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CONTROL OF AMPLITUDE TURBULENCE IN DELAYED DYNAMICAL SYSTEMS

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An experiment on Bénard–Marangoni time-dependent convection shows evidence of an amplitude turbulent regime in the temperature signal which is modeled by a delayed dynamical system. Application of a control procedure, which perturbs the value of the delay time, leads to the control of such dynamical regime, by suppression of phase defects and stabilization of the regular oscillations. The control technique is robust against the presence of large amounts of noise.

1. Introduction

The original idea of Ott, Grebogi and Yorke [Ott *et al.*, 1990] on chaos control has generated many different theoretical schemes and experimental applications facing the problem of controlling unstable periodic orbits (UPO's) in chaotic concentrated systems (CS), that is in systems modeled by ordinary differential equations.

Controlling spatially extended systems (ES), i.e. systems ruled by partial differential equations whose order parameter y is a m-dimensional vector ($m \geq 1$) in phase space, with k components ($k \geq 1$) in real space, is still an open problem. Even though some proposals have been put forward for the case $k = 2$ [Lu *et al.*, 1996], experimentally implementable tools have not yet been introduced for the control of unstable periodic patterns (UPP) in extended systems.

The link between CS and ES is provided by delayed dynamical systems (DS), i.e. systems ruled by

$$\dot{y} = \mathcal{F}(y, y_d)\,, \tag{1}$$

$y = y(t)$, dot denotes temporal derivative, $\mathcal{F}$ is a nonlinear function, $y_d \equiv y(t - T)$, and T is a time delay.

The evidence of the analogy between DS and ES was given experimentally for a CO_2 laser with delayed feedback [Arecchi *et al.*, 1992] and supported by a theoretical model [Giacomelli & Politi, 1996].

The DS to ES conversion is based on a two variable time representation, defined by $t = \sigma + \theta T$, where $0 \leq \sigma \leq T$ is a continuous space-like variable and θ is a discrete temporal variable [Arecchi *et al.*, 1992]. In this framework, the long range interactions introduced by the delay can be reinterpreted as short range interactions along the θ direction ($y_d \equiv y(\sigma, \theta - 1)$) and the formation and propagation of *space-time* structures, as defects and/or spatiotemporal intermittency can be identified [Arecchi *et al.*, 1992; Giacomelli & Politi, 1996].

For delays T larger than the period of oscillation of the system, the behavior of a DS is analogous to that of an ES with $k = 1$. Namely, DS may display phase defects, i.e. points where the phase suddenly changes its value and the amplitude goes to zero. In this paper we show evidence of these phase defects in a recent experiment on Bénard–Marangoni convection [Mancini & Maza, 1997], and we propose a control technique to suppress them. The control restores regular patterns within an amplitude turbulent regime, which implies the presence of a large number of defects. The control efficiency persists even in presence of a large amount of noise.

2. The Experiment and The Delayed Dynamical Model

The experimental setup is depicted in Fig. 1. A cylindrical cell (diameter 128 mm) confines a fluid layer of silicon oil (Pr $\simeq$ 3000) with the free surface open to the atmosphere and heated from the bottom. The heater does not cover the whole of the container giving open boundaries for the heating. A convective instability driven by buoyancy and temperature dependent surface tension (80% and 20%, respectively), takes place as the heating is increased giving rise to a stationary planform [Fig. 1(a)]. This pattern is composed of four convective cells located on the heater region, but the flow is developed over all the container size.

Following an imaginary drop of fluid traveling with the flow in one of these cells, the drop is heated while traveling near the bottom over the heater, rises up to the centre, travels out near the surface until it becomes cold and then falls down near the lateral boundaries. Finally, the drop is fed back to the heater region completing a round trip in a mean time T [Fig. 1(b)].

If the heating is further increased a time-dependent regime arises, generating spatio-temporal modulations of the stationary velocity and temperature fields. The origin of this behavior is related with a thermal boundary layer instability which give rise to *thermals* or "hot plumes" which are dragged by the flow along the cell. This behavior can be seen in the space-time image of Fig. 1(c). An experimental measurement of the temperature at the center of the cell shows modulated oscillations which have a power spectrum composed by two frequencies clearly differentiated (plus their nonlinear combinations terms). One of these frequencies corresponds to a relaxation oscillation inside the thermal boundary layer, the other one corresponds to the characteristic time necessary for

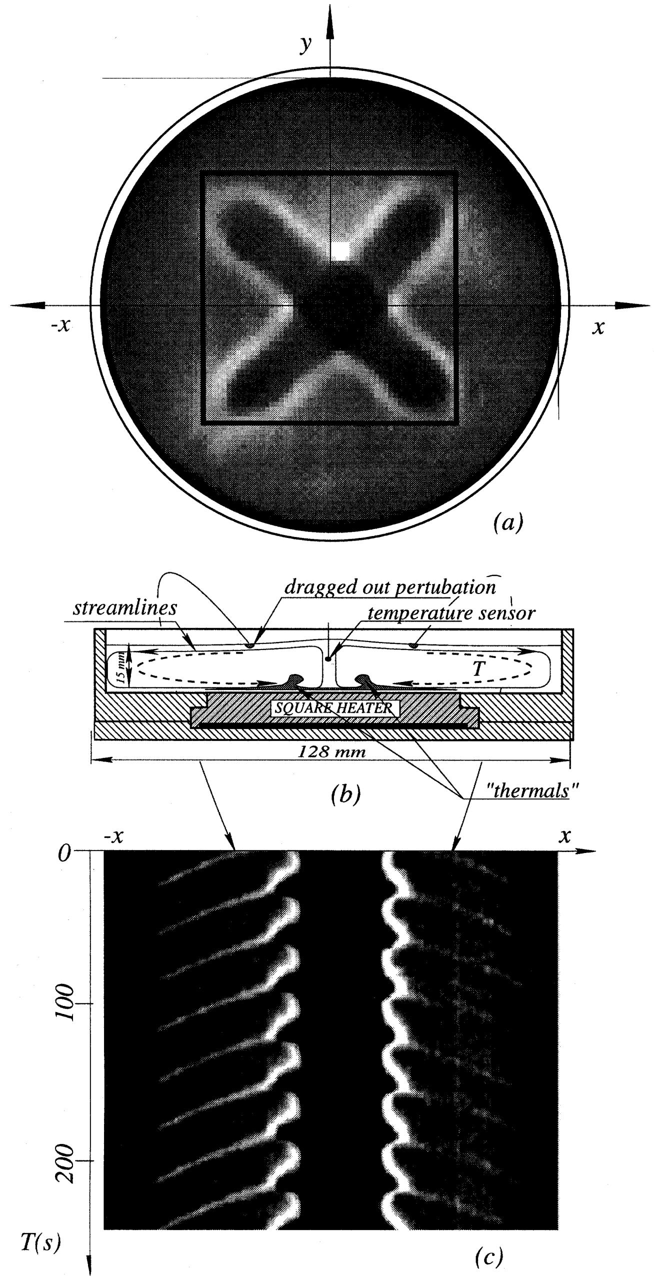


Fig. 1. (a) Image of the stationary planform below the time-dependent regime. (b) Cross-section of the experimental setup. Above the time-dependent threshold, the thermals coming from the thermal boundary layer generate hot drops which are dragged by the flow along the convective cells in a mean time T. (c) Time-dependent regime observed from the planform. The white traces in the spatiotemporal diagram correspond to the effect of the hot drop traveling near the surface in the x-axis direction. See [Mancini & Maza, 1997] for futher details.

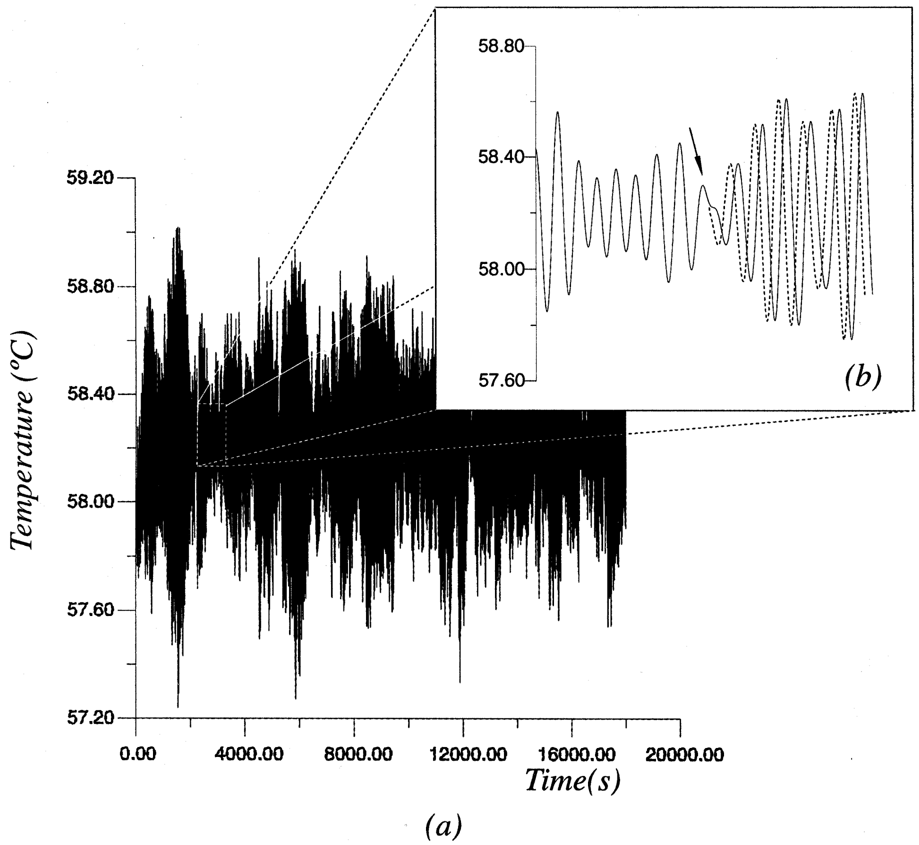


Fig. 2. (a) Experimental time behavior of the temperature signal. Vertical axis reports the temperature at the center of the cell near the bottom and horizontal axis reports the time in seconds ($T \simeq 330$ sec.). (b) Expanded view of the signal within the box which exhibits a phase jump (indicated by the arrow).

a round trip of a thermal along the convective cell. Further details on the experiment and a detailed discusion of this mechanism can be found in [Mancini & Maza, 1997].

If the temperature of the heater is further increased, a chaotic regime is reached. In this regime, an experimental measurement of the temperature at the center of the cell yields the data in Fig. 2. The signal shows trains of modulated oscillations, interrupted by localized events (phase defects), wherein the phase changes suddenly and the amplitude decreases to zero. Figure 2(b) highlights the presence of a phase defect within the data.

The experimental configuration provides a natural delayed interaction between thermals and thermal boundary layer since it reiterates at each position the local value of the order parameter after a *time delay* T, which equals the time lag necessary for the trip of the cell. Moreover, it involves a pertubated state far from the time dependent convection threshold. We propose a nonlinear model for the experimental temperature signal:

$$\dot{A} = \varepsilon A + \beta_1 \left(\int_0^\infty A^2(t - t')f(t')dt' \right) A + \beta_2 \left(\int_0^\infty A^4(t - t')f(t')dt' \right) A \,, \qquad (2)$$

$$\dot{\varepsilon} = \mu \left(S - \frac{\mu_1}{\mu}\varepsilon - kA^2 \right) . \qquad (3)$$

Here, all quantities are real. A represents the temperature, ε is a time dependent control parameter, β_1, β_2, μ_1, k are suitable fixed parameters, μ is a measure of the ratio between the characteristic time scales for A and ε, and S is a measure of the power provided to the system.

The relaxation oscillations of the temperature in Fig. 1(b) are represented by the normal form of a Hopf bifurcation [Eq. (2)], in which the saturating terms include a delayed function modulated by $f(t')$ to account the delayed action of the thermal inside the convective cell. Equation (3) models the slow evolution ($\mu < 1$) of the linear gain ε, which is enhanced by the external heating (S) and depressed by the convective motion ($-kA^2$) which tends to uniformize top and bottom temperatures. In general $f(t')$ is a Gaussian-type function which expresses the lateral heat diffusion of the thermal to the main flow. However, we will consider the case of a perfectly localized pulse using a Dirac delta function located a $t' = T$. The model can be written as:

$$\dot{A} = \varepsilon A + \beta_1 A^2(t-T)A + \beta_2 A^4(t-T)A\,, \quad (4)$$

$$\dot{\varepsilon} = \mu\left(S - \frac{\mu_1}{\mu}\varepsilon - kA^2\right)\,. \quad (5)$$

Even though Eqs. (4) and (5) have been here introduced for modeling a specific experimental situation (a chaotic transition associated with quasiperiodicity), they are in fact rather general. When $T = 0$, $S < 0$, $\beta_1 > 0$, $\beta_2 < 0$, $\mu > 0$, $\mu_1 > 0$, $k > 0$, they model an excitable system, producing the so-called *Leontovitch* bifurcation, evidence of which has been shown experimentally on a CO_2 laser with intracavity saturable absorber [Plaza *et al.*, 1997]. For $T \neq 0$, they are similar to the models already introduced to describe self-sustained oscillations of confined jets [Villermaux & Hopfinger, 1994], or memory induced low frequency oscillations in closed convection boxes [Villermaux, 1995], or even the pulsed dynamics of a fountain [Villermaux, 1994].

Adjusting pump and delay parameters (S and T) in Eqs. (4) and (5), the system enters the chaotic region. This region, in fact, is split into two different regimes. For low T values, chaos is due to a local chaotic evolution of the phase, whereas no appreciable amplitude fluctuations are observed. This regime is called *phase turbulence* (PT). By increasing T, a transition toward *amplitude turbulence* (AT) is observed. In AT, the dynamics is dominated by the amplitude fluctuations, and a large number of defects is present. Both PT and AT have counterparts in a one-dimensional complex Ginzburg–Landau equation, for which the parameter space shows a transition from a regime of stable plane waves toward PT (Benjamin–Fair instability), followed by another transition to AT with evidence of space-time defects [Montagne *et al.*, 1996] .

3. The Control

The aim of the present paper is to control AT by an adaptive technique recently introduced for chaos recognition [Arecchi *et al.*, 1994], and applied to chaos control on CS [Boccaletti & Arecchi, 1995], chaos synchronization [Boccaletti *et al.*, 1997], targeting of chaos [Boccaletti *et al.*, 1997] and filtering of noise from chaotic data sets [Boccaletti *et al.*, 1997]. A direct application of such a technique to Eqs. (4) and (5) has been already provided in [Boccaletti *et al.*, 1997]. In that case, a small continuous perturbation $U(t)$ of the local value of the temperature leads to the suppression of the phase defects, and restores the regular Hopf oscillations. Here we show an alternative strategy for the control of AT, whereby tiny continuous modifications of the parameter T lead to a local control of the phase of the signal. In order to prove the efficacy of our method we use the system in Eqs. (4) and (5) in the AT regime. Here, the time delay is proportional to the spatial extension of the system. We will show that very small perturbations of the time delay are sufficient for the control of phase defects.

Let us, therefore, consider the modified system

$$\begin{aligned}\dot{A} = \varepsilon A &+ \beta_1 A^2(t-(T+U(t)))A \\ &+ \beta_2 A^4(t-(T+U(t)))A\,, \end{aligned} \quad (6)$$

$$\dot{\varepsilon} = \mu\left(S - \frac{\mu_1}{\mu}\varepsilon - kA^2\right)\,. \quad (7)$$

The control algorithm which selects $U(T)$ can be summarized as follows. At time $t_{n+1} = t_n + \tau_n$ (τ_n being an adaptive observation time interval to be later specified), the observer defines the variation $A(t_{n+1} - T_H) - A(t_{n+1})$ between the actual and the delayed values of A (T_H being the Hopf period). The corresponding variation rate

$$\lambda_{n+1} = \frac{1}{\tau_n}\log\left|\frac{A(t_{n+1}-T_H) - A(t_{n+1})}{A(t_n - T_H) - A(t_n)}\right| \quad (8)$$

allows to select a new time interval

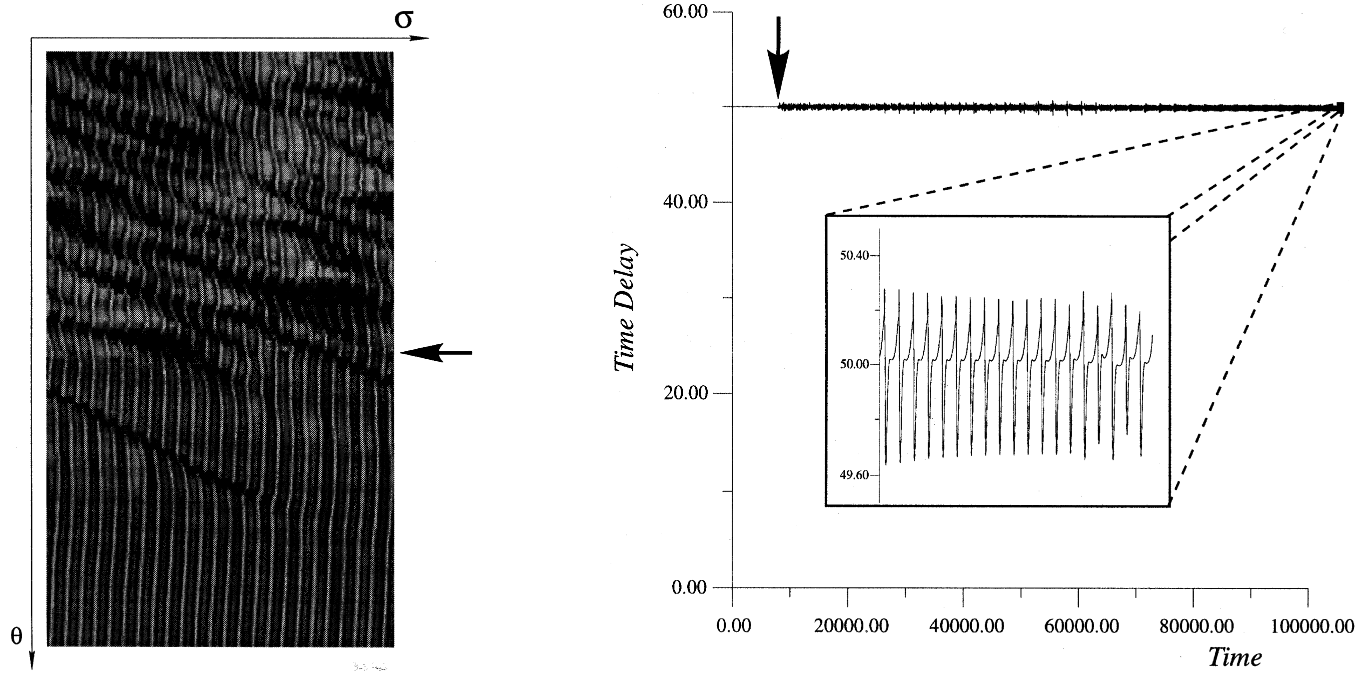
$$\tau_{n+1} = \tau_n(1 - \mathrm{tgh}(g\lambda_{n+1}))\,, \quad g > 0 \quad (9)$$

and consequently a new observation at the time $t_{n+2} = t_{n+1} + \tau_{n+1}$. In the following we perturb T by adding iteratively to it a controlling term given by

$$U(t) = \frac{1}{\tau_{n+1}}(A(t-T_H) - A(t))\,. \quad (10)$$

The details of the algorithm have been given in [Arecchi *et al.*, 1994; Boccaletti *et al.*, 1997]. For relatively small perturbations, the following approximation holds. Let $\langle\tau\rangle$ denote the average of the $\{\tau_n\}$ set, then Eq. (9) can be written as

$$\tau_{n+1} \simeq \langle\tau\rangle(1 - g\lambda_{n+1}) \quad (11)$$



(a) (b)

Fig. 3. (a) Space(σ)-time(θ) representation of the controlling process for Eqs. (6) and (7). $\beta_1 = 1$, $\beta_2 = -1/16$, $\mu = 0.8$, $\mu_1 = 0.8$, $k = 11$, $S = 7$, $T_H = 1.95$. $T = 50$, AT regime. The dynamics is dominated by amplitude fluctuations, with the presence of defects. Phase defects appear as dislocations in such a representation. The algorithm ($K_1 = 0.3$, $K_2 = 0.07$) suppresses the defects and restores the regular oscillation. Arrow indicates the instant at which control is switched on. (b) The behavior of the time delay $T + U(t)$.

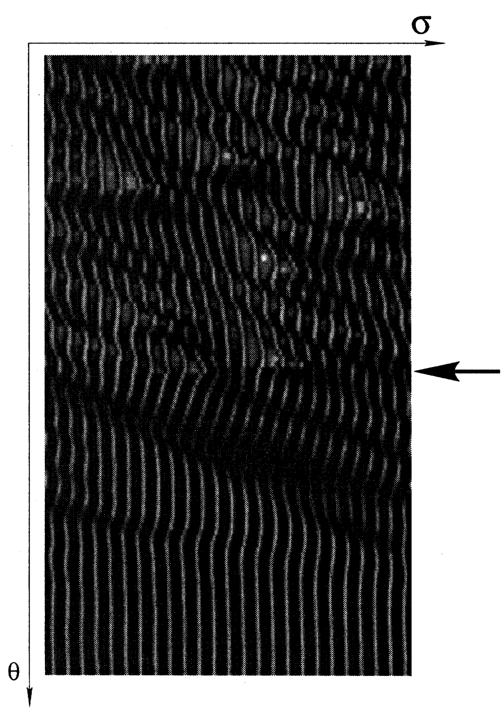


Fig. 4. $T = 50$, AT with 10% noise. Control with $K_1 = 0.3$, $K_2 = 0.07$. Same stipulations and parameters as in the caption of Fig. 2. Arrows indicate the instant at which control is switched on.

where (i) τ_n has been replaced with its ensemble average, and (ii) the tgh function has been linearized. In the same way, Eq. (8) can also be linearized as

$$\lambda(t) \simeq \frac{1}{\langle\tau\rangle} \frac{\dot{A}(t) - \dot{A}(t - T_H)}{A(t) - A(t - T_H)} \tag{12}$$

where the discretized stroboscopic observations have been approximated with a continuous inspection. Combining Eqs. (11) and (12) into Eq. (10), this reduces to

$$\begin{aligned} U(t) &= K_1(A(t - T_H) - A(t)) \\ &\quad + K_2(\dot{A}(t - T_H) - \dot{A}(t)) \end{aligned} \tag{13}$$

with $K_1 = 1/\langle\tau\rangle$ and $K_2 = g/\langle\tau\rangle^2$. The consequences of this approximation are relevant. First of all, for $K_2 = 0$ one recovers the Pyragas control method [Pyragas, 1992]. However, in our case, K_1 and K_2 can be independently selected, and this introduces an extra degree of freedom with respect to [Pyragas, 1992].

In Fig. 3 we report the application of our method to Eqs. (6) and (7). The desired oscillation, which in the space-time representation gives rise to a roll set, is controlled in AT [Fig. 2(a)]. Figure 2(b) reports the behavior of $U(t)$, which is vanishing up to the instant at which control is switched on.

Let us now discuss the robustness of our procedure against external noise. For this purpose, we add white noise to the measured A values before the onset of the adaptive feedback control. In such a case, the noise does not act additively, since it affects the calculation of $U(t)$, hence the local value of the time delay. As a consequence, the noise acts dynamically on the evolution of the system. A relevant result is that our method is robust against large amounts of noise, as it can be appreciated in Fig. 4 where the control is achieved within AT for 10% noise.

4. Conclusion

We show that is possible to control delayed dynamical systems applying small perturbations on the time delay variable. The control algorithm is easily implementable. The robustness of this method against noise has been verified. The proposed procedure would imply an experimental setup wherein the control is achieved by modifying the cell length.

Acknowledgments

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