 1
Comparison between experimental data and theoretical models. The first four columns report the experimental data corresponding to self-oscillation, whose frequency f is in the sixth column. The seventh column shows the frequency f' given by our model, while the eighth column reports the integer multiplets of $1/\tau$ according to the heuristic model of refs. [5–7]. In the fifth column we give the reciprocal of the delay interval for which we obtain again self-oscillation at the same frequency. Notice that each $1/\Delta\tau$ matches quite accurately its corresponding frequency.

$\Omega/2\pi$ (MHz)	$\Gamma/2\pi$ (MHz)	B	τ (ns)	$1/\Delta\tau$ (MHz)	f (MHz)	f' (MHz)	k/τ (MHz)
1189	95	0.227	2.65	943	1053	1088	1132
1189	95	0.227	3.71		1053	1053	1078
1496	80	0.162	3.55	1351	1376	1382	1408
1496	80	0.162	4.29	1408	1376	1378	1399
1496	80	0.162	5.00		1376	1381	1400
1781	50	0.103	2.92	1724	1690	1692	1714
1781	50	0.103	3.50	1724	1690	1686	1712
1781	50	0.103	4.08	1613	1690	1695	1716
1781	50	0.103	4.70	1724	1690	1686	1702
1781	50	0.103	5.28	1695	1690	1691	1704
1781	50	0.103	5.87		1690	1691	1704

frequency source, e.g., as a clock in a fast signal regenerator in digital data transmission, and in all optical data processing. It does not require electronic circuitry, and can be "in principle" completely built in an integrated optics device.

As a conclusion, our model considerations describe accurately the experimental results, yielding an agreement of a few parts in 10^3 whereas previous models give a rough agreement within a few percent. The introductory considerations (see eq. (1)) have shown that our dynamics implies more than one degree of freedom. Thus the available phase space has the right dimension to account for chaotic phenomena. Since however no amplification was added on the feedback loop, we did not explore the nonlinear domain sufficiently far from the onset of the instability to overcome the threshold for chaos. This is also shown by the stability diagram in the parameters space B–τ (fig. 4). An operating condition as that indicated by the arrow shows the coexistence of two independent frequencies. If a sufficient amplification permits to increase the B value for the same τ, the system should be forced far beyond the instability threshold, and a competition between two (or more) frequencies should occur. We plan to explore this nonlinear dynamics in a successive work.

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