DETERMINATION OF A SMALL PHOTON NUMBER BY STATISTICAL AMPLIFYING TRANSIENTS

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The first observation of a statistical spread in the leading edge of a transient He-Ne laser pulse was associated with the appearance of a large peak in the variance of the transient photon number distribution[1]. This fact was explained in terms of an approximately deterministic decay out of a macroscopic unstable state, to be averaged over the statistical distribution of the initial states[2]. The assumption of deterministic evolution neglected the role of fluctuations along the build-up with respect to the initial ones. A quest for a discrimination between fluctuations on the initial condition and those along the path led to a new observation method, based on the statistics of passage times at a given threshold[3,4,5].

We generate a transient dynamics in a single-mode CO_2 (Class B) laser by switching an intracavity modulator (EOM) from absorption to transparency in a time shorter than the build-up time of the giant pulse. As we switch the EOM voltage from V_0 to V_1, the cavity loss rate k decreases from k_0 to k_1 with a time constant of 0.6 μs as shown in fig. 1. A laser pulse builds from a low photon number n_0 (laser below threshold for loss k_0) up to peak value, at $t_2 \simeq 3$ μs. The pulse develops over a time much shorter than the molecular decay time, thus, the population change during the pulse is mainly due to the radiative iteraction. We notice from fig. 1 that the time jitter already built in the first part of the leading edge is preserved along the whole pulse. This shows that noise in the initial condition is more important than noise along the trajectory. Indeed, ref. 3 proves that a constant time variance is the indicator of no relevant noise along the trajectory.

In order to remain in the linear regime we consider the evolution up to t_1 when the photon number n_1 is still smaller than the saturation value. Up to t_1, we can take the population inversion as fixed at its pre-pulse value. As appears in fig. 1, superposition of many successive pulses gives a spread in the build-up time. By setting a threshold circuit on the photo-detector output at the photon number $n=n_1$ and using a time-to-amplitude converter, we measure the statistical distribution

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$p(t_1)$ of arrival times to n_1. An example is shown in fig. 2. From this we can evaluate accurately $\bar{t}_1 = \langle t_1 \rangle$ and $\delta t = \langle \Delta t^2 \rangle^{1/2}$ and higher-order cumulants of $p(t_1)$, as skewness and kurtosis.

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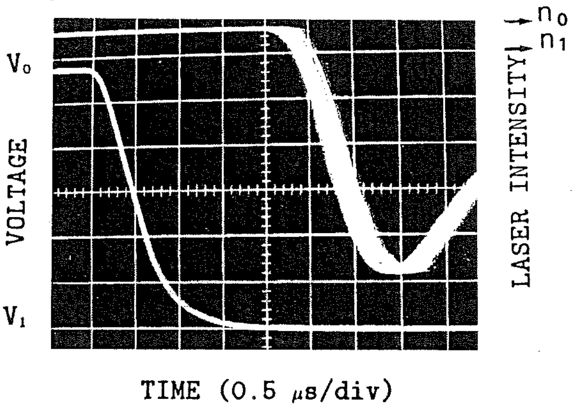

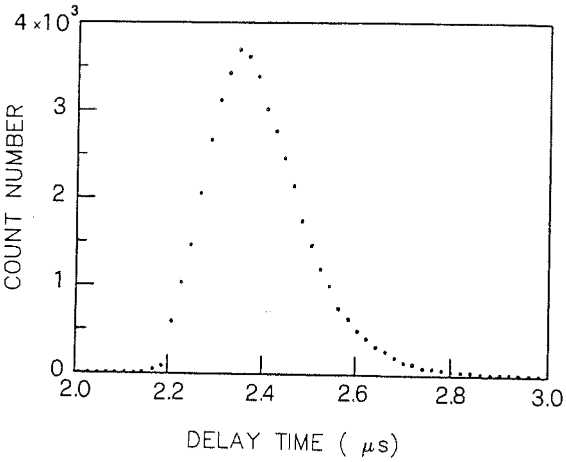

Fig. 1. Oscilloscope plots of the EOM voltage driven from V_0 to V_1 and of a set of transient intensity traces starting from the off-state (laser intensity increasing downwards from a very low initial value n_0, practically coincident with the baseline). The exposure corresponds to 10^4 successive transients. The time jitter is uniform along the evolution. To avoid nonlinearities, the threshold n_1 for the time distribution $p(t_1)$ (see fig. 2) is adjusted below the saturation photon number, that is rather close to the off-state.

Fig. 2. Statistical distribution $p(t_1)$ of the passage times through the threshold photon number n_1. This distribution corresponds to experiment 3 of the two successive figures.

A simple argument gives the connection between the time statistics $p(t_1)$, and the statistical spread $p(n_0)$ of the initial photon numbers n_0. Since very below threshold the field is Gaussian, $p(n_0)$ is a simple exponential

$$p(n_0) = (1/\langle n_0 \rangle) \exp[-n_0/\langle n_0 \rangle]. \tag{1}$$

A noise-free linear amplification with constant gain a could be ruled by the equation $\dot{n} = a.n$ (n = photon number). Stopping the evolution at time t_1, we would measure

$$n_1 = n_0 \exp[at_1]. \tag{2}$$

A spread in n_0 will induce a corresponding spread in t_1. Deriving $p(t_1)$ via $p(n_0)$, eqs. (1) and (2) provide a relation between the first moments $\langle n_0 \rangle$ and $\langle t_1 \rangle$, of $p(n_0)$ and $p(t_1)$, respectively,

$$\langle n_0 \rangle = n_1 \exp[-a\bar{t}_1]. \tag{3}$$

This is the core of the "statistical microscope" method, that is, accurate calibrations of n_1, a and $\bar{t}_1$ (all reliable laboratory operations as we shall see) permit to retrieve the small initial number $\langle n_0 \rangle$, otherwise unaccessible. At a fixed threshold n_1, t_1 is spread around $\bar{t}_1$ with an r.m.s. spread t. Via eqs. (1) and (2) we evaluate

$$\overline{\delta t} = \frac{\sqrt{\psi'(1)}}{a}, \tag{4}$$

where $\psi'(x)$ is the trigamma function[8]. Thus a is inferred from the experiment without having to rely on a model for coupling coefficients or population inversion. Inclusion, however, of noise along the amplification process is equivalent to an additive contribution to initial noise as shown in ref.(6). This equivalence means that the time spread $p(t_1)$ can still be seen as the result of a noise-free amplification process as eq. (2), starting, however, from a modified initial distribution $p(n_0)$, where $\langle n_0 \rangle$ of eq. (1) is replaced by the sum N_0 of the initial photon number $\langle n_0 \rangle$ plus the contribution n_p due to noise along the path ($N_0 = \langle n_0 \rangle + n_p$). Thus the statistical microscope equation becomes

$$N_0 = n_1 \exp[-a\bar{t}_1]. \tag{5}$$

In the experiment the switch takes a finite time t_s. This has to be accounted for in the last equation by a time translation. The complete expression is then

$$N_0 = n_1 \exp[-a(\bar{t}_1 - t_s) - \psi(1)], \tag{5'}$$

where $\psi(1) = -0.577...$ is the digamma function[8] $\psi(x)$ at $x = 1$. This way, we have found a method to retrieve a small photon number N_0 (of the order of 10^3 or 10^2) by measuring a large photon number $n_1 = 1.7\times10^{11}$ and the statistics of the passage times across the threshold n_1.

Now, it is a well-known result that the equilibrium variance for a statistical process can be expressed as the ratio of the noise correlation amplitude D to the dissipation rate a. Applying this to our case we find

$$N_0 = \langle n_0 \rangle + n_p = \frac{D}{|a_0|} + \frac{D}{a}. \tag{6}$$

Here, $D = \gamma_{sp} N_2 \eta$ is a diffusion coefficient due to spontaneous noise ($\gamma_{sp} = 0.3s^{-1}$ is the spontaneous emission rate, N_2 the total population of the upper laser level and $\eta \simeq 10^{-6}$ an angular coupling efficiency); and a and a_0 are the net gain rates, that is the difference between gain and cavity loss rate k. k is related to the EOM voltage via a relation as $k = \bar{k}(1 + f(V))$, where $\bar{k} = 2.0.10^7 s^{-1}$ and f(V) is the modulator characteristic. As the voltage V is switched from V_0 (corresponding to a high loss k_0) to V_1 (low loss k_1), the gain rate is switched from a negative value a_0 to a positive one a.

In fig. 3 and 4 we give data for three experimental situations, corresponding to the same initial photon number $\langle n_0 \rangle = D/|a_0|$, that is, (see inset in fig. 3) same discharge current and initial EOM voltage

V_0 = 778 V. We notice from Fig. 4 that we can retrieve $\langle n_0 \rangle$ free from n_p.

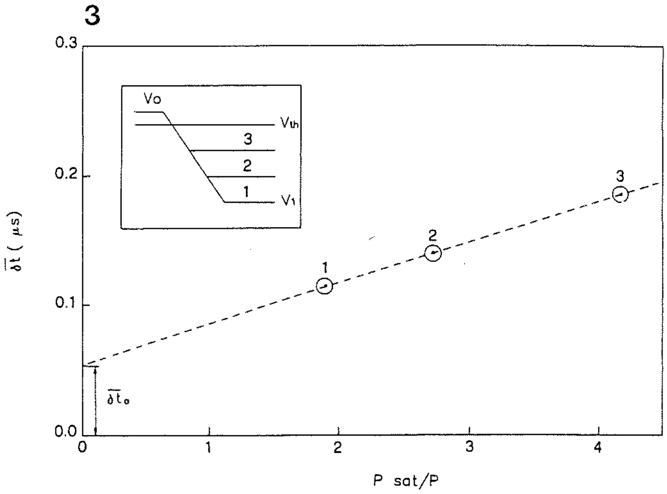

Fig. 3

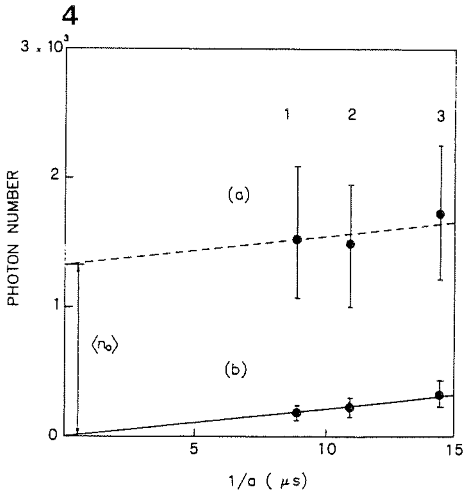

Fig. 4

Fig. 3. Plot of the average time spread $\overline{\delta t}$ vs. the reciprocal of the output power P. P is evaluated for long times, after the transient has died out, the normalization P_s is the saturation power. The error bars are smaller than the black dots (the circles around the dots are just indicators). The intercept $\overline{\delta t}_0$ is the asymptotic uncertainty for large powers. The inset shows the relative EOM voltage settings in the three experiments (V_o and V_1 initial and final voltages, respectively, V_{th} threshold voltage).

Fig. 4. a) Total photon number N_0, and b) equivalent photon number n_p corresponding to noise along the path, vs. the reciprocal of the gain rate a. The vertical intercept of N_0 for 1/a = 0 is the initial photon number $\langle n_0 \rangle$.

In conclusion, our "statistical microscope" permits to retrieve a small photon number via a macroscopic measurement separating the initial photon number from the fluctuations along the amplification process. Furthermore, knowledge of the first four cumulants of the passage time distribution permits a calibration of residual errors, and these ones can be reduced by increasing the count number. In the limit of a large number of counts, the procedure here outlined seems to imply a precise determination of $\langle n_0 \rangle$ if $\langle n_0 \rangle \gg 1$. In fact, in order to arrive at the crucial assumption eq. (6), we have considered an amplification factor $\exp(at_1) \gg 1$. We recall that in our case $\bar{a}t_1 = \bar{t}_1/\overline{\delta t} \simeq 20$. For $\bar{a}t_1$ less than 1, that is in the very early part of the growth process, our procedure fails. However, that region has no interest for the method here proposed, which instead is based on a large amplification factor.

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