Information encoding in homoclinic chaotic systems

I. P. Mariño,[a] E. Allaria, R. Meucci,[b] S. Boccaletti, and F. T. Arecchi[c]
Istituto Nazionale di Ottica Applicata, I50125 Florence, Italy

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We present a simple method for real-time encoding of information in the interspike intervals of a homoclinic chaotic system. The method has been experimentally tested on a CO_2 laser with feedback displaying Sil'nikov chaos and synchronized with an external pulsed signal. Information is encoded by the length of the temporal intervals between consecutive pulses of the external signal. This length is varied each time a new pulse is generated.
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In this paper, we introduce a novel information-encoding scheme that exploits homoclinic chaotic dynamics. The feasibility of the method is experimentally demonstrated with the help of a CO_2 laser synchronized by means of a weak external pulsed signal. It is shown how the interspike intervals peculiar to this chaotic system are enslaved to follow a prescribed time sequence. This fact allows one to encode messages from arbitrary sources, both analog and discrete, in the timing between spikes by applying pulses of very low power. For discrete sources, the length of the interspike intervals is quantified in order to establish a correspondence with the discrete symbols, whereas for analog sources a continuous mapping can be used. The basic mechanism that supports this scheme is the phase synchronization phenomenon between the external signal, in which the desired message has been encoded, and the corresponding temporal series of the laser intensity. Since this method only adjusts the phases of the perturbed signal and does not act in any way on the chaotic amplitudes, it can be useful in order to provide a robust communication system. In particular, we show how to encode a binary message in a homoclinic chaotic signal and how robust the signal is to the presence of additive white Gaussian noise (AWGN) in a communication channel. Even though specifically applied to Sil'nikov chaos in the present paper, the proposed encoding method has potential application in all cases in which phase synchronization of a chaotic system is achieved by means of an external perturbation.

Chaotic carriers are exploited to encode and transmit signals via two distinct approaches. The first method consists of applying control techniques in order to stabilize one chaotic orbit embedded within the chaotic attractor, chosen such that its intersections with the Poincaré section can be mapped into a desired sequence of bits.[1,2] The second method, which has been used more extensively, consists of exploiting the synchronization properties of coupled chaotic systems in order to modulate a chaotic carrier with a message signal at the transmitter, and then recover the message by demodulating the synchronized state at the receiver.[3] In the past, the latter method has mainly focused on the problem of warranting privacy in the communication,[4] but it has recently found applications for communication with chaotic time-delayed optical systems.[5] Alternative schemes for encoding information on laser pulses have been presented by Colet and Roy[6] and by Alsing *et al.*[7] It has also been observed by Sukow and Gauthier[8] that laser spike events can be controlled by external perturbations.

In this paper we discuss a procedure that exploits the properties of phase synchronization of chaotic systems[9] to encode desired messages into the interspike interval sequences of a homoclinic chaotic system. In this way, the information is coded only in the time intervals at which spikes occur, and does not affect any geometrical property of the chaotic flow, thus resulting in a better performance against unwanted perturbations, as, e.g., additive noise contamination in the communication channel. We will discuss the performance of the proposed method in an experiment on a CO_2 laser displaying a regime of Sil'nikov chaos.

Sil'nikov chaos[10] has been observed in many systems, such as chemical[11] and laser[12] experiments. This kind of behavior shows striking similarities with the electrical spike trains traveling on the axons of animal neurons.[13,14] More generally, chemical oscillators based on an activator-inhibitor competition, which rule biological clocks controlling living rhythms, such as the heart pacemaker, hormone production, metabolism, etc., display rhythmical trains of spikes with erratic repetition frequency as shown in Refs. 11 and 12.

Homoclinic chaos of the Sil'nikov type normally appears when a parameter is varied toward the homoclinic condition associated with a saddle focus.[10,15,16] Its peculiarity consists of an astonishing regularity of the geometric trajectory in phase space. The chaotic motion is characterized by large fluctuations in the return time of consecutive spikes, so that only an average return period can be defined. Thus, an appropriate indicator of chaos may be the distribution of the return times to a given threshold, and the strength of chaos is

[a]Also at Departamento de Ciencias Experimentales e Ingeniería, Universidad Rey Juan Carlos, Móstoles, Madrid, Spain.
[b]Electronic mail: ric@ino.it
[c]Also at the Department of Physics, University of Firenze, Italy.

associated with the amount of decorrelation between successive returns.[17] Such decorrelation occurs around the saddle focus, when the system displays a large susceptibility, i.e., a large response to an external perturbation;[18] we will exploit this susceptibility for information encoding.

Recently, different phase locking regimes between an initial homoclinic chaotic signal and a sinusoidal periodic external modulation have been reported.[19] The initial homoclinic chaotic behavior can be suppressed by the action of a periodic external forcing, giving rise to different phase locking domains (1:1, 1:2, 1:3 and 2:1), depending on the frequency of the applied periodic forcing. The applied forcing readjusts the evolution of the dynamics of the homoclinic chaotic signal in such a way that the return period is locked in the same ratio with the external perturbation frequency.

Based on this phase synchronization, we propose an innovative and viable method of controlling the temporal interval between spikes. In this way, the return period of each cycle changes according to external information. In order to control the length of the interspike intervals (ISIs), it is crucial to reduce the residual phase fluctuations between the system and the forcing signal. For this purpose, an external signal consisting of short pulses provides better results than a sinusoidal perturbation. Thus, synchronization between both signals is obtained in a very fast way, achieving a robust control on the individual interspike interval of a homoclinic chaotic system, and hence encoding the desired information in the sequence of inter-spikes. This is tantamount to introducing the coding scheme on top of a phase synchronization phenomenon, thus not implying in principle any disturbance to chaotic amplitudes, as opposed to previously studied method for communicating with chaos.[1]

The experiment has been performed on a single mode CO_2 laser with a feedback proportional to the output intensity (see Ref. 19). The control parameters, bias voltage, and gain of the feedback are set in a regime of homoclinic chaotic behavior [see Fig. 1(a)]. Figure 1(b) shows the phase space reconstruction of the chaotic attractor. The modulating external signal is applied, in an additive way, on the bias parameter with a wave form generator Tektronix TM5003, which is controlled by a real time PC board (PCI-7030/6040E) from National Instruments. The perturbation signal is a train of square wave forms. The interpulse intervals can be controlled, in real time, by means of an adequate computer program. Each time a new pulse is generated, this distance in time is changed, to encode in these time intervals the desired information. The interpulse interval is varied inside the Arnold tongue describing the domain of 1:1 phase locking for the external perturbation and the laser intensity. This time variation range corresponds to a frequency of repetition of pulses that can take values in a quite large interval. The time duration of each pulse is 10% of the interpulse interval. The amplitude of the pulses is 4% of the value of the amplitude of the main spikes of the temporal evolution of the laser intensity. The laser output is recorded by a digital oscilloscope with a sampling time of 1 μs.

The first performance index we have studied is the correlation between the different time (or frequency) intervals of consecutive pulses of the external signal and the corresponding return periods (or frequencies) in the temporal series of the laser intensity. This index provides information on the amount of synchronization between the external forcing and the signal, initially in a homoclinic chaotic regime. Performance has been evaluated for a random uniform frequency distribution of the forcing signal centered in the middle value of the corresponding Arnold tongue associated with the 1:1 phase locking, that is, centered at $f_0 = 1.7$ kHz (see Ref. 19). Thus, the frequency of the external modulation takes values in the interval $(f_0 - f_s, f_0 + f_s)$ provided that the 1:1 phase locking is guaranteed, f_s being the half width of the Arnold's tongue at the modulation height chosen for the pulses in our experiment (20 mV). It results that $f_s \sim 0.6$ kHz, as one can infer from Fig. 3 of Ref. 19.

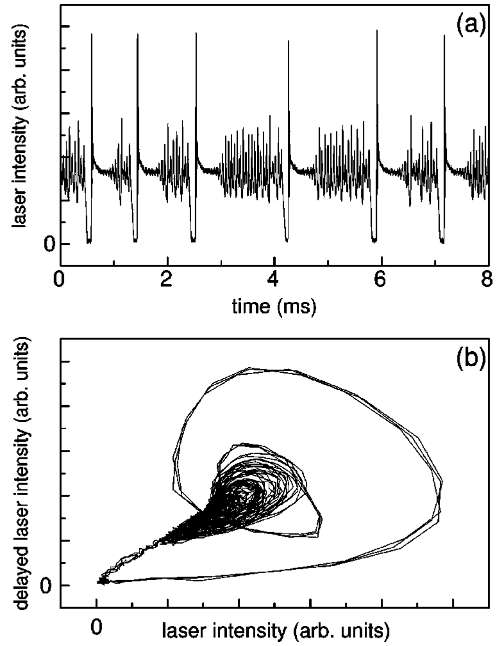

FIG. 1. (a) Experimental time series of the laser intensity for a CO_2 laser with feedback, in the regime of homoclinic chaos. The intensity modulation has been amplified and lifted for visual convenience. Homoclinic chaotic behavior in a CO_2 laser. (b) Phase space trajectory built by an embedding technique with appropriate delay.

As shown in Fig. 2(a), the phase of the temporal series of the laser intensity follows almost perfectly the external signal, with the laser spikes adjusting in each orbit to the corresponding external signal. This is obtained because the laser parameters are adjusted to the homoclinic chaos regime where the system crosses a region of large susceptibility.[18] Figure 2(b) shows the interspike intervals of the laser intensity versus the interpulse intervals of the modulating signal. Synchronization is expressed by the straight line of 45°. A better estimation of the error in time between the interspike intervals of the laser and the timing between pulses of the external modulating signal is shown in Fig. 2(c), where an histogram of these differences in time is plotted. Such histogram is fitted by a zero-mean Gaussian distribution with a standard deviation of 7.8 μs. In Fig. 2(c) we also appreciate that the difference between the corresponding return periods of the external signal and the laser intensity is always below a small value ($\Delta t \leq 25$ μs).

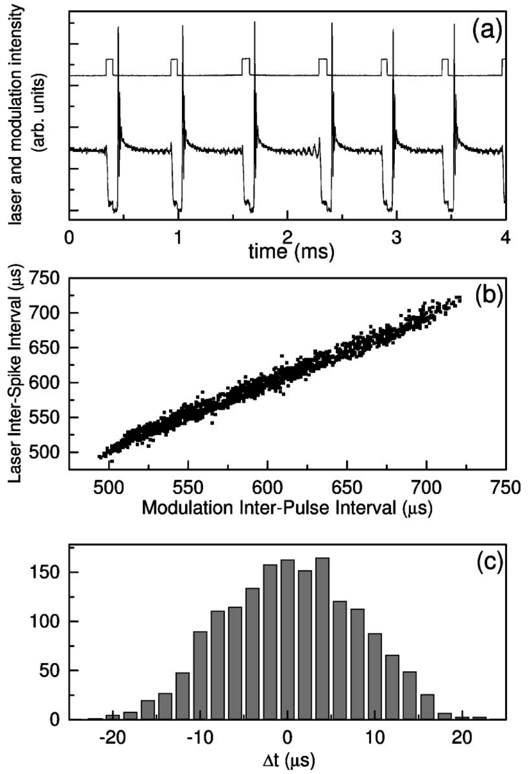

FIG. 2. (a) Experimental temporal evolution of the laser intensity driven by an external pulsed signal. (b) Interspike intervals of the laser intensity vs interpulse intervals of the forcing signal. This plot indicates synchronization between the two variables. (c) Histogram of the time differences (Δt) between interpulse intervals of the external signal and interspike intervals of the laser intensity.

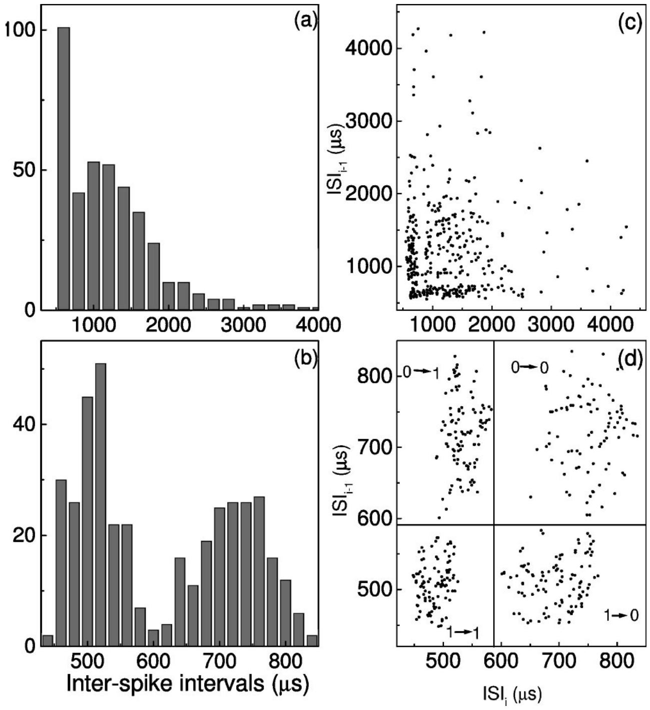

FIG. 3. (a) Histogram of the frequencies of the free-running signal before applying any perturbation. (b) Histogram when information has been encoded in the signal. The information consists of a uniform distribution of bits "1" and "0." (c) and (d): Phase portraits ISI_{i-1} vs ISI_i corresponding to (a) and (b), respectively.

From this analysis, we conclude that it is possible to enslave the phase of the laser spikes with pulses of very low power. Thus, it is possible to encode in the return periods of the laser intensity messages of binary, ternary, quaternary nature, etc. The fact of encoding information in the timing between spikes can be very useful in order to design a robust communication system[20] or to achieve a better understanding in the communication phenomenon among cells in the central nervous system.[21] It is well known that the pattern of spike times provides a large capacity for conveying information beyond that due to the code commonly assumed by physiologists, the number of spikes fired.[22]

As an example of how to encode information in the timing between spikes, we present the simple case in which a binary message is encoded in the return period of the spikes in the CO_2 laser intensity. The homoclinic chaotic behavior of the laser can be used to encode a desired message in such a way that the bits "1" and "0" are easily identified as timing between spikes less or greater than a threshold value. The encoding procedure is the following. First of all, an external pulsed signal that modulates the laser intensity is synthesized with an interpulse frequency changed after each pulse according to a uniform frequency distribution centered at the mean repetition frequency, $f_0 = 1.7$ kHz. When the value of the frequency is larger than f_0, a bit "1" is encoded in the external signal; when this value is lower than f_0, a bit "0" is encoded in the external signal. Since the error in time in the synchronization process between the external signal and the laser intensity is below 25 μs, in order to avoid errors around the mean frequency, we consider variations in the frequency in the interval $(f_0 - f_s, f_0 - f_a)$ to encode the bit "0," and in the interval $(f_0 + f_a, f_0 + f_s)$ to encode the bit "1." It is obvious that for $f_a \geqslant 0.08$ kHz no errors in the transmission are guaranteed, since this is the variation in the frequency associated with a time variation of 25 μs ($\Delta f = \Delta t f_0^2$). Figure 3(a) shows the histogram of the interspike intervals of the free-running signal before applying any perturbation, and Fig. 3(b) shows the same when information, consisting of a uniform distribution of bits "1" and "0", has been encoded in the signal. We can appreciate in these histograms how a left shift in time (right shift in frequency) is produced when the control is applied. In this way, the orbit is shortened and the time difference between consecutive spikes is reduced. Notice also that the frequency range of the modulating signal to achieve 1:1 phase locking is centered around the most probable value of the frequency corresponding to the homoclinic behavior of the laser intensity. In Figs. 3(c) and 3(d) the corresponding phase portraits of the ISI_{i-1} vs ISI_i are reported. The effect of the applied encoding is to separate the pseudo-phase-space of ISI in four distinct regions corresponding, respectively, to sequences $0 \rightarrow 1$, $0 \rightarrow 0$, $1 \rightarrow 1$, and $1 \rightarrow 0$. This is analogous to the results obtained by Hayes *et al.*[1] in phase space.

As an illustration, we have experimentally verified how the laser output signal correctly tracks a given binary message. In particular, we have taken some lines of text,[23] translated them into a binary sequence according to the ASCII

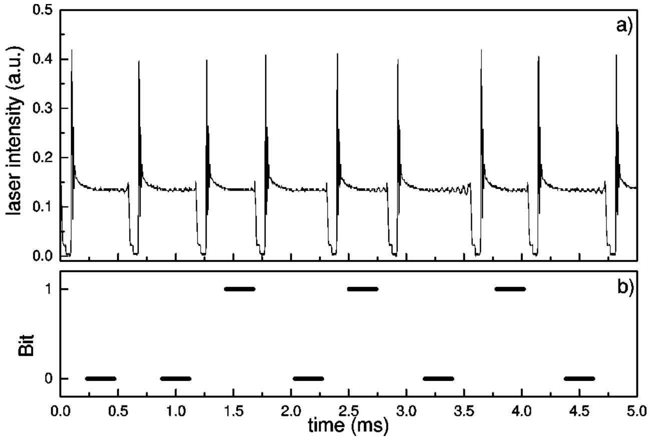

FIG. 4. (a) Portion of the laser output intensity containing the encoding of the binary conversion of the letter "T." (b) Binary decoding according to a threshold level set at 75% of the peak height.

code (8 bits per character), and encoded this sequence into the laser signal using the proposed method. The whole message was decoded with zero errors. Figure 4 shows a short fragment of the laser signal in the experiment and the corresponding bits obtained after decoding at the receiver.

The information decoding process in the laser intensity can be carried out by observing the instant when the laser intensity is higher than a threshold level (see Fig. 4). The fact that the spike amplitude is very high in comparison with the signal elsewhere, and the fact that the system can be controlled with small perturbations of very low power, make it suitable for information transmission in a communication system. Since the information is contained entirely in the timing between spikes, channel distortions that affect the pulse shape will not significantly influence the information transmission.

From this point of view, we analyze how robust the system is to contamination with zero-mean additive white Gaussian noise (AWGN) in the communication channel. Notice that the channel noise, denoted as $n(t)$, is added to the transmitted signal as $s(t)+n(t)$ in the transmission channel and sent to the receiver. Figure 5 shows the bit error rate (BER) attained by the receiver versus the E_b/N_0 ratio, where

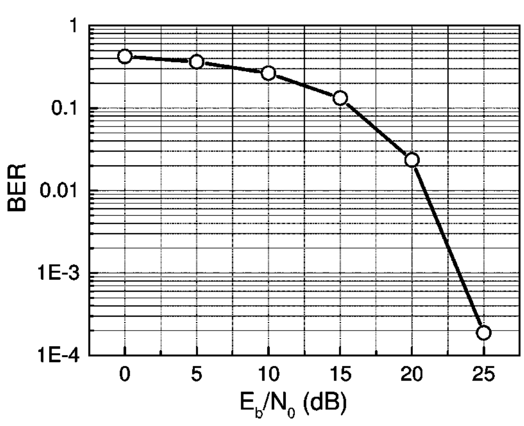

FIG. 5. BER vs E_b/N_0 of the proposed communication system when additive white Gaussian noise is contaminating the channel.

E_b is the energy per bit and $N_0/2$ represents the power spectral density of the channel noise. We can see in this Fig. 5 how the BER strongly decreases as E_b/N_0 increases. For $E_b/N_0 \geqslant 25$ dB, the BER is already below 10^{-3}. Our method provides a BER consistently lower than other recent proposals,[24,25] which are based on the synchronization properties of coupled chaotic systems (compare Fig. 5 in the present article with Fig. 3 in Ref. 25). The improved performance of our method is due to the fact that we encode information in the ISI, rather then in the amplitude of the signal. Since the AWGN in the channel distorts the amplitude of the transmitted signal, but not the ISI, our scheme turns out to be very robust to this type of noise. Detection errors at the receiver might occur when occasional noise bursts are mistaken with true signal spikes, but the receiver is not sensitive to the amplitude distortion of the true spikes. Another advantage of our method is its generality, since it is applicable to any chaotic dynamical system where phase synchronization can be achieved by means of an external perturbation.

As stated previously, our encoding is highly redundant since wide frequency intervals are associated with bits 0 and 1, respectively,

$$(f_0-f_s, f_0-f_a), \quad (f_0+f_a, f_0+f_s), \tag{1}$$

where f_a is a conservative property to account for the time synchronization error plotted in Fig. 2(c). If any other source of frequency jitter might occur, as, e.g., a jitter in the pulse generator, then a commutative shrinkage of the encoding intervals can be introduced to account for the extra noise sources. Precisely, we would replace the above-mentioned coding intervals by

$$(f_0-f_s+\Delta f, f_0-f_a-\Delta f) \quad \text{and}$$

$$(f_0+f_a+\Delta f, f_0+f_s-\Delta f), \tag{2}$$

respectively, where Δf is a conservative safety span which cuts the error probability below a preassigned value.

In conclusion, we have shown experimentally how it is possible to enslave the interspike intervals of a homoclinic chaotic system to follow a prescribed bit sequence. This fact allows one to encode information in the timing between spikes. The basic mechanism is the selection of instantaneous perturbation frequencies within the range described by the Arnold tongue of phase synchronization. Even though here specifically applied to Sil'nikov chaos, the encoding method can be of potential application in all cases in which phase synchronization of a chaotic system is achieved by means of an external perturbation. In particular the method only adjusts the phases of the perturbed signal, but does not act in any way on the chaotic amplitudes. This can be useful in order to achieve a better understanding in the communication phenomenon among cells in the central nervous system as well as to design a robust communication system. In particular, we have shown a simple method to encode a binary message in a homoclinic chaotic signal that can be used in communication applications.

ACKNOWLEDGMENT

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[1] S. Hayes, C. Grebogi, and E. Ott, Phys. Rev. Lett. **70**, 3031 (1993); S. Hayes *et al.*, *ibid.* **73**, 1781 (1994); S. Boccaletti *et al.*, Phys. Rep. **329**, 103 (2000).

[2] I. P. Mariño, E. Rosa, Jr., and C. Grebogi, Phys. Rev. Lett. **85**, 2629 (2000); I. P. Mariño, C. Grebogi, and E. Rosa, Int. J. Bifurcation Chaos Appl. Sci. Eng. **11**, 2631 (2001).

[3] For an overview of synchronization phenomena see S. Boccaletti, L. M. Pecora, and A. Pelaez, Phys. Rev. E **63**, 066219 (2001), and references therein.

[4] K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993); G. Pérez and H. A. Cerdeira, *ibid.* **74**, 1970 (1995); J. H. Peng *et al.*, *ibid.* **76**, 904 (1996); S. Boccaletti, A. Farini, and F. T. Arecchi, Phys. Rev. E **55**, 4979 (1997).

[5] G. D. Van Wiggeren and R. Roy, Science **279**, 1198 (1998); G. D. Van Wiggeren and R. Roy, Phys. Rev. Lett. **81**, 3547 (1998).

[6] P. Colet and R. Roy, Opt. Lett. **19**, 2056 (1994).

[7] P. M. Alsing *et al.*, Phys. Rev. E **56**, 6302 (1997).

[8] D. W. Sukow and D. J. Gauthier, IEEE J. Quantum Electron. **36**, 1765 (2000).

[9] M. G. Rosenblum, A. S. Pikovski, and J. Kurths, Phys. Rev. Lett. **76**, 1804 (1996).

[10] L. P. Sil'nikov, Sov. Math. Dokl. **6**, 163 (1965); Math. USSR Sbornik **10**, 91 (1970).

[11] F. Argoul, A. Arnéodo, and P. Richetti, Phys. Lett. A **120**, 269 (1987).

[12] F. T. Arecchi, R. Meucci, and W. Gadomski, Phys. Rev. Lett. **58**, 2205 (1987).

[13] A. L. Hodgkin and A. F. Huxley, J. Physiol. (London) **117**, 500 (1952).

[14] U. Feudel *et al.*, Chaos **10**, 231 (2000).

[15] S. Wiggins, *Global Bifurcations and Chaos—Analytical Methods* (Springer, New York, 1984).

[16] Y. A. Kuznetsov, *Elements of Applied Bifurcation Theory* (Springer, New York, 1995).

[17] F. T. Arecchi, *Lecture Notes at the E. Fermi School on Recent Advances in Metrology and Fundamental Constants, Varenna, Italy 25 July–4 August 2000*, edited by T. J. Quinn and S. Leschiutta (IOS, Amsterdam, 2001).

[18] F. T. Arecchi, A. Lapucci, and R. Meucci, Physica D **62**, 186 (1993).

[19] E. Allaria, F. T. Arecchi, A. Di Garbo, and R. Meucci, Phys. Rev. Lett. **86**, 791 (2001).

[20] M. Sushchik, Jr., N. Rulkov, and L. Larson, IEEE Commun. Lett. **4**, 128 (2000).

[21] S. R. Schultz and S. Panzeri, Phys. Rev. Lett. **86**, 5823 (2001).

[22] D. MacKay and W. S. McCulloch, Bull. Math. Biophys. **14**, 127 (1952).

[23] The text used is a paragraph of more than 400 characters extracted from the web page of the Istituto Nazionale di Ottica Applicata (http://www.ino.it/presentation.htm, The National Insistute .... Didactics).

[24] K. Murali, Phys. Rev. E **63**, 016216 (2000).

[25] M. N. Lorenzo *et al.*, Int. J. Bifurcation Chaos Appl. Sci. Eng. **10**, 2863 (2000).