Pattern dynamics in a large Fresnel number laser close to threshold

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A high Fresnel number laser close to threshold can be set at a detuning value where it displays only the first transverse pattern with zero intensity in its center. However, the availability of a large phase space gives rise to a new type of dynamics. Precisely, for this highly symmetric situation, the intensity pattern undergoes a transition between stable and oscillating superpositions of traveling and standing waves. Such a transition corresponds to the so-called Takens-Bogdanov bifurcation. We report experimental evidence of such a behavior. [S1050-2947(97)01409-1]

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I. INTRODUCTION

The growing interest in nonlinear dynamics and pattern formation in optical systems [1,2] has led to careful investigations on lasers operating at high Fresnel numbers. From a theoretical point of view, different models, originated from the standard Maxwell-Bloch scheme, have been proposed to describe spatiotemporal instabilities, such as the Kuramoto-Shivasinsky [3] and the Swift-Hohenberg models [4,5].

From an experimental point of view, several difficulties arise when a detailed investigation of patterns is required [6–9]. In particular, when the spatial structures include points of zero intensity or topological defects [10,11], a high space resolution of the detecting apparatus is required. In this framework, the availability of different methods for optical detection and their comparison can increase the reliability of the measurements.

In this paper we report on the dynamics of pattern formation in a highly symmetric CO_2 laser. Even though the large Fresnel number allows for high order modes, one can select the first transverse profile containing a stationary intensity zero (phase singularity) in its center by working close to threshold and adjusting the cavity detuning. We characterize the dynamics by measurements of both the spatial profiles and the temporal evolution of the intensity at a given point, showing that the main features can be described in terms of the interaction between two azimuthal traveling waves. In particular, the spatial pattern undergoes a transition between stable and oscillating regimes, corresponding to the so-called Takens-Bogdanov bifurcation [12–14]. The aperiodic character of the oscillations can be explained in terms of the noise due to spontaneous emission. From a dynamical point of view, the noise acts as an external signal breaking the temporal invariance of the system.

In Sec. II we describe the experimental setup and present the measurements which justify our claim for a Takens-Bogdanov phenomenon. To our knowledge, this is the first evidence of such a behavior in optics.

In Sec. III we discuss the theoretical model which supports the experimental findings and explain in detail the role of noise in coupling two disjoint parameter regions.

II. THE EXPERIMENT

The experimental setup (Fig. 1) consists of a laser discharge tube of 40 cm length with internal diameter 26 mm, closed at one end by a flat total reflective mirror $M1$ with 38 mm diameter, and at the other end by an antireflection coated ZnSe flat window W (99.5% transmittance at normal incidence). The electrodes are specifically designed so as to preserve the cylindrical geometry along the discharge column. The optical cavity, 57.5 cm long, is completed by a Ge outcoupler $M2$ (25 mm diameter, 5 m radius of curvature, 90% reflection) mounted on a piezo translator (PZT) which controls the cavity length (0.5 μm/volt). With this configuration, the Fresnel number is above 100, thus ensuring highly symmetric conditions in the central part of the resonator occupied by the laser beam. The gas mixture is 82% He, 13.5% N_2, and 4.5% CO_2 and the flow is maintained at an average pressure of about 23 mbar. The medium is pumped by a dc discharge provided by a current stabilized power supply with a residual ripple of 0.01% and stability of 0.02% per hour. In the following, we define the normalized pump parameter as $P = I/I_{\mathrm{th}}$, that is, as the ratio between the operating discharge current and the corresponding value for which the observed mode is at threshold.

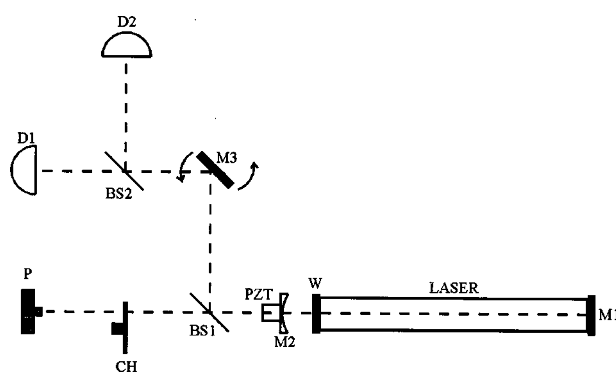

FIG. 1. Experimental setup. $M1$: total reflective mirror; W: ZnSe antireflection window; $M2$: Ge outcoupler mirror mounted on a piezo translator (PZT); BS1 and BS2: ZnSe beam splitters; CH: mechanical chopper; P: pyroelectric array detector; $D1$ and $D2$: single element $Hg_xCd_{1-x}Te$ detectors.

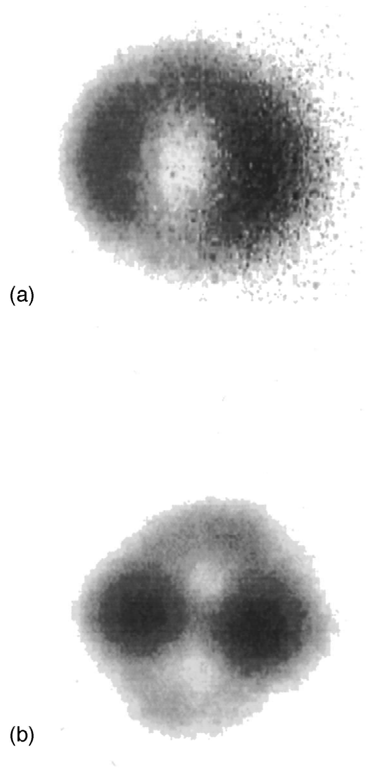

FIG. 2. Spatial patterns observed on an infrared image plate for two slightly different values of the cavity detuning.

The output laser beam is split by a ZnSe 50% beam splitter BS1. One beam goes to a linear pyroelectric array P (Spiricon LP-256-11), composed of 256 elements spaced (center to center) by 100 μm, for precise measurements of the beam transverse profile. The other beam is sent to a horizontal scanning mirror $M3$ and then divided by another 53% ZnSe beam splitter BS2; the two rays are sent to two single-element (100×100 μm) $Hg_xCd_{1-x}Te$ fast detectors ($D1$ and $D2$) that can be micrometrically translated perpendicularly to the beams. By this arrangement, one can measure the spatial profiles of the beam as $M3$ rotates; if $M3$ is stopped, one can detect the temporal evolution of the laser intensity at a single point of the pattern.

With the above configuration and fixed cavity detuning, controlled by the PZT voltage, it is possible to select several different patterns, including the fundamental Gaussian mode. We focus our attention on the onset of the first off-axis transverse mode (Fig. 2), which occurs at a detuning of about 28 MHz from the fundamental mode, in good agreement with the bare cavity frequency separation between two successive transverse modes with the same longitudinal index.

In Fig. 3 we compare three different methods of detecting the spatial profile. Figure 3(a) shows the beam profile as obtained by the linear pyroelectric array, while Fig. 3(b) reports the beam profile as obtained by using the scanning mirror and $D1$ with a 200 μm pinhole placed just in front of the detector window. The role of the pinhole is crucial to avoid undesired effects due to stray reflections inside the housing of the sensitive element. Such effects may lead to dramatic alterations of the shape of the effective profile, inducing integration of the signal modulation and the disappearance of the central zero as shown in Fig. 3(c). We have also verified that further reduction of the pinhole diameter up to 50 μm does not lead to any relevant change in the shape of the detected profiles, thus indicating that the resolution of Figs. 3(a) and 3(b) is sufficient.

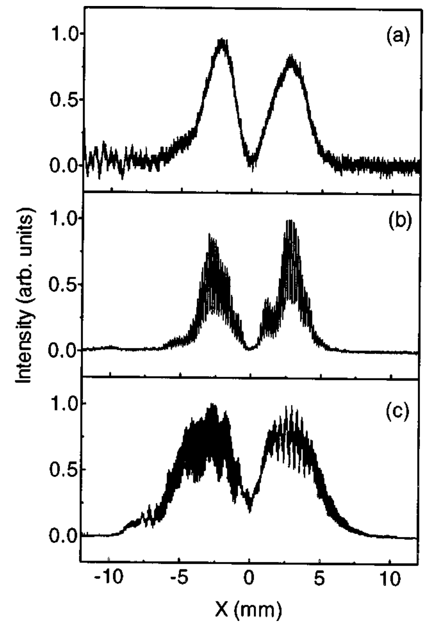

FIG. 3. Experimental transverse profiles of the pattern of Fig. 2(a) detected by (a) the linear pyroelectric array, (b) the $Hg_xCd_{1-x}Te$ detector with a 200 μm pinhole, and (c) the $Hg_xCd_{1-x}Te$ detector without pinhole.

Figure 4 shows that the intensity is zero in the central part of the three profiles corresponding to discharge currents of 2.21, 2.37, and 2.65 mA, that is, to above threshold conditions of 5%, 13% and 26%, respectively, since the threshold current is $I_{th} = 2.10$ mA. Moving far from threshold ($I > 2.80$ mA), the central zero is no longer maintained, but there is still a deep depression at the center with an increasing number of rings.

Another important property of the observed pattern concerns the presence of temporal oscillations (never observed on the fundamental Gaussian mode), clearly shown in Fig. 3(b). If a certain value of the cavity detuning is overcome (after a shift of about 3 MHz), these oscillations disappear

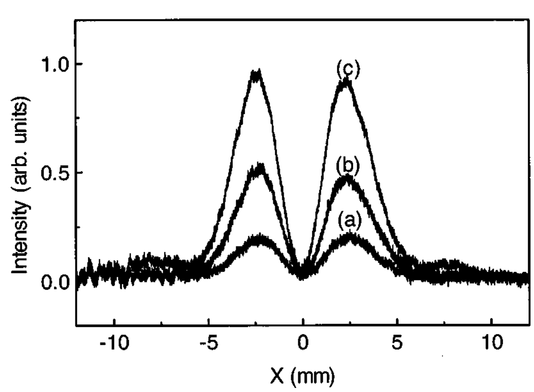

FIG. 4. Pattern profile recorded by the pyroelectric array as a function of the discharge current: (a) 2.21 mA, (b) 2.37 mA, (c) 2.65 mA.

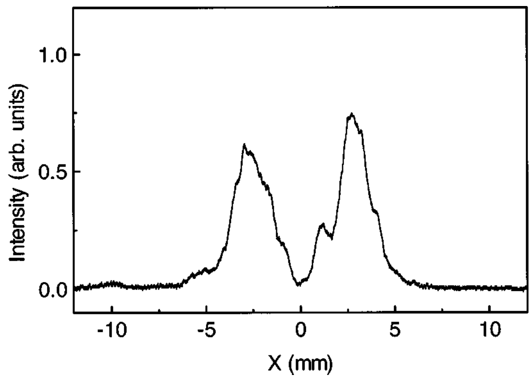

FIG. 5. Experimental transverse profile obtained as in Fig. 3(b) and showing the absence of temporal oscillations.

completely. The signal recorded by the detector $D1$ in this condition is shown in Fig. 5. Notice that the spatial patterns corrresponding to Figs. 5 and 3(b) are those of Figs. 2(b) and 2(a), respectively. To further characterize this phenomenon, we have blocked $M3$ and placed $D1$ and $D2$, with the 200 μm pinholes, in correspondence to the two peaks of the transverse profile (continuously monitored through the pyroelectric array), where the intensity fluctuations are larger (although they are present throughout the whole pattern). The detected signal for $I=2.21$ mA (5% above threshold) is strongly aperiodic and the corresponding power spectrum (Fig. 6) displays a single broad peak, in the range 200–400 kHz; numerical analysis indicates that the fractal dimension estimated following the method introduced by Grassberger and Procaccia [15] does not saturate when the embedding dimension is increased up to 12, thus revealing the presence of noise.

By micrometrically tilting BS2 when $M3$ is moving, it has been possible to perform several horizontal sections of the mode at different heights. These measurements show that the laser intensity, even when oscillating, never reaches zero (Fig. 7), thus revealing the absence of a pure standing wave in the transverse section, at variance with a previously reported situation [11]. Moreover, all the above mentioned results remain substantially unchanged if a polarizer is inserted just at the appropriate angle on the beam output. This fact proves that the experiment can be explained in terms of a linearly polarized electric field.

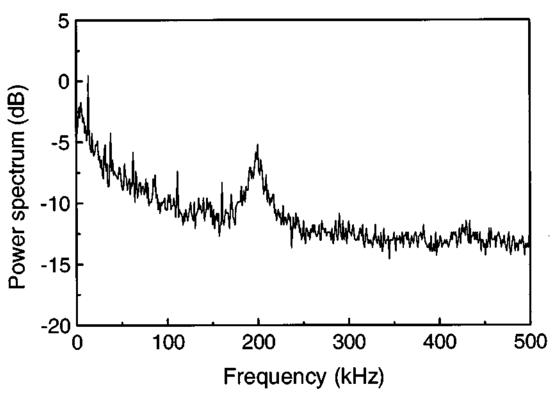

FIG. 6. Power spectrum of the local laser intensity.

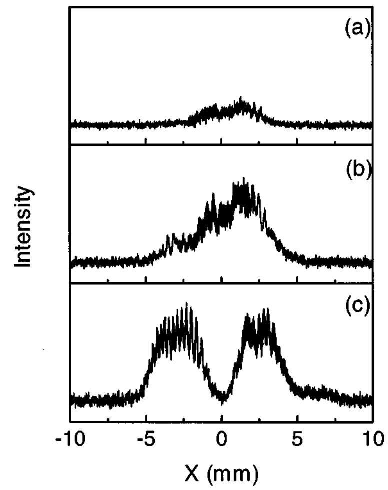

FIG. 7. Horizontal sections of the pattern of Fig. 4(a) performed at different heights: (a) on the top edge, (c) on the diameter, and (b) in the middle between (a) and (c).

Hence the main feature of the experiment is that, even close to threshold and below a given cavity detuning, aperiodic fluctuations appear at any point of the pattern, except in the center (where the zero intensity state is stationary), while the temporally averaged spatial pattern remains highly symmetric. The transition between stationary and oscillating behaviors is a signature of the so-called Takens-Bogdanov bifurcation [12–14], which has already been reported in fluid dynamics, in a Bénard-Marangoni convection experiment [16].

III. THE THEORETICAL MODEL

In order to provide a theoretical background for the above observations, we note that temporal oscillations in the range of hundreds of kilohertz may suggest a coupling with the population inversion. This fact has been considered in a recent report [11] showing regular oscillations, but for conditions well above threshold, and on both sides of the resonance curve. In our case, fluctuations are observed very close to threshold, where the coupling with population is not much relevant since the laser output is far below the saturation intensity [17], and only on one side of the resonance curve. On the other hand, the presence of a central zero in the intensity pattern does not allow us to interpret our experiment in the framework of a bifurcation from a homogeneous nonzero solution [6]. Thus we consider the bifurcation from the homogeneous zero intensity solution toward a pattern containing a stationary intensity zero at its center [12,13]. In this case, the cavity field can be expressed in terms of the modes arising from a Hopf bifurcation from the homogeneous zero solution:

$$E=P(r)L(z)e^{i\omega t}(z_1e^{im\theta}+z_2e^{-im\theta}), \tag{1}$$

where $P(r)$ $[L(z)]$ accounts for radial (longitudinal) dependence of the bifurcating modes, ω is the temporal frequency of the modes, and m denotes the angular momentum and θ the angular variable. According to the selected detuning we can set $m=1$ and neglect the presence of the Gaussian mode, since a single radial maximum (as reported in Figs. 3, 4) does not allow $m>1$ [18]. Starting from the Maxwell-Bloch description, it is possible to derive the following equations for the complex amplitudes [11–13]:

$$\dot{z}_1=\mu z_1-A(z_1z_1^*+2z_2z_2^*)z_1+\epsilon z_2,$$
$$\dot{z}_2=\mu z_2-A(z_2z_2^*+2z_1z_1^*)z_2+\epsilon z_1, \quad (2)$$

which represent an imperfect $O(2)$ symmetry, the parameter ϵ accounting for imperfections in the optical cavity that break the rotational symmetry.

References [12] and [13] show in great detail how Eqs. (2), under suitable assumptions, lead to the normal form of the so-called Takens-Bogdanov bifurcation. We herewith summarize that approach. Equations (2) can be recast in terms of the variables $(\rho_1,\rho_2,\delta,\psi)$, where $z_i=\rho_i e^{i\phi_i}$, $\delta=\phi_2-\phi_1$, $\psi=\phi_2+\phi_1$. Due to the symmetries of the problem, the equations for ρ_1,ρ_2, and δ constitute a closed set of three ordinary differential equations. The primary bifurcation from $\rho_1=\rho_2=0$ leads to two standing waves $(\rho_1=\rho_2)$ which are stable, provided $|\mathrm{Re}(\mu)|\leqslant|\epsilon|$, i.e., close to threshold. Increasing the gain to $|\mathrm{Re}(\mu)|>|\epsilon|$ new solutions appear. The relevant motion in the phase space is approximately confined over the surface $B^2=\rho_1^2+\rho_2^2=$const, which is stable since the corresponding eigenvalue is $\lambda=-2[\mathrm{Re}(\mu)\pm\mathrm{Re}(\epsilon)]$ (the $\pm$ refers to the expansion in proximity of the two standing waves). The reduced two-dimensional model can be written in the so-called Takens-Bogdanov form (TBF, Eqs. (16) and (17) of Ref. [13]). In a certain region Σ of the parameter space close to a double-zero eigenvalue, the TBF presents traveling wave solutions $(\rho_1\neq\rho_2)$ which undergo a transition, called the Takens-Bogdanov bifurcation [12,13], from stable to periodic oscillating regime. Moreover, for parameters in Σ, it has also been proved [12] that the regular oscillations become chaotic if a very small phase symmetry breaking term (say k) is added. Indeed, although the manifold $B=$const remains stable, the dynamics is strongly perturbed by k, which couples again the original set (ρ_1,ρ_2,δ) with ψ so that the system evolves in a three-dimensional phase space. As a matter of fact, the saddle loops persist in the limit $k\rightarrow 0$ and $\epsilon\neq 0$, but in a region of the parameter space which is not accessible to a real system [12].

So far we have summarized the approach of Ref. [13]. In our case, accounting for spontaneous emission [19], we should perturb Eqs. (2) with noise terms as

$$\dot{z}_1=\mu z_1-A(z_1z_1^*+2z_2z_2^*)z_1+\epsilon z_2+k\xi_1,$$
$$\dot{z}_2=\mu z_2-A(z_2z_2^*+2z_1z_1^*)z_2+\epsilon z_1+k\xi_2, \quad (3)$$

where $\langle\xi_i(t)\rangle=0$ and $\langle\xi_i(t),\xi_j(t')\rangle=\delta_{(i,j)}(t-t')$ ($\langle\cdots\rangle$ denotes an ensemble average). This noise perturbation clearly breaks the phase symmetry of the system, and thus it can induce large fluctuations in proximity of the bifurcation. Since in the experiment we do not detect standing waves, we argue that the residual asymmetries of the laser are so small that it never gets into the region $|\mathrm{Re}(\mu)|\leqslant|\epsilon|$ and it always stays at $|\mathrm{Re}(\mu)|>|\epsilon|$, even for the smallest gain that can be set in the experiment. Indeed, we have verified that the standing waves corresponding to the $\mathrm{TEM}_{0,1}$ and $\mathrm{TEM}_{1,0}$ modes can be easily induced in our laser if a pin is inserted radially in the beam region inside the cavity, thus inducing a large symmetry break ϵ.

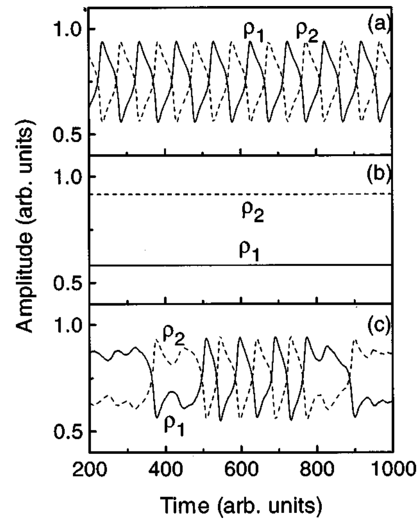

FIG. 8. Solutions of the theoretical model (3), without and with noise. Temporal evolution of the two amplitudes ρ_1 (full line) and ρ_2 (dashed line) for (a) $k=0, A=1.0+i0.4554$, (b) $k=0, A=1.0+i0.4669$, and (c) $k=5\times10^{-4}, A=1.0+i0.4554$.

Figure 8 shows the result of numerical simulation of Eqs. (3) (performed with appropriate algorithms [20]) in a parameter region corresponding to Σ, with $\mu=1.4489$ and $\epsilon=0.2448+i0.4360$. In Fig. 8(a), with $A=1.0+i0.4554$ and $k=0$, we see periodic oscillations of the amplitudes ρ_1 and ρ_2, while Fig. 8(b) corresponds to the stable regime (for $A=1.0+i0.4669, k=0$). In both cases $\rho_1\neq\rho_2$ so that the solution is a traveling wave with and without oscillations, respectively. Figure 8(c) shows the dramatic effect introduced by a small amount of noise when $k=5\times10^{-4}$ [the ratio μ/k is consistent with previous works on CO_2 lasers [19], and the other parameters are the same as for Fig. 8(a)]. In this last case, the signal becomes strongly aperiodic (as in the experiment) and the power spectrum displays a broad peak centered around a frequency which is about $\mu/130$: this is in agreement with the experiment where the oscillation frequency is about hundreds of kilohertz, that is, two orders of magnitude smaller than the rescaled gain parameter $G\sim 10$–100 MHz, roughly corresponding to the resonator buildup time.

IV. CONCLUSIONS

We have shown that in a highly symmetric laser the intensity pattern associated with the lowest order transverse mode undergoes a transition between stable and oscillating superpositions of standing and traveling waves. Such a transition is the Takens-Bogdanov bifurcation when the dynamics is confined to a two-dimensional subspace. The aperiodic character of the oscillations is explained by adding noise terms that account for spontaneous emission. From a physical point of view, the noise acts as an external signal with all frequencies at random phases, thus breaking the temporal invariance of the system without inducing any locking between the modes. This model is valid also for class A lasers, which represent the limit of class B lasers near threshold, where the laser intensity is much smaller than the saturation value.

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