MEASUREMENT OF THE TIME EVOLUTION OF A RADIATION FIELD BY JOINT PHOTOCOUNT DISTRIBUTIONS*

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This Letter reports the measurement of the time evolution of a stationary Gaussian field by means of joint photocount distributions, which can be considered as a generalization of the single-time photocount distributions used thus far in all previously reported experiments.[1-6]

In those experiments, photoelectric counting measurements are performed for an observation time, T, much shorter than the coherence time of the field, so that, when dealing with a stationary field, one measures the probability distribution $W_1(n)$ of sorting out a given count number from the available statistical ensemble, or, using the theory of the photoelectric measurement,[7,8] the probability distribution of sorting out a given field intensity from its ensemble.[9]

The one-time probability distribution W_1 does not completely describe a random field, unless one has separate information on the law of motion.[10] One way of measuring the time evolution of a field would be observing the photocounts for increasing times T up to (or larger than) the relaxation times of the field and then correlating the field evolution to the various shapes of the photocount distribution. This has been treated theoretically for a Gaussian field evolving as a Markoff process (Lorentzian spectrum),[11] and asymptotic formulas have been given for the photocount distribution when T is much longer than the coherence time.[8,12] Experimental results have been given by Arecchi (Table II of Ref. 2) and Johnson, McLean, and Pike.[1] Evidently, this is a smoothing procedure which averages out the relevant statistical information over long integration times.

The procedure we report there corresponds instead to spanning a long time interval by separate, correlated observations, each one lasting for a time, T, much shorter than the coherence time, so that it can be taken as an "instantaneous" observation. The measurement consists essentially in repeating twice the operation described in Ref. 2 and correlating the two observations. The photoelectron pulses from a single photomultiplier charge a capacitor for an interval, T, around time t_1 and again for an interval, T, around time t_2 (the time interval t_2-t_1 being controlled by an electronic clock). Voltage outputs from the capacitor are sent, respectively, to the first and second address of a bidimensional, multichannel pulse-height analyzer. The measurement is classified on a 32×32 matrix. The 1024 numbers of this matrix yield the joint photocount distribution $W_2(n_1, t_1; n_2, t_2)$. Defining a conditional probability, P_C, through the relation[9]

$$W_2(n_1, t_1; n_2, t_2) = P_C(n_1, t_1 | n_2, t_2) W_1(n_1, t_1), \quad (1)$$

one easily realizes that P_C is given by the 32 readings on the row corresponding to a chosen value of n_1. The marginal distribution $W_1(n_1) = W_1(n_2)$ corresponding to an uncorrelated experiment is obtained by summing for each column (or row) the values corresponding to all rows (columns) belonging to that column (row).

A direct measurement of P_C can be obtained using the count number n_1 at t_1 to operate a single-channel amplitude analyzer, which in turn gates the second count number n_2 at t_2, allowing it to be classified by a standard multichannel analyzer, only when n_1 has corresponded to the selected value. Moreover, by this second technique 512 channels are available for each P_C distribution instead of only 32.

Measurements are reported in Figs. 1 and 2, in the case of a Gaussian field obtained by random superposition from uncorrelated scatterers.[2] The results are interpreted with great accuracy by the following considerations. Defining a joint-field distribution function in the Glauber representation by[13]

$$W_2(E_2, t_2; E_2, t_2) = \int P(\{\alpha_k\}) \delta^2[E_1 - E(t_1, \{\alpha_k\})] \delta^2[E_2 - E(t_2, \{\alpha_k\})] \Pi_k d^2\alpha_k, \quad (2)$$

one easily obtains for the case of a stationary Gaussian-Markovian field,

$$W_2(E_2, 0; E_2, \tau) = \frac{1}{\pi^2 \langle n \rangle^2 [1-\exp(-2\beta\tau)]} \exp \frac{-|E_1|^2 + |E_2|^2 - 2\,\mathrm{Re}E_1 E_2^* \exp(-\beta\tau)}{\langle n \rangle [1-\exp(-2\beta\tau)]}, \quad (3)$$

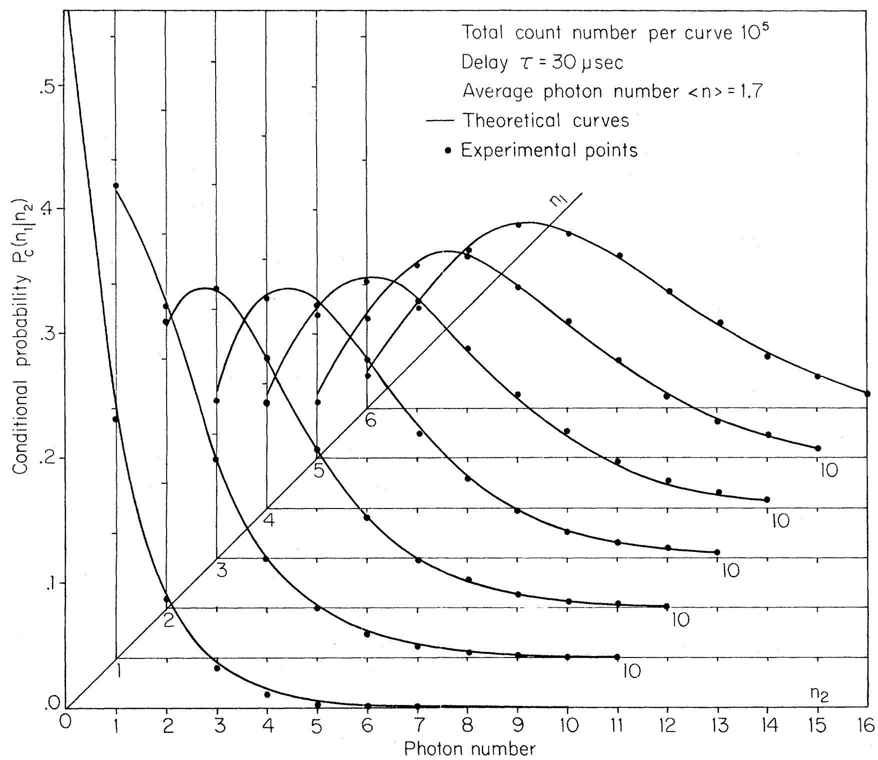

FIG. 1. Conditional photocount probabilities for a Gaussian field with a coherence time of 700 μsec and for a delay $\tau = 30$ μsec ($|g^{(1)}(\tau)|^2 = 0.925$). Each curve has unit area.

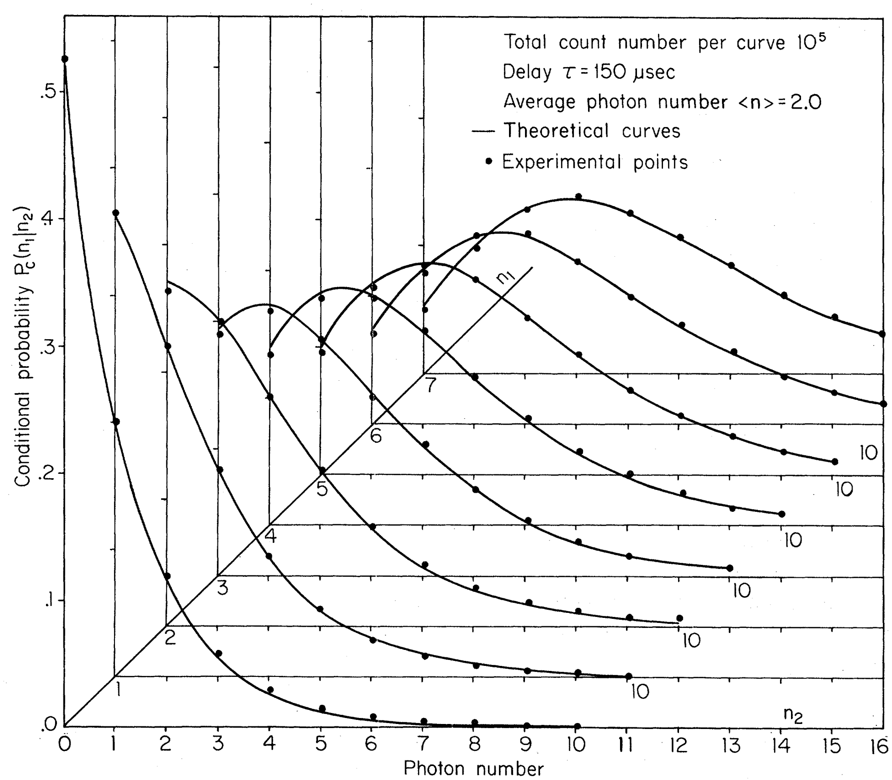

FIG. 2. Conditional photocount probabilities for the same field of Fig. 1 and for a delay $\tau = 150$ μsec $[|g^{(1)}(\tau)|^2 = 0.65]$. Similar measurements for a delay $\tau = 2500$ μsec yield a set of identical geometric (Bose) distributions similar to the ones presented in Ref. 2. This must be expected since $P(n_1, 0; n_2, \tau) \to W_1(n_2)$ as $\tau \to \infty$.

where[13]

$$\langle n\rangle = G^{(1)}(r, t; r, t)T$$

is the average photocount number per observation time (for simplicity we have taken unit quantum efficiency) and $|g^{(1)}(\tau)| = \exp(-\beta\tau)$ is the normalized autocorrelation function. As for the first-order distribution functions,[8] one can define here a generating function[14] as the following average over the joint photocount distribution:

$$Q(\lambda_1, \lambda_2) = \langle (1-\lambda_1)^{n_1}(1-\lambda_2)^{n_2}\rangle. \quad (4)$$

It is easily shown[15] that Q is related to the joint-field distribution function by

$$Q(\lambda_1, \lambda_2) = \int \exp(-\lambda_1 |E_1|^2 T) \exp(-\lambda_2 |E_2|^2 T) \times W_2(E_1, E_2) d^2E_1 d^2E_2, \quad (5)$$

which allows connection of the field to the photocount distribution by means of the relation

$$W_2(n_1, t_1; n_2, t_2) = \frac{(-1)^{n_1+n_2}}{n_1! n_2!}\left[\frac{\partial^{n_1+n_2}}{\partial\lambda_1^{n_1}\partial\lambda_2^{n_2}} Q(\lambda_1, \lambda_2)\right]_{\lambda_1=\lambda_2=1}. \quad (6)$$

Substitution of (3) into (5) yields

$$Q(\lambda_1, \lambda_2) = \{1 + (\lambda_1+\lambda_2)\langle n\rangle + \lambda_1\lambda_2\langle n\rangle^2[1-\exp(-2\beta\tau)]\}^{-1}. \quad (7)$$

In the case $\tau \to \infty$, this factorizes into the product of two independent generating functions for geometric distributions. Using (6), one then finds that $W_2(n_1, n_2)$ is given for any value of τ by

$$W_2(n_1, 0; n_2, \tau) = \frac{(n_1+n_2)!}{n_1! n_2!} \frac{(\langle n\rangle B)^{n_1+n_2}}{A^{n_1+n_2+1}} \times {}_2F_1(-n_1, -n_2; -n_1-n_2; C), \quad (8)$$

where $A = 1 + 2\langle n\rangle + \langle n\rangle^2[1-\exp(-2\beta\tau)]$, $B = 1 + \langle n\rangle \times [1-\exp(-2\beta\tau)]$, $C = [1-\exp(-2\beta\tau)]A/B^2$, and ${}_2F_1$ is the Gaussian hypergeometric function.[16] For instance when $\tau = 0$ (fully correlated measurements) one obtains (Fig. 1)

$$W_2(n_1, 0; n_2, 0) = \frac{(n_1+n_2)!}{n_1! n_2!} \frac{\langle n\rangle^{n_1+n_2}}{(1+2\langle n\rangle)^{n_1+n_2+1}},$$

whereas, for $\tau \to \infty$, one obviously obtains the product of two geometric distributions referring, respectively, to n_1 and n_2 counts.

As shown in Figs. 1 and 2, the agreement between theoretical curves and experimental points is within the experimental error. These results complete the investigation on the random scatterer used in Ref. 2, showing that not only the associated ensemble distribution is Gaussian, but also that the associated time evolution is of Markovian type, as one would expect in an ideal Brownian motion.[17]

The applications of the method introduced here are much more general than the investigation of a stationary Gaussian field presented here. Use of the linear method to get rid of dead-time effects[18] permits the use of T intervals as short as a few nanoseconds, and thereby allows investigation of relaxation phenomena in the nanosecond region. Knowledge of the joint photocount distributions allows the evaluation of interesting correlation functions such as $\langle n^k(t_1)n^h(t_2)\rangle$. For $k = h = 1$, this method has already been used elsewhere[19] to show the bunching associated with a Gaussian field, and the result is equivalent to a Hanbury-Brown and Twiss experiment. When $k, h > 1$ these functions are very useful in the investigation of coherence properties of multiple photon processes.

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[1]C. Freed and H. A. Haus, in Physics of Quantum Electronics, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Book Company, Inc., New York, 1965), p. 715; F. A. Johnson, T. P. McLean, and E. R. Pike, ibid. p. 706.

[2]F. T. Arecchi, Phys. Rev. Letters 15, 912 (1965).

[3]F. T. Arecchi, A. Berné, and P. Burlamacchi, Phys. Rev. Letters 16, 32 (1966).

[4]C. Freed and H. A. Haus, Phys. Rev. Letters 15, 943 (1965).

[5]A. W. Smith and J. A. Armstrong, Phys. Letters 19, 650 (1966).

[6]In the previous references we have reported only the first experiments. However, many more experimental papers have appeared in recent times.

[7]L. Mandel, Proc. Phys. Soc. (London) 72, 1037 (1958); in Progress in Optics, edited by E. Wolf (North-Holland Publishing Company, Amsterdam, 1963, Vol.

21, p. 181.

[8]R. J. Glauber, in Quantum Optics and Electronics, edited by C. De Witt, A. Blandin, C. Cohen-Tannaudji (Gordon and Breach Publishers, Inc., New York, 1965), p. 63.

[9]We use the notation of M. C. Wang and G. E. Uhlenbeck, Rev. Mod. Phys. 17, 323 (1945), i.e., we call $W_1(y)$ the probability distribution of finding a random variable y (either photoelectric count number n or field amplitude E) in given range $(y, y+dy)$ at time t; then call the joint probability distribution of finding y "around" y_1 and y_2 at times t_1 and t_2, etc.

[10]Since the field we have been investigating is a stationary Gaussian field, a measurement of the frequency spectrum would be sufficient to determine the whole process (see, e.g., Wang and Uhlenbeck, Ref. 9). The application of our method to non-Gaussian fields, however, supplies information which is not contained in the spectrum.

[11]D. Slepian, Bell System Tech. J. 37, 163 (1958).

[12]T. P. McLean and E. R. Pike, Phys. Letters 15, 318 (1965).

[13]Since we make observations within a coherence area of the field, we specialize Eq. (14.61) of Ref. 8 for equal-space positions and different times t_1, t_2.

[14]Use of a bidimensional generating function has been suggested to us by Professor R. J. Glauber.

[15]Derivation of Eq. (5) proceeds as follows: By slight modification of Eq. (17.23) of Ref. 8 for the case of very short T, one finds

$$Q(\lambda) = \int P(\{\alpha_k\}) \exp[-\lambda |E(t, \{\alpha_k\})|^2 T] \Pi_k d^2\alpha_k.$$

This can be easily generalized at two different times t_1, t_2, under the assumption (usually realized) that $t_2 - t_1$ is long enough compared with the atomic relaxation times of the photodetector. Furthermore, one should recall, that by the same definition of the distribution function $W(E)$ for the field, an average over $P(\{\alpha_k\})$ is

[16]E. T. Whittaker and G. N. Watson, Modern Analysis, (Cambridge University Press, New York, 1940), Chap. XVI.

[17]S. Chandrasekhar, Rev. Mod. Phys. 15, 1 (1943).

[18]F. T. Arecchi, A. Berné, P. Burlamacchi, and A. Sona, in Proceedings of the IV International Conference on Quantum Electronics, Phoenix, 12-15 April 1966 (unpublished).

[19]F. T. Arecchi, E. Gatti, and A. Sona, Phys. Letters 20, 27 (1966); in Fig. 1 of that paper, we have presented the function $P(\tau) = \langle n(0)n(\tau)\rangle / \langle n\rangle^2$ which is a normalized correlation function, and have referred to it somewhat loosely as a "conditional probability".

EVIDENCE FOR A SOURCE OF PRIMARY GAMMA RAYS*

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Since Morrison[1] suggested the possibility of gamma-ray astronomy in 1958, there has been a growing interest in the field. Many theoretical reasons for expecting a measurable primary γ-ray flux at the top of the earth's atmosphere have been discussed. These are summarized in four excellent reviews of the field[2-5] which have been published in the past two years. To date, several experiments have been reported[6-13] in which a variety of instruments have been used to search for point sources. None of these experiments yielded any definite evidence for the existence of localized source intensities. Cobb, Duthie, and Stewart[11] have set upper limits of 5×10^{-5} cm^{-2} sec^{-1} from the Crab Nebula and a few times 10^{-4} cm^{-2} sec^{-1} from three other celestial objects. Frye and Smith[12] have also set upper limits of a few times 10^{-4} cm^{-2} sec^{-1} from a variety of celestial objects. Similar results were reported by Kraushaar et al.[6] In this Letter we wish to report the existence of an anomalously high count of gamma rays from the direction of the constellation Cygnus. This high count was associated with an energy spectrum which appears to differ significantly from the spectrum of secondary γ rays generated by cosmic rays interacting in the atmosphere above the balloon-borne detection system.

The present detection system is similar to that described elsewhere[11] except for a change in the location of the anticoincidence counter. A scintillation-and-Cherenkov telescope was used as the trigger for the detection of gamma rays converting in a $\frac{1}{16}$-in. lead radiator placed between two spark chambers. The system is estimated to become very inefficient at detecting gamma rays with energies less than 50 MeV. The conversion efficiency for vertically incident γ rays approaches 19% at high energies. The area solid angle factor was 25.8 cm^2 sr. The two spark chambers were used to identify the γ rays and to determine the direction of the incident photon. The data reported here rep-