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Polygon pattern formation in a nonlinear optical system with 2D feedback

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Experimental evidence is presented of polygon-type pattern formation in a 2D feedback system using a nonlinear interferometer consisting of a liquid crystal light valve (LCLV) plus a polarizer and a diffraction length. A mathematical model of this system is also presented.

1. Introduction

Spontaneous pattern formation in nonlinear optical systems is one of the most interesting directions in nonlinear optics. This phenomenon is closely connected to fundamental problems of modern science such as selforganization and spatio–temporal chaos [1–3].

A simple optical scheme consisting of a Kerr slice and feedback mirror can be used to produce rolls and hexagon excitation [4]. Hexagon pattern formation was theoretically investigated using numerical methods by Firth et al. [4,5]. Hexagons and rolls have been observed experimentally in optical systems using a nonlinear Kerr medium [6–8] and in a 2D optical system using a liquid crystal light valve (LCLV) [9,10]. With pattern formation, different boundary conditions lead to the excitation of different polygon types. This has been numerically analyzed in the Kerr slice/feedback mirror system [11].

Here we discuss experimental observations of polygon patterns in a nonlinear interferometer using 2D feedback and a LCLV.

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2. Optical scheme and mathematical model

The experimental set-up is shown in fig. 1. The system we used is a passive nonlinear resonator with 2D feedback, with the key element being the LCLV.

As shown in refs. [9,12], the LCLV can be modeled as a Kerr slice having a large nonlinear parameter of $n_2 \sim 0.1$ cm^2/mW. The long axis of the LC molecules (director) in the LCLV is oriented at an angle α with the input beam polarization (fig. 2). In our experiments $\alpha = \pi/4$.

We represent the complex amplitude of the input wave A_0 as the sum of two components, one along the director and one orthogonal to it, as:

$$A_0 = \boldsymbol{j} A_0^{\parallel} + \boldsymbol{i} A_0^{\perp} ,$$

where $A_0^{\parallel} = A_0 \cos\alpha$, $A_0^{\perp} = A_0 \sin\alpha$ and $(\boldsymbol{i}, \boldsymbol{j})$ are unit vectors.

After reflection from the LCLV mirror and passing twice through the LC layer, the input field component $\boldsymbol{j} A_0^{\parallel}$ (extraordinary wave) acquires the additional nonlinear phase modulation $u(\boldsymbol{r}, t) = 2n_2 lk\Delta n$, which depends on the intensity of the controlling light incident on the LCLV photosensitive layer $I_{FB}(\boldsymbol{r}, t)$ in the plane P_{FB}. Here $k = 2\pi/\lambda$, and $\Delta n = n_{\parallel} - n_{\perp}$ is the variation in refractive index due to illumination of the LCLV's photoconductive layer for ordinary $n_{\perp}$ and extraordinary $n_{\parallel}$ waves. The thickness of the LC layer l is approximately 10 μm;

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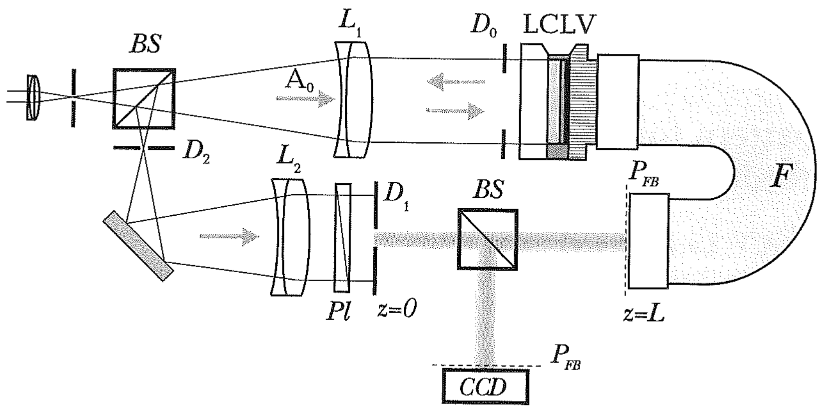

Fig. 1. Experimental set-up. BS is beam splitter, D_0–D_2 are diaphragms, Pl is a polarizer, CCD is the CCD camera, F is an optical fiber bundle and L_1 and L_2 are confocal lenses. The LCLV is placed in the focal plane of lens L_1. Diaphragm D_2 is used for filtering additional reflections from the optical surfaces. LCLV has an applied dc voltage through it. It is composed of (from left to right) a liquid crystal cell, a mirror which reflects back the light entering from L_1, and a photoconductor layer yielding a large or small DC voltage drop depending on whether it receives light from the right through the fiber bundle. Within the spots illuminated from the right, the dc voltage is mostly applied to the liquid crystal, orienting its molecules and thus providing a giant Kerr effect.

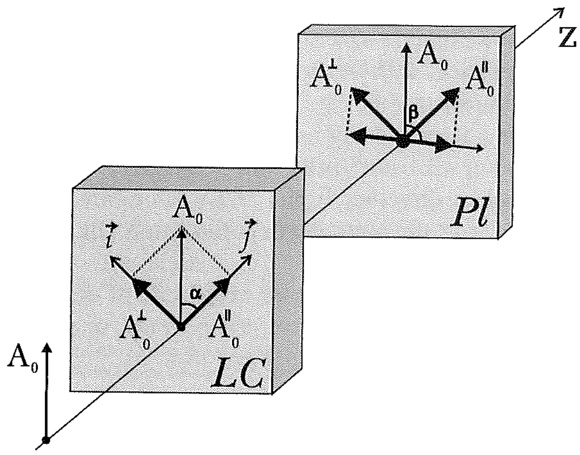

Fig. 2. Geometry of polarization vector in the optical scheme.

therefore we neglect diffractive effects within the LC slice.

The complex amplitude of the field reflected from the LCLV mirror is

$$\boldsymbol{A}_r = \boldsymbol{i}A_0^{\parallel}\exp[\mathrm{i}(u+\phi)] + \boldsymbol{j}A_0^{\perp}\exp(\mathrm{i}\phi)\,,$$

where $\phi = 2lkn_{\perp}$ is the constant phase shift of the ordinary component of the input field.

We use the polarizer P in the optical feedback. The angle between the orientation of the polarizer and the direction of the input beam polarization is β. In our experiments we set $\beta=\pi/2$.

After passing though the polarizer and the diaphragm D_1 the complex amplitude of the field in the feedback circuit becomes:

$$A = \kappa A_0\, t(\boldsymbol{r})\, \{\exp[\mathrm{i}(u+\phi)] - \gamma\exp(\mathrm{i}\phi)\}\,, \qquad (1)$$

where $\kappa=\cos(\alpha)\cos(\beta-\alpha)$, $\gamma=\tan(\alpha)\tan(\beta-\alpha)$ and $t(\boldsymbol{r})$ is the aperture function of the diaphragm. Note that when $\alpha=\pi/4$ and $\beta=\pi/2$, $\gamma=1$ and $\kappa=1/2$.

In the plane of diaphragm ($z=0$) we now have interference between the two beams $\kappa A_0 t(\boldsymbol{r}) \times\exp[\mathrm{i}(u+\phi)]$ and $-\gamma\kappa A_0\, t(\boldsymbol{r})\exp(\mathrm{i}\phi)$, one of which has a controlling phase shift of $u(\boldsymbol{r}, t)$.

The new field formed in the optical feedback loop diffracts after passing through the diaphragm. The length of the optical path from the plane of the diaphragm to the plane of the LCLV's photoconductor layer is $L=1$ m. In our experiments the diaphragm diameter was varied from $d=2.5$ mm down to $d=0.5$ mm. Thus the distance L normalized by the Rayleigh distance $z_d=kd^2$ yields a value $Z=L/z_d$ lying within the interval $[Z_{\min}, Z_{\max}]$, where $Z_{\min}=0.005$ and $Z_{\max}=0.3$.

Free space propagation over the distance $z=L$ is given by

$$-2ik\frac{\partial A}{\partial z}=\nabla_\perp^2 A\ , \qquad (2)$$

where $\nabla_\perp^2$ is the laplacian in the x and y directions. Using the simplest mathematical model for LCLV dynamics [13] nonlinear phase modulation can be expressed by

$$\tau\,\partial u/\partial t+u=l_D^2\nabla_\perp^2 u+K\,|A_{FB}|^2\ . \qquad (2)$$

This equation includes transverse diffusion of photoelectrons in the LCLV photosensitive layer, with a diffusion length l_D, response time τ, and nonlinearity parameter K. The controlling intensity in the plane P_{FB} is

$$|A_{FB}|^2=|A(\boldsymbol{r},z=L,t)|^2\ . \qquad (4)$$

Compare this model to a known mathematical model for a Kerr slice with a feedback mirror [5]. Our model is a generalization of this previous one, since by changing the angle β of the polarizer orientation we can realize the condition $\beta=\alpha$ and $\gamma=0$. This condition indicates a lack of interference, as well as Kerr slice/feedback mirror system (assuming a plane wave input).

3. Experimental results

In our experimental set-up we use an argon laser ($\lambda=514.5$ nm). The power of the input laser beam is 40 mW. The influence of boundary conditions on the pattern formation in nonlinear interferometer with diffraction was investigated for two situations (fig. 1):

(a) Diaphragm D_1 is open and diffraction effects are only due to diaphragm D_0.

(b) Diaphragm D_0 is open and all effects are con-

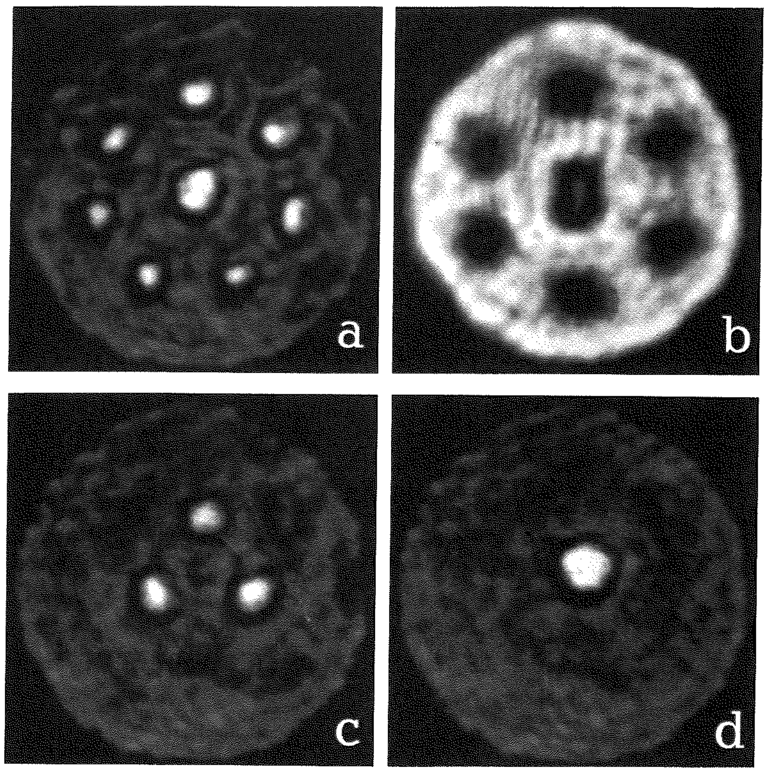

Fig. 3. Patterns as formed in the nonlinear interferometer with 2D feedback and diffraction. Diaphragm D_0 is open, and the diameter d of the diaphragm D_1 is varied. (a) $d=2.1$ mm, $Z=0.019$, (b) $d=1.8$ mm, $Z=0.025$, (c) $d=1.2$ mm, $Z=0.057$, (d) $d=0.8$ mm, $Z=0.13$. The intensity reversal in case (b) is related to uniform phase shifts.

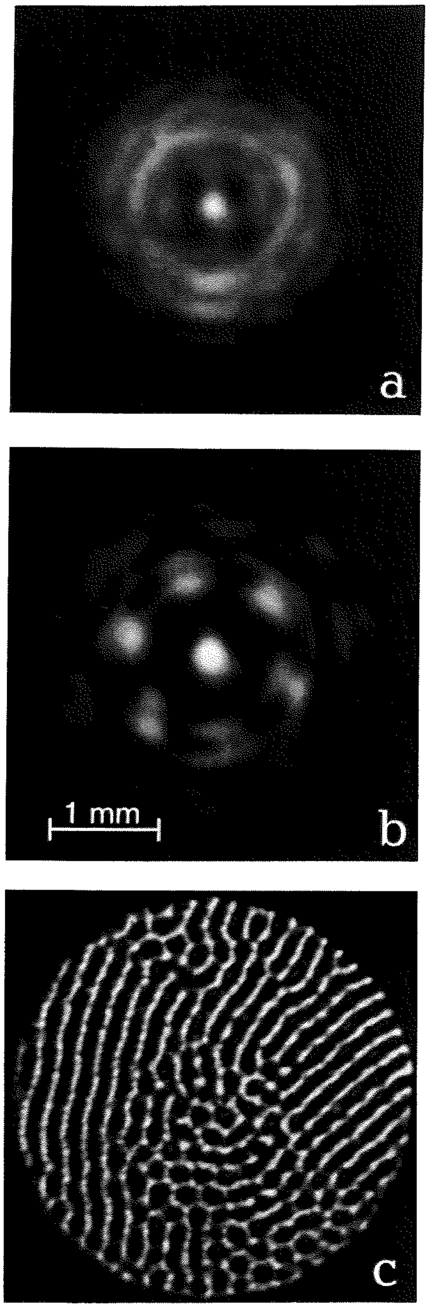

Fig. 4. Hexagonal patterns and competition of hexagons and rolls. Diaphragm D_1 is open. (a) $d=1.8$ mm, $Z=0.025$, closed feedback (i.e., opaque screen placed in the plane P_{FB}), (b) $d=1.8$ mm, $Z=0.025$, open feedback, (c) $d=10$ mm, $Z=0.0008$, open feedback.

nected with diffraction of the laser beam on diaphragm D_1.

Similar results were obtained for both cases, although a larger variety of polygon patterns were seen with case (b).

Changing the spatially uniform phase modulation in the interferometer with diffraction offers an additional way of controlling pattern formation. This can be achieved by varying the voltage applied to the LCLV.

Figure 3 shows a selection of polygon-type patterns we observed in our experiments. By varying the LCLV voltage the intensity of the polygon patterns was reversed (compare fig. 3b and figs. 3a, 3c and 3d), just due to a uniform phase shift.

The order of the polygon pattern (i.e., the number of spots) depends on the diffractive parameter $Z=L/(kd^2)$ and on the intensity distribution of the laser beam in the plane of the diaphragm. Some of the polygon-type patterns (for example hexagons and heptagons) are formed by small variations in intensity distribution.

Polygon formation is sensitive to diaphragm quality and system adjustment. Patterns in the plane of the LCLV's photosensitive layer are shown in fig. 4 for closed (a) and opened (b) feedback. Imperfections in the diaphragm and a nonuniform intensity distribution of the input beam lead to azimuthal asymmetry in the beam's diffractive pattern (fig. 4a), which in turn causes one of several possible polygon patterns to appear (fig. 4b). By opening the diaphragm it is possible to excite rolls having different types of defects (fig. 4c). The excitation of rolls in the external feedback loop scheme may be associated with small transverse shifts of the beam due to field rotation, as caused by reflections from mirrors in the optical feedback loop. To decrease this effect we use an optical fiber bundle with rotatable end in the feedback loop (fig. 1). However, aberrations in the optical system itself can also cause small transverse shifts, thus preventing the formation of periodic hexagonal structures on large apertures.

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