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ADAPTIVE CONTROL OF CHAOTIC AND HYPERCHAOTIC DYNAMICS

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Exploiting the information provided by the local variation rates of a dynamical system, we introduce an adaptive technique able to recognize and stabilize the unstable periodic orbits embedded in a chaotic attractor, based on additive corrections to the dynamics whose relative weight is adjusted to the size of the local expansion rate. Application of this method to chaotic and hyperchaotic systems is discussed.

1. Introduction

In this paper we report the results of recent studies [1, 2], which have introduced an adaptive strategy for recognition and control of chaos in dynamical systems, which implements a stroboscopic inspection of the system through an adaptive windowing controlled by the local variation (expansion or contraction) rates. This not only provides useful indicators for chaos recognition, but it can also be applied for chaos control, even for systems displaying more than one positive Lyapunov exponent (hyperchaos).

Controlling chaos consists in perturbing the considered systems in order to stabilize a given unstable periodic orbit (UPO) embedded in the chaotic attractor [3].

Many theoretical methods for controlling chaos have been proposed, based on slight readjustment of a control parameter each time the trajectory crosses the Poincaré section (PS) [4]; or on periodic [5, 6] and stochastic [7] perturbations to the system; or on a continuous application of a delayed feedback term [8].

On the other hand, experimental chaos control has been achieved in many circumstances [9 – 13].

Here we review a new method whose influence holds upon time scale T_s intermediate between the short resolution T_1 of continuous methods [8] and the long one T_2 corresponding to the delay between two successive PS crossings used in Ref. [4].

2. The Adaptive Algorithm

Let us consider a general dynamical dissipative system ruled by

$$\dot{\mathbf{x}} = \mathbf{G}(\mathbf{x}, \mu), \tag{1}$$

where $\mathbf{x}$ is a D-dimensional vector, $\mathbf{G}$ is a nonlinear function and μ is a set of control parameters chosen in such a way as to produce chaos.

For each component i ($i = 1, 2, \ldots, D$) of x and at the time $t_{n+1} = t_n + \tau_n$ (τ_n being the adaptive observation interval to be specified later), one defines the variation over n-th observation time interval τ_n as

$$\delta x_i(t_{n+1}) = x_i(t_{n+1}) - x_i(t_n), \tag{2}$$

and the local variation rates λ of δx_i as

$$\lambda_i(t_{n+1}) = \frac{1}{\tau_n} \log \left| \frac{\delta x_i(t_{n+1})}{\delta x_i(t_n)} \right| . \tag{3}$$

By use of rates λ, it is possible to fix the new observation at the time $t_{n+2} = t_{n+1} + \tau_{n+1}$, where $\tau_{n+1} = \min_i \tau_{n+1}^{(i)}$ and

$$\tau_{n+1}^{(i)} = \tau_n^{(i)} [1 - \tanh(g\lambda_i(t_{n+1}))] . \tag{4}$$

Equation (4) arises from the following considerations. In order to obtain a series of δx_i ranging over a small interval, we contract (expand) the observation time interval whenever the actual value of δx_i is greater (smaller) than the previously observed one. $g > 0$ represents the sensitivity of the method. A complete discussion on the algorithm and on the criteria for the choice of g is provided in Ref. [1].

The sequence of stroboscopic times (obtained starting from t_0 and $\tilde{\tau}$) t_0, $t_1 = t_0 + \tilde{\tau}$, $t_2 = t_1 + \tau_1$, ..., $t_{n+1} = t_n + \tau_n$, ...now contains the relevant information on the dynamics. Thus, characterization and recognition of chaos can be done by the study of such a time sequence.

In the following we will summarize the application of such a method to the chaotic LORENZ (Lo) [14] model and to the hyperchaotic four-dimensional ROESSLER (Ro4) [15] model.

3. Recognition and Stabilization of UPO's

Here we are intrested in stabilizing a periodic dynamics, so that we need to extract the periods of UPO's embedded in the chaotic attractor. For this purpose, we construct the maps τ_{n+k} vs. τ_n, $k = 1, 2, \ldots$ and we plot the r.m.s. η of the point distribution around the diagonal of such maps as functions of the step interval k.

Since for a chaotic dynamics temporal self-correlation lasts only for a finite time, one should expect to obtain a monotonically increasing function. In fact the chaotic dynamics brings the trajectory in the phase-space to shadow neighbourhoods of different UPO's. As the trajectory gets close to an UPO of period T_j, temporal self-correlation is rebuilt after T_j and the distribution of τ includes windows of correlated values appearing as minima of η vs. k around $k_j = T_j / <\tau>$, $<\tau>$ being the average of the τ distribution.

To give an example, we report in Fig. 1 the $\eta - k$ plot for Ro4, from which one can extract the different UPO's periods looking for the minimum of a suitable cost function introduced in Ref. [2] around each k_j.

When periods T_j ($j = 1, 2, \ldots$) of the UPO's have been measured, stabilization of each one can be achieved when the system naturally shadows neighbourhoods of that UPO.

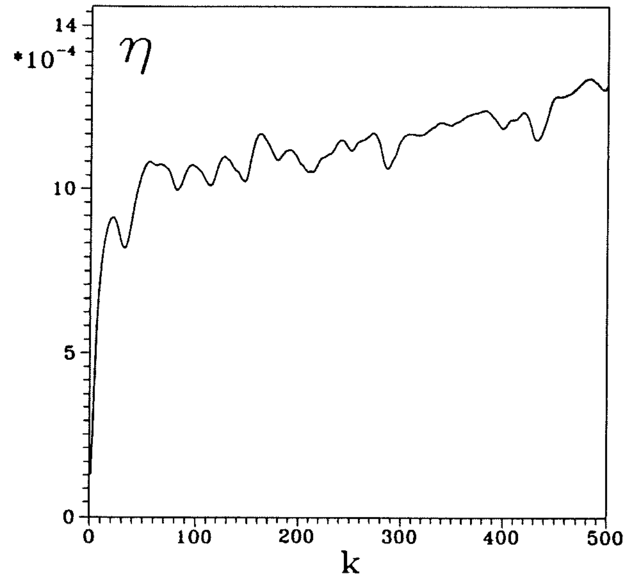

FIG. 1. $\eta - k$ plot for Ro4 attractor. Ro4 is described by $\dot{x}_1 = -x_2 - x_3$, $\dot{x}_2 = x_1 + 0.25x_2 + x_4$, $\dot{x}_3 = 3 + x_1x_3$, $\dot{x}_4 = -0.5x_3 + 0.05x_4$. For initial conditions $x_1(0) = -20$, $x_2(0) = x_3(0) = 0$, $x_4(0) = 15$, Ro4 produces a hyperchaotic dynamics with two positive Lyapunov exponents [15]. The recognition task has been performed with $g = 0.01$. Data on vertical axis have to be multiplied by 10^{-4}.

The control procedure is done by use of the following modified algorithm. At each new observation time $t_{n+1} = t_n + \tau_n$ and for each component i of the dynamics, instead of Eq. (2), we evaluate the differences δ between actual and desired values:

(7) $$\delta x_i(t_{n+1}) = x_i(t_{n+1}) - x_i(t_{n+1} - T_j),$$

and the local variation rates λ now read

(8) $$\lambda_i(t_{n+1}) = \frac{1}{\tau_n} \log \left| \frac{x_i(t_{n+1}) - x_i(t_{n+1} - T_j)}{x_i(t_n) - x_i(t_n - T_j)} \right| .$$

Equation (4) and choice of the minimum are kept for the updating process of τ.

Defining $\mathbf{U}(t)$ as the vector with i-th component (constant over each observation time interval) given by

(9) $$U_i(t_{n+1}) = \frac{1}{\tau_{n+1}} \Big(x_i(t_{n+1} - T_j) - x_i(t_{n+1}) \Big),$$

we add such a vector to the evolution equation, which now reads

(10) $$\frac{d\mathbf{x}}{dt} = \mathbf{G}(x, \mu) + \mathbf{U}(t).$$

Now, λ are measuring how the separation of actual trajectory from the desired one evolves; indeed, λ negative (positive) means that locally the true orbit is collapsing into (diverging away) the desired one.

As a consequence, contraction or expansion of τ now reflects the necessity to perturb the dynamics more or less often in order to stabilize the desired UPO, as well as it

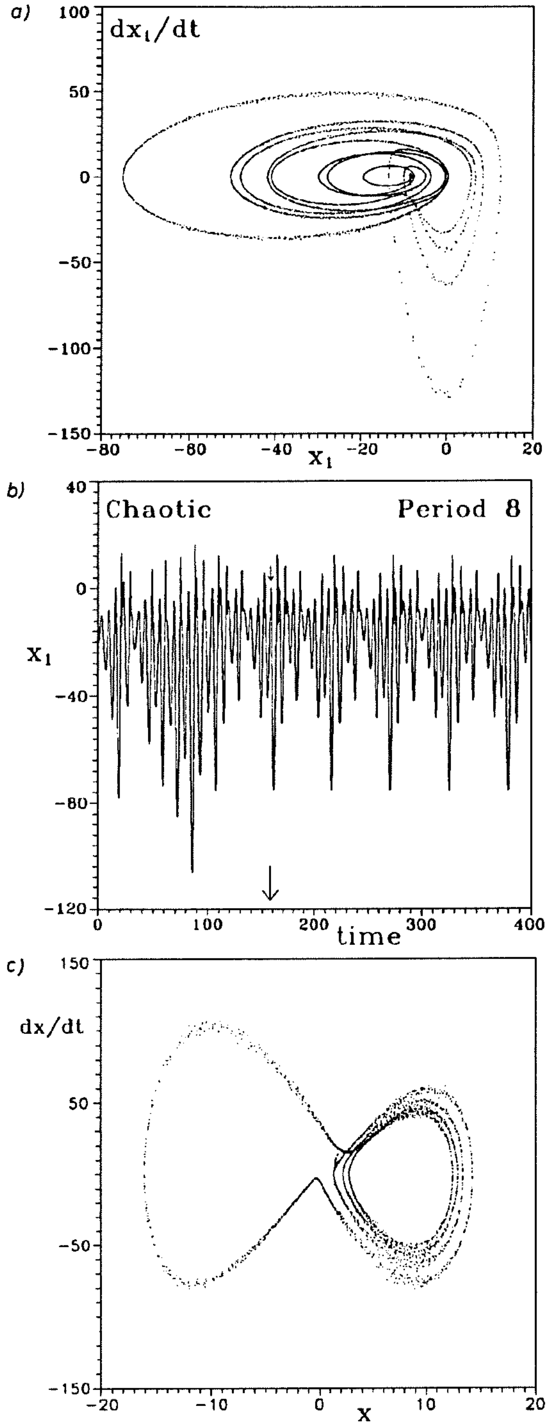

FIG. 2. a) $(x_1, \dot{x}_1)$ representation for the controlled period 8 of Ro4 attractor. Control task has been performed with period-8 extracted from Fig. 1 and $g = 10^{-5}$. b) Time evolution of the first component x_1 of Ro4 before and after control. Arrows indicate the instance at which control task begins. c) $(x, \dot{x})$ representation for the period 5 of Lo ($\sigma = 10$, $b = 8/3$, $r = 25$). In this latter case control has been switched on when the distance between true orbit and desired period was quite large in order to highlight the shadowing process.

fixes the weight of the correction to be done in order to constrain the true orbit to continue shadowing the desired UPO. Notice that, once a given T_j has been chosen by the operator, the weight to the perturbation in Eq. (9) is selected by the same adaptive dynamics.

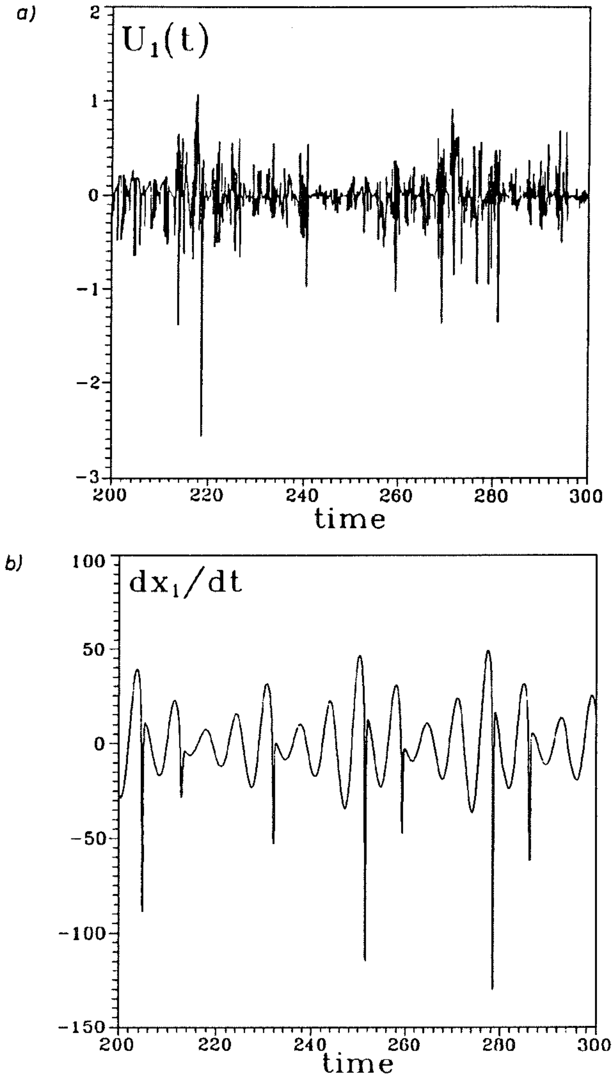

FIG. 3. a) Temporal evolution of the first component of the additive controlling term U_1 during the control of period 8 of Ro4, and b) temporal evolution of the uncontrolled dx_1/dt. The adaptive correction term is between two and three orders of magnitude smaller than the natural evolution of the dynamics. Same stipulation for controlling task as mentioned in the caption of Fig. 2.

In Fig. 2 we show the control of period-8 of Ro4 and of period-5 of Lo. In the latter case a bigger initial δ has been selected in order to highlight the shadowing process; in Fig. 3 we report the perturbation $U_1(t)$ and the unperturbed dynamics G_1 for the Ro4 model during the control task of period-8, in order to show that the former is between two and three orders of magnitude smaller than the latter, as assured by the adaptive weighting procedure.

As for time scales, notice that while in Eq. (7) differences δx_i are evaluated with respect to the goal dynamics (thus over the period T_j), in Eq. (8) all λ are evaluated over the adaptive τ, which, as discussed in Ref. [1], has to be much larger than the Runge-Kutta integration interval (about 100 time larger) but much smaller than the UPO's period (as it is evident from Fig. 1 where all k_j are much above the unity). This way the method introduces a natural adaptation time scale intermediate between minimum resolution time and time scale of the periodic orbits.

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References

1. F.T. Arecchi, G. Basti, S. Boccaletti and A.L. Perrone, Europhys. Lett., **26**, 327, 1994.
2. S. Boccaletti and F.T. Arecchi, Europhys. Lett., **31**, 127, 1995.
3. D. Auerbach, P. Cvitanovic, J.-P. Eckmann, G. Gunaratne and I. Procaccia, Phys. Rev. Lett., **58**, 2387, 1987.
4. E. Ott, C. Grebogi and J.A. Yorke, Phys. Rev. Lett., **64**, 1196, 1990.
5. R. Lima and M. Pettini, Phys. Rev., **A41**, 726, 1990; Y. Braiman and I. Goldhirsch, Phys. Rev. Lett., **66**, 2545, 1991.
6. A. Azevedo and S.M. Rezende, Phys. Rev. Lett., **66**, 1342, 1991.
7. H. Herzel, A. Angew. Math. Mech., **68**, 11, 1988; S. Fahy and D.R. Hamann, Phys. Rev. Lett., **69**, 761, 1992.
8. K. Pyragas, Phys. Lett., **A170**, 421, 1992.
9. E.R. Hunt, Phys. Rev. Lett., **67**, 1953, 1991.
10. J. Singer, Y.-Z. Wang and H.H. Bau, Phys. Rev. Lett., **66**, 1123, 1991.
11. R. Roy, T.W. Murphy Jr., T.D. Maier, Z. Gills and E.R. Hunt, Phys. Rev. Lett., **68**, 1259, 1992.
12. B. Peng, V. Petrov and K. Showalter, J. Phys. Chem., **95**, 4957, 1991; V. Petrov, V. Gaspar, J. Masere and K. Showalter, Nature, **361**, 240, 1993.
13. R. Meucci, W. Gadomski, M. Ciofini and F.T. Arecchi, Phys. Rev., **E49**, R2528, 1994.
14. E.N. Lorenz, J. Atmos. Sci., **20**, 130, 1963.
15. O.E. Roessler, Phys. Lett., **71A**, 155, 1979.

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