

## CO<sub>2</sub> laser with swept pump parameter: The nonlinear regime

S. Balestri, M. Ciofini, R. Meucci, and F. T. Arecchi\*

*Istituto Nazionale di Ottica, Largo Enrico Fermi 6, 50125 Firenze, Italy*

P. Colet, M. San Miguel, and S. Balle

*Departament de Física, Universitat de les Illes Balears, E-07071 Palma de Mallorca, Spain*

(Received 20 May 1991)

We show that a CO<sub>2</sub> laser driven from below to above threshold by pump sweeping displays a linear dependence of the peak intensity on the switch-on time. This feature is a consequence of the small spread of the switch-on times. Experimental results for the slope of the linear relation are well reproduced by a calculation based on the rate equations.

PACS number(s): 42.50.—p, 42.55.Em

### I. INTRODUCTION

The study of transient dynamics in laser systems has given several fundamental results in the past few years. A delay in the laser switch-on time when the net gain is swept from below to above threshold has been predicted theoretically [1] and observed experimentally [2]. Moreover the important role played by quantum noise in determining macroscopic fluctuations has been investigated in the linear regime of laser amplification for both class-*A* and -*B* lasers [3–6]. The nonlinear regime, where saturation phenomena become relevant, has been extensively treated for class-*A* lasers [7], while for class-*B* only few recent works are available on semiconductor lasers [8,9]. In any case, in semiconductors the transient times are so fast that only the statistical envelope of trajectories can be observed. Dealing instead with a CO<sub>2</sub> laser provides a more accessible time scale and hence an experimental characterization of the single transient.

In this paper we report on the dynamics of a single-mode CO<sub>2</sub> laser when the population inversion is swept in time by applying a linear ramp to the excitation current. After the laser net gain overcomes the dynamical threshold (amplification regime) a large spike in the intensity occurs, followed by relaxation oscillations. The peak intensity depends on the value reached by population inversion at the time at which the laser amplification takes place. For this reason the quantum noise, which determines a statistical spread in the switch-on times, is also responsible for the spread in the peak intensity values. A linear relation between the switch-on time and the peak intensity has been shown to be a rather general result when the statistical spread in the switch-on time is small compared with the mean value [9]. The outline of this paper is the following. In Sec. II the experimental setup and results are reported. Section III deals with the theoretical analysis which provides a simple linear relationship between the peak intensity and the switch-on time. In Sec. IV we compare the experimental data with the results of the preceding section. Finally we draw our conclusions in Sec. V.

### II. EXPERIMENTAL SETUP AND RESULTS

In our experiment (Fig. 1) we use a CO<sub>2</sub> laser tube, terminated with Brewster angle windows, placed inside a resonator 1.5 m long. One of the reflectors of the laser cavity is a grating (150 lines / mm), selecting the *P*(20) line at 10.6 μm. The other is a partially reflecting Ge mirror (*R* = 90%), with a 5-m radius of curvature. This mirror is mounted on a hollow cylindrical piezoelectric translator (PZT), in order to control the detuning between the center of the molecular line and the frequency of the cavity mode. The inflow CO<sub>2</sub> laser is pumped by a

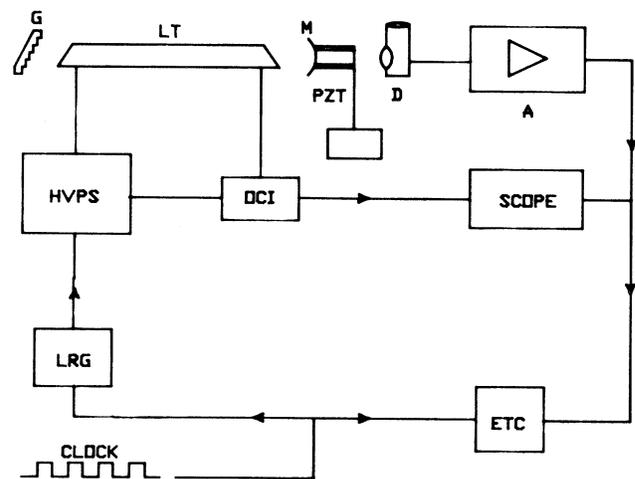


FIG. 1. Experimental setup. LT, laser tube; G, grating; M, mirror; PZT, piezoelectric translator; HVPS, high voltage power supply; OCI, optically coupled isolator; LRG, linear ramp generator; ETC, electronic time counter; D, Hg-Cd-Te detector; A, low-noise amplifier.

TABLE I. Mean value ( $\bar{T}$ ) and standard deviation ( $\delta T$ ) of the switch-on time distribution for two different slopes of the linear ramp.

$\bar{T}$ ( $\mu\text{s}$ )	$\delta T$ ( $\mu\text{s}$ )
$\theta = 3.80$ A/s	
1182.38	2.22
1182.37	2.21
1182.32	2.14
1182.32	2.01
1182.79	2.12
$\theta = 4.69$ A/s	
966.73	1.58
966.73	1.61
966.69	1.58
966.73	1.67
966.70	1.67

dc discharge. The discharge length is about 40 cm, and the gas mixture is composed of CO<sub>2</sub> (14.4 vol %), H<sub>2</sub> (2 vol %), N<sub>2</sub> (14.2 vol %), and He (69.4 vol %) at a total pressure of 21 mbar, measured at the gas inlet of the laser tube. The power supply is current stabilized to better than 0.05% and it can be driven by an external linear ramp generator with variable slopes [6]. The output current is directly monitored by means of an optically coupled isolator in series with the discharge tube. The laser output intensity is detected by a liquid-N<sub>2</sub>-cooled Hg-Cd-Te detector, with a rise time faster than 10 ns. The photodetector signal is amplified and sent, together with the current signal, to a digital oscilloscope, interfaced via general purpose interface board with a computer. An electronic time counter is used to measure the time interval between the start of the linear ramp and the instant at which the laser intensity reaches a given threshold below the saturation value. The counter repeats 1000 measurements and displays the mean value

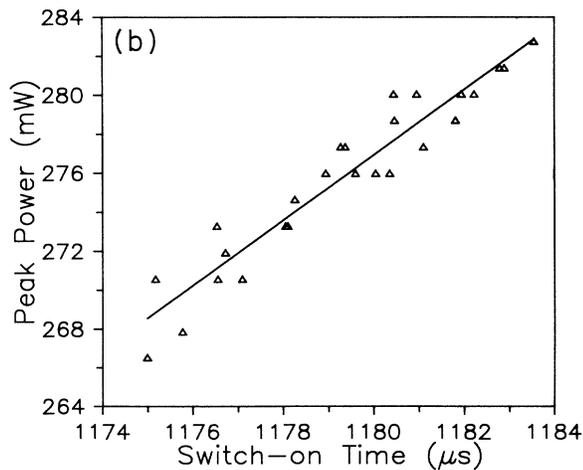
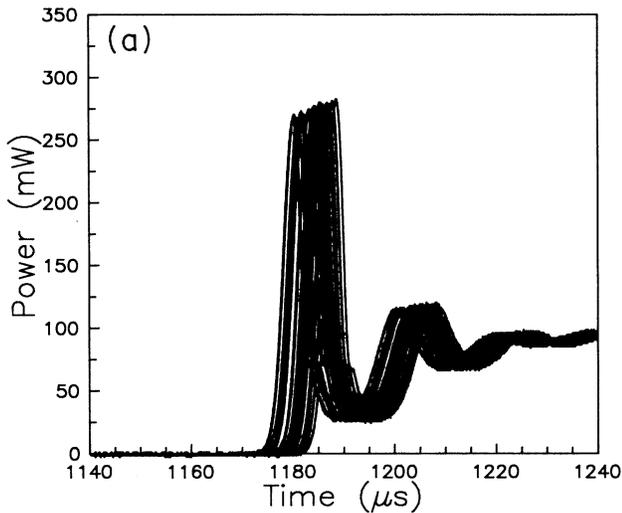


FIG. 2. (a) Switching events obtained for  $\theta_1 = 3.80$  A/s. (b)  $I_m$  vs the switch-on time  $T$  for  $\theta_1 = 3.80$  A/s ( $I_{th} = 5$  mW). The best fit line slope is 1673 W/s.

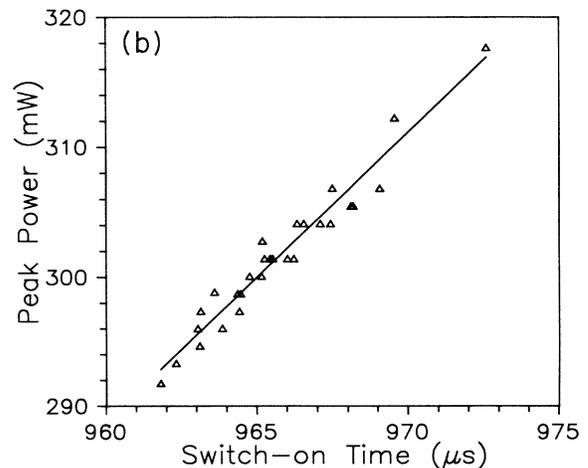
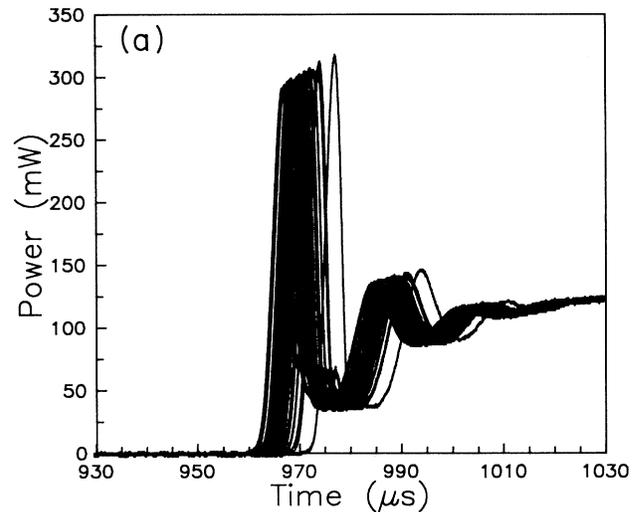


FIG. 3. (a) Switching events obtained for  $\theta_2 = 4.69$  A/s. (b)  $I_m$  vs the switch-on time  $T$  for  $\theta_2 = 4.69$  A/s ( $I_{th} = 5$  mW). The best fit line slope is 2223 W/s.

and the standard deviation of the switch-on time distribution.

We have performed two sets of measurements for different values of the slope  $\theta$  of the discharge current. In Fig. 2(a) are reported several trajectories of the laser intensity for  $\theta_1 = 3.80$  A/s. The corresponding linear dependence of the peak intensity from the switch-on time is presented in Fig. 2(b). Figures 3(a) and 3(b) refer to  $\theta_2 = 4.69$  A/s. In Table I are reported the results of five statistical measurements performed for each value of  $\theta$ .

### III. THEORETICAL ANALYSIS

Our results can be explained in terms of the single-mode class-B laser equations.

$$\dot{E} = \left[ \frac{G\Delta}{2} - K \right] E + (\epsilon)^{1/2} \xi(t), \quad (1)$$

$$\dot{\Delta} = -\gamma(\Delta - \Delta_0) - 2G|E|^2\Delta, \quad (2)$$

where  $E$  is the dimensionless complex amplitude of the electric field,  $\Delta$  the population inversion between the two resonant levels,  $K = 0.85 \times 10^7$  s<sup>-1</sup> is the field decay rate estimated including 7% of diffraction losses and  $\gamma = 5.0 \times 10^4$  s<sup>-1</sup> is the value used for the population inversion decay rate [10].  $\Delta_0$  is the population inversion provided by the pump mechanism and  $G = 5.37 \times 10^{-8}$  s<sup>-1</sup> is the field-matter coupling constant [6].  $\xi(t)$  is a complex Gaussian white-noise stochastic process with zero mean. The last term in Eq. (1) describes a random process of strength  $\epsilon$  accounting for spontaneous emission, which is responsible for the spread in the laser switch-on time  $T$ . The noise strength  $\epsilon$  is proportional to the population of the upper level. Our results below depend on  $\epsilon$  through the value of the switch-on time.  $T$  is defined as the time when the intensity  $|E|^2$  reaches 20% of the saturation value  $\gamma/2G$ . Thus the time range  $t < T$  corresponds to a linear amplification regime.

As the excitation discharge current is linearly modulated,  $\Delta_0$  can be considered a linear function of time

$$\Delta_0 = \Delta_0(t) = \beta t.$$

In the linear regime  $t < T$ , the last term in Eq. (2) can be neglected and the solution is

$$\Delta(t) = \beta t - \frac{\beta}{\gamma} [1 - \exp(-\gamma t)]. \quad (3)$$

In the time interval  $T \leq t \leq T_m$ , where  $T_m$  is the time corresponding to the maximum peak intensity, the term  $\xi(t)$  in Eq. (1) can be omitted and Eq. (2) can be rewritten neglecting the first term on the right-hand side as

$$\dot{\Delta} = -2G|E|^2\Delta. \quad (4)$$

Integrating the solution of Eq. (4) combined with the deterministic part of Eq. (1) between  $T$  and  $T_m$ , we obtain [9]

$$I_m = I_{\text{th}} - \frac{1}{2} [\Delta(T_m) - \Delta(T)] + \frac{K}{G} \left[ \ln \frac{\Delta(T_m)}{\Delta(T)} \right], \quad (5)$$

where the term  $I_{\text{th}} = |E|^2(T)$  represents the threshold intensity and the term  $I_m = |E|^2(T_m)$  the laser peak intensity.

In order to obtain  $I_m$  as a function of the switch-on time we consider  $T$  as the upper limit of validity of the linear approximation leading to Eq. (3). Moreover as  $T \approx 1$  ms the exponential term can be neglected with respect to unity, obtaining

$$\Delta(T) = \beta T - \frac{\beta}{\gamma}.$$

Since  $\dot{E}(T_m) = 0$ ,  $\Delta(T_m)$  corresponds to the threshold population inversion

$$\Delta(T_m) = \frac{2K}{G}.$$

Defining

$$I_0 = I_{\text{th}} - \frac{\beta}{2\gamma} - \frac{K}{G} \left[ 1 - \ln \frac{2K}{G} \right], \quad (6)$$

Eq. (5) can be written in its final form

$$I_m = I_0 + \frac{\beta}{2} T - \frac{K}{G} \ln \left[ \beta T - \frac{\beta}{\gamma} \right]. \quad (7)$$

According to the experimental data which show a small dispersion of the switch-on times around the mean value  $\bar{T}$ , we can expand Eq. (7) in powers of  $T$

$$I_m = I_m(\bar{T}) + \left. \frac{\partial I_m}{\partial T} \right|_{T=\bar{T}} (T - \bar{T}).$$

The final linear relation between the height of each intensity peak and the corresponding switch-on time is

$$I_m = I_{om} + \alpha T \quad (8)$$

where

$$I_{om} = I_0 - \frac{K}{G} \ln \left[ \beta \bar{T} - \frac{\beta}{\gamma} \right] + \frac{K \bar{T}}{G(\bar{T} - 1/\gamma)} \quad (9)$$

and

$$\alpha = \frac{\beta}{2} - \frac{K}{G[\bar{T} - 1/\gamma]}. \quad (10)$$

### IV. INTERPRETATION OF DATA

The experimental slope of the linear relationship between the intensity peak and the switch-on time  $T$  [see Figs. 2(b) and 3(b)] is 1673 and 2223 W/s, respectively, with a precision better than 6%. Since in our formalism the electric field is dimensionless, we have to multiply the above values for the fixed parameter  $M = 2L/hc\nu T_r$  (where  $L = 1.5$  m is the cavity length,  $h$  is the Planck constant,  $c$  is the light speed,  $\nu$  is the laser frequency, and  $T_r = 0.1$  is the coupling mirror transmission coefficient), obtaining  $\alpha_1 = 8.92 \times 10^{15}$  s<sup>-1</sup> and  $\alpha_2 = 1.18 \times 10^{16}$  s<sup>-1</sup>, respectively.

On the other side, to evaluate the theoretical slope  $\alpha$  defined in Eq. (10), we suppose that the population inver-

sion  $\Delta_0(t)$  and the discharge current  $i(t)$  are proportional for small modulation amplitudes

$$i(t) = \rho \Delta_0(t) = \rho \beta t = \theta t,$$

where  $1/\rho = 7.639 \times 10^{16} \text{ A}^{-1}$  [6] and  $\theta$  is the slope of the linear ramp defined in Sec II. Substituting  $\beta = \theta/\rho$  in Eq. (10), we finally obtain  $\alpha_1 = 8.72 \times 10^{15} \text{ s}^{-1}$  and  $\alpha_2 = 1.19 \times 10^{16} \text{ s}^{-1}$ . The agreement with the experimental data is within 2%.

The two level model of Eqs. (1) and (2) neglects some physical processes occurring in a CO<sub>2</sub> laser as the effect of rotational levels, which can be taken into account adding a correction term that does not affect  $T$  [11]. The good quantitative agreement between our experimental and theoretical results might be understood noting that the value of the slope given by Eq. (10) only depends on the values of  $\bar{T}$  and  $\beta$ . As a check of the general validity of Eq. (10) we have performed numerical simulations of Eqs. (1) and (2) with  $\gamma = 5 \times 10^4 \text{ s}^{-1}$ ,  $\gamma = 2.3 \times 10^5 \text{ s}^{-1}$ . We have also performed numerical simulations of an ap-

propriate three-level model. In all cases considered the variations on the value of the switch-on time are less than 1% and the values obtained for the slope  $\alpha$  are within 5% of the experimental values.

## V. SUMMARY AND CONCLUSIONS

A linear relationship between  $I_m$  and the switch-on time  $T$  in a CO<sub>2</sub> laser with slowly swept pump parameter has been found. Our results can be explained in term of class-B laser equations with suitable approximation in order to explain the linear regime, where the laser switches on, and the highly nonlinear regime, where the laser intensity peak reaches the maximum.

## ACKNOWLEDGMENTS

The authors thank Dr. P. Spano, Dr. A. Mecozzi, Dr. A. Lapucci, Dr. Peng-ye Wang, and Professor J. R. Tredicce for helpful discussions.

\*Also at Department of Physics, University of Florence, Florence, Italy.

- [1] P. Mandel and T. Erneux, *Phys. Rev. Lett.* **53**, 1818 (1984); P. Mandel, *Opt. Commun.* **64**, 549 (1987).
- [2] F. T. Arecchi, W. Gadomski, R. Meucci, and J. A. Roversi, *Opt. Commun.* **70**, 155 (1989); W. Scharpf, M. Squicciarini, D. Bromley, C. Green, J. R. Tredicce, and L. Narducci, *ibid.* **63**, 344 (1987).
- [3] R. Roy, A. W. Yu, and S. Zhu, *Phys. Rev. Lett.* **55**, 2794 (1985); *Phys. Rev. A* **34**, 4333 (1986); F. De Pasquale, J. M. Sancho, M. San Miguel, and P. Tartaglia, *Phys. Rev. Lett.* **56**, 2473 (1986); M. R. Young and S. Singh, *Phys. Rev. A* **35**, 1453 (1987); A. Mecozzi, S. Piazzolla, D. D'Ottavi, and P. Spano, *ibid.* **38**, 3136 (1988).
- [4] G. Broggi, A. Colombo, L. A. Lugiato, and P. Mandel, *Phys. Rev. A* **33**, 3635 (1986); N. G. Stocks, R. Mannella, and P. V. E. McClintock, *ibid.* **40**, 5361 (1989).
- [5] M. C. Torrent and M. San Miguel, *Phys. Rev. A* **38**, 245 (1988); F. T. Arecchi, W. Gadomski, R. Meucci, and J. A. Roversi, *ibid.* **39**, 4004 (1989).
- [6] M. Ciofini, R. Meucci, and F. T. Arecchi, *Phys. Rev. A* **42**, 482 (1990). The active medium volume of the used laser is about 46 cm<sup>3</sup>.
- [7] F. T. Arecchi, V. Degiorgio, and B. Querzola, *Phys. Rev. Lett.* **19**, 1168 (1967); F. T. Arecchi and V. Degiorgio, *Phys. Rev. A* **3**, 1108 (1971).
- [8] P. Spano, A. Mecozzi, and A. Sapia, *Phys. Rev. Lett.* **64**, 3003 (1990); A. Mecozzi, P. Spano, and A. Sapia, *Opt. Lett.* **15**, 1067 (1990).
- [9] S. Balle, P. Colet, and M. San Miguel, *Phys. Rev. A* **43**, 498 (1991).
- [10] Peng-ye Wang, A. Lapucci, R. Meucci, and F. T. Arecchi, *Opt. Commun.* **80**, 42 (1990); E. Arimondo and P. Glorieux, *Appl. Phys. Lett.* **33**, 49 (1978).
- [11] G. L. Oppo, J. R. Tredicce, and L. Narducci, *Opt. Commun.* **69**, 393 (1989).