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Chaos in a CO_2 waveguide laser due to transverse-mode competition

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We report on the spontaneous onset of temporal chaos in a waveguide CO_2 laser due to the interaction between two transverse modes. A nonstandard configuration of the resonator has been used in order to obtain multitransverse mode operation. Starting from a mirror setting where two modes have a stable coexistence, further tiny variations of the cavity length induce a period-doubling cascade to chaos.

As it is well known, the dynamics of a single-mode class-B laser is ruled by two equations, thus there is no chance for the onset of chaos, unless one forcefully introduces a third degree of freedom.[1] The search for the spontaneous appearance of chaotic instabilities implies multimode operation, either longitudinal or transverse. It is much easier to couple different modes if they are transverse, and thus pile up in a narrow frequency range.

A large amount of theoretical work has been recently provided on this issue.[2] Experimental evidence of chaotic instabilities when more than two transverse modes are excited is reported on a cw CO_2 laser.[3] A recent experiment on a CO_2 laser whose cavity configuration is swept from quasiconfocal to nearly planar has shown evidence of spatiotemporal instabilities due to transverse-mode coupling but no chaos has been observed.[4] Preliminary evidence of spatio-temporal chaos has been provided for a Na_2 ring laser,[5] involving Gaussian TEM_{00} and TEM_{01} modes, however, the route to chaos is not explored and only evidence of developed chaos is given.

In the experimental reports,[2–5] a standard Fabry-Pérot cavity was used, and suitable adjustments of the confocal parameter[4] provide coexistence of several transverse modes. We have devised a gas laser with a waveguide gain tube, so that many off-axis modes will be confined with almost equal losses. The mode selection is then provided by the angular acceptance of an external curved mirror (CM in Fig. 1). When the CM is far away from the right tube end, we have a standard single-mode operation, as CM is put very close to the tube end, the laser oscillates over a large number of modes. By carefully setting CM at an intermediate position, we can select a stable two-mode operation.

Here we report clear evidence of the spontaneous onset of chaos due to the nonlinear interaction of the two transverse modes when the cavity length is driven slightly away from the stable operation. Waveguide lasers are generally used for single longitudinal and transverse-mode operation, on the lowest-order hybrid mode, EH_{11},[6] and for this purpose there exist three (theoretical) configurations for the resonator.[7,8] We have modified one of these configurations to achieve multitransverse mode emission. Our system, shown in Fig. 1, consists of a CO_2 waveguide laser resonator that contains a hollow Pyrex guide of circular section with 4-mm inner diameter and 88-cm length, which is terminated by a ZnSe window W (Ar coated on both surfaces). The optical cavity consists of the waveguide, coupled directly on the left-hand side to a diffraction grating (100 lines/mm) mounted at Littrow's angle, selecting the $P(20)$ rotational line at 10.6 μm, and on the right-hand side, via W, to an external coupling mirror CM (80% reflectivity). This spherical mirror with 2 m of radius of curvature can be placed at various distances (up to 1.5 m) from the the end of the guide. The coupling mirror is mounted on a piezoelectric translator (PZT) which allows micrometric control of the cavity length, over more than one free spectral range. The free spectral range of the optical cavity can be varied with the distance of the external mirror down to a minimum of about 80 MHz, that is still large enough to avoid simultaneous multilongitudinal mode emission. The gas flows in the guide and in a case including the grating at a pressure of 40 mbar, providing a homogeneously broadened linewidth for the laser transition ($\Delta\nu \cong 160$ MHz). The excitation of the medium is obtained by means of a dc discharge inside the guide, current stabilized to 0.1%. The value of the current is set to 10 mA. The output power of the laser is

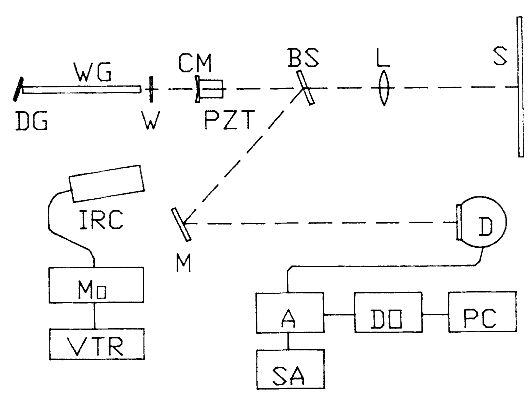

FIG. 1. Experimental setup: DG, diffraction grating; WG, Pyrex hollow cylindrical guide; W, ZnSe window; CM, coupling mirror (Ge 80% reflectivity); PZT, piezotranslator; BS, ZnSe beam splitter (10%); L, Ge lens; S, screen; M, mirror; IRC, infrared camera; Mo., monitor; VTR, videotape recorder; D, Hg-Cd-Te detector; A, video-band amplifier; DO, digital oscilloscope; PC, personal computer; SA, digital spectrum analyzer (20–40 MHz).

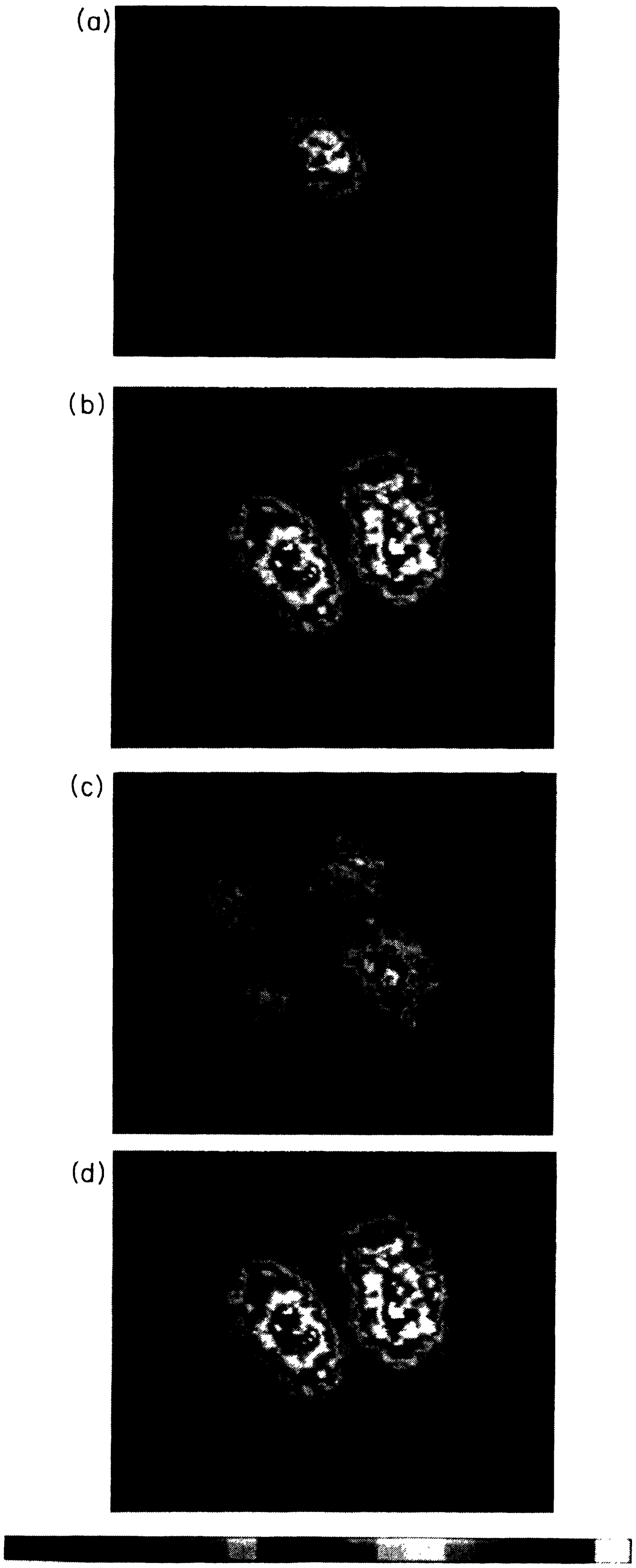

FIG. 2. (a) EH_{11} far-field spatial pattern, obtained for a mirror distance of 1 m. (b),(c) Two TEM-like mode patterns obtained for mirror distances of (b) 40 cm and (c) 30 cm, respectively. (d) Spatial pattern of the emission (stable in time) obtained for a mirror distance of 25 cm, before the transition to chaos.

monitored using the zeroth-order reflection of the grating. In order to rule out noise contributions, the laser is mounted on an antivibrating table, the mirrors are spaced by Invar rods, and the airborne fluctuations are avoided by a tight case. The spatial profile of the modes has been visualized with an infrared camera (Inframetrics model 600) by projecting on a wide (30×40 cm^2) screen the laser beam expanded by a germanium lens. The size of the screen has been chosen in order to avoid distortions of the image due to the speckle pattern (typical of laser-diffused radiation). We follow the temporal behavior of the intensity by means of an Hg-Cd-Te photodiode with a 100-MHz bandwidth. The signal is then amplified and sent to an eight-binary-digit (bit) digital oscilloscope (LeCroy 9400), controlled by a PC.

The pattern shown in Fig. 2(a) refers to the single EH_{11} mode operation, obtained by placing the coupling mirror at 1 m from the end of the guide. Multimode operation has been obtained by reducing the distance of the coupling mirror in the range 2–60 cm. There is no theoretical prediction for the behavior of this kind of resonator within this range of conditions. We have tested emission on

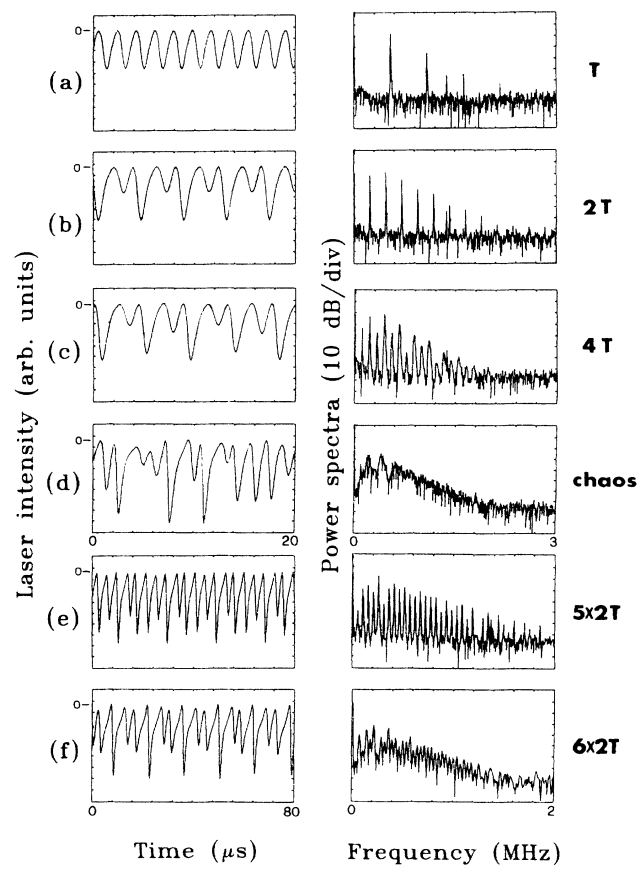

FIG. 3. Temporal behavior of the laser intensity (left) and their related power spectra (right) for increasing values of the PZT voltage starting from the spatial pattern of Fig. 2(d). The sequence shows a subharmonic route to chaos, plus two periodic windows beyond the onset of chaos. T indicates the period. Left-hand side: (a)–(d) 2 μs/div; (e) and (f) 8 μs/div. Right-hand side: (a)–(d) 300 kHz/div; (e) and (f) 200 kHz/div (the spectral line at about 1.4 MHz is due to external noise).

modes different from EH_{11}; some of these modes [Figs. 2(b) and 2(c)] closely resemble TEM Gaussian modes.

By careful setting of the coupling mirror, we can obtain simultaneous oscillation of two or more transverse modes, characterized by a complication in the spatial profile of the laser emission and by the presence of beat notes in the output intensity, in the range 1–40 MHz. The beatings have been measured using a digital spectrum analyzer. In particular, for a mirror distance of 25 cm, we observe the pattern reported in Fig. 2(d), which corresponds to a stationary emission. Starting from this pattern we find, by a slight increase of the cavity length with the PZT, the onset of a single note beat at about 2 MHz (two-mode operation). Then this frequency slows down to 400 kHz, loosing in this process the dc component typical of beating phenomena. Thus, at the end of this slowing down the beating has disappeared and we have instead a nonlinear competition between two modes. On increasing our control parameter, the system undergoes a subharmonic bifurcation cascade[9] leading to a chaotic regime, within which we detected the odd-period periodic windows predicted by the theory. This situation is shown in Fig. 3, where in the left-hand side we report the temporal behavior for increasing values of the control parameter, and on the right-hand side the associated power spectra, obtained by fast Fourier transform (FFT) of the digitized time data.

A further increase of the cavity length drives the system through the inverse bifurcation sequence, until a new stable situation is reached. The whole instability window covers about $\frac{1}{40}$ of the free spectral range, that corresponds to a cavity length variation of 0.13 μm (or to a detuning of 3.3 MHz).[10] The low dimensionality of this chaos has been confirmed by measuring the correlation dimension of the attractor with the Grassberger and Procaccia algorithm[11] using 32000 points and finding a value $D_2 = 2.6 \pm 0.2$.

In conclusion, we have provided experimental evidence of transverse-mode interaction inducing a spontaneous chaos in the output intensity of a CO_2 waveguide laser for critical values of the cavity length.

More complex regimes occur when more than two modes simultaneously compete, thus leading to spatiotemporal instability phenomena similar to those observed in hydrodynamic systems. Such a behavior will be reported elsewhere.

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[10]Since the cavity detuning represents our control parameter, we have carefully checked for stability and resettability. The stability is provided by the Invar spacers between the cavity mirrors. The resettability is provided by the lack of hysteresis of the PZT that yields a length variation of 4.38 μm/kV. Since we have applied a PZT voltage variation of 30 V from Figs. 3(a) to 3(f), that corresponds to a maximum detuning of 3.3 MHz, where 132 MHz is the mode spacing at the selected cavity length (89+25 cm)

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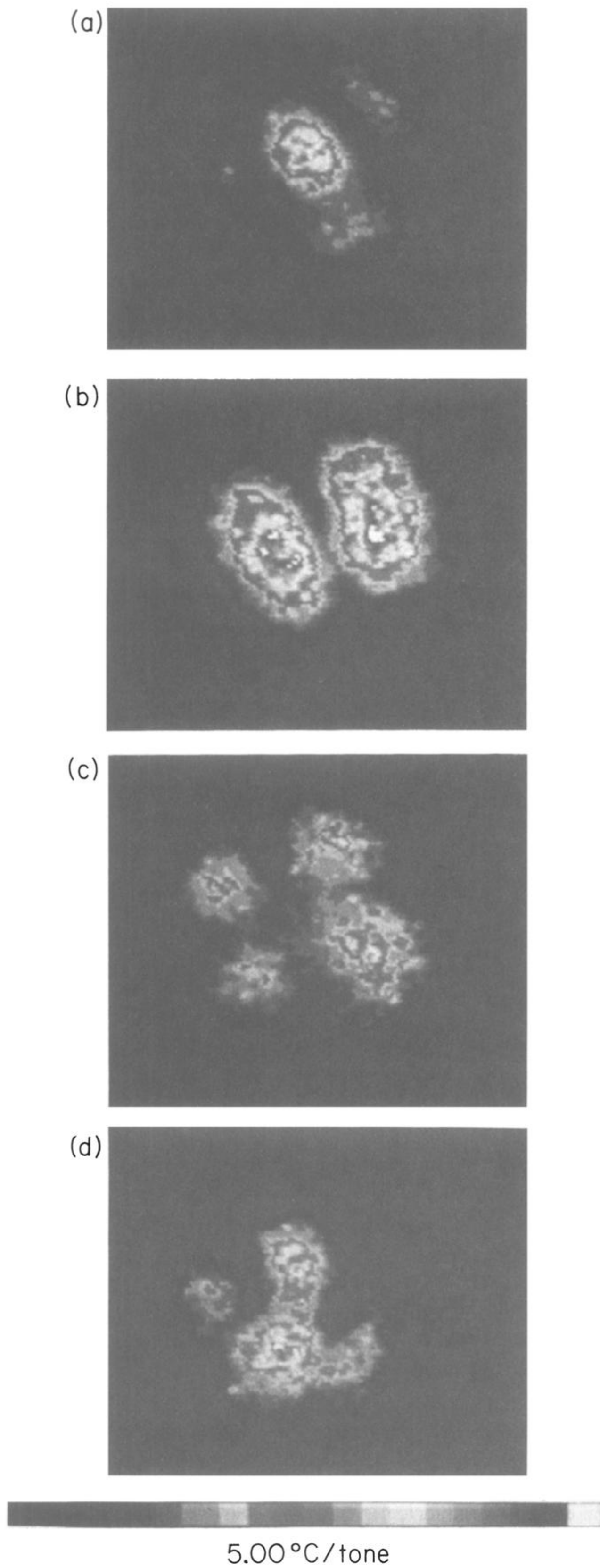

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