Dynamical Control of Decay and Decoherence in Complex Quantum Systems

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Abstract

A unified theory is given of dynamically modified decay and decoherence in driven quantum systems that are coupled to arbitrary finite-temperature reservoirs and undergo random phase fluctuations. Criteria for the optimization of the suppression and the limitations of this approach are obtained. Decay *acceleration* by frequent measurements (interruptions of the coupling), known as the anti-Zeno effect (AZE) is argued to be much more ubiquitous than its inhibition in one- or two-level systems coupled to reservoirs (continua). In multilevel systems, frequent measurements cause accelerated decay by destroying the multilevel interference, which tends to inhibit decay in the absence of measurements.

1 Introduction

The prevailing view until recently has been that successive frequent measurements (interruptions of the evolution) *must slow down the decay of any unstable* system [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. This is known as the Quantum Zeno Effect (QZE), introduced by Misra and Sudarshan [3], following the early work of Khalfin [1] and Fonda [2].

We have recently shown [14] that, in fact, the *opposite* is commonly true for decay into open-space continua: the anti-Zeno effect (AZE), i.e. decay acceleration by frequent measurements [15, 16, 17], is far more ubiquitous than the QZE ([18, 19]). How can this conclusion be understood and what was missing in standard treatments that claimed the QZE universality?

Our general approach [14] to dynamical control of states coupled to an arbitrary *zero-temperature* "bath" or continuum has reaffirmed the intuitive anticipation that, in order to suppress their decay, we must modulate the system-bath coupling at a rate exceeding the spectral interval over which the coupling is significant. The spectra of baths (continua) corresponding to vibrational or collisional decay or decoherence typically allow *dynamical suppression*, using realistic rates of modulation [14].

These results leave several basic questions open: Would *the two-level system (TLS) model hold* at all for modulation rates, that are comparable to the TLS transition frequency ω_a (between its states $|e\rangle$ and $|g\rangle$) which may invalidate the standard rotating-wave approximation (RWA) [20]? Would *temperature effects*, which are known to incur *upward* $|g\rangle \rightarrow |e\rangle$ transitions [21], further complicate the dynamics and perhaps hinder the suppression of decay? How to control decay in an efficient, *optimal* fashion? In Sec. 2 we address these questions within a general theory of a TLS that is coupled to an *arbitrary bath* and is driven by an *arbitrary* time-dependent field. In Sec. 3 we review the QZE and AZE in single-state decay. In Sec. 4 we introduce a new element: the measurement-assisted decay of multi-level systems, where quantum interference between the levels inhibits the decay, whereas its destruction by phase-randomizing measurements can accelerate it.

2 Dynamically Modified Decay and Decoherence in Two-Level Systems Coupled to Baths (Reservoirs)

The Hamiltonian in question is the sum of the system (S), reservoir bath (B) and system-bath interaction (I) terms,

$$H = H_S(t) + H_B + H_I(t), \quad H_I(t) = S(t)B(t). \quad (1)$$

Here $H_S(t)$ is the driven (and modulated) system Hamiltonian, $S(t)$ is a system operator and $B(t)$ is a bath operator, whose choice depends on the system-bath coupling (linear or quadratic, diagonal or off-diagonal). These operators vary with time due to the external fields. This general form of $H_I(t)$, unlike common treatments, *does not invoke the RWA* [20], which may fail for ultrafast modulation. The combined state of the system and the

bath is described by the density matrix $\rho_{S+B}(t)$. Let $\rho_{S+B}(0) = \rho_S(0) \otimes \rho_B$, $\rho_S(t)$ being the density matrix of the system and $\rho_B = Z^{-1}\exp[-(\beta/\hbar)H_B]$ the density matrix of the bath in equilibrium, with Z as the normalization factor, $\beta = \hbar/k_BT$ the inverse temperature (in frequency units), and k_B the Boltzmann constant. Using the projection-operator technique, we have derived, to *second order* in the coupling, the quantum ME in the following *differential* form

$$\dot{\rho} = -\frac{i}{\hbar}[H_S(t),\rho] + \int_0^t dt'\{\Phi(t,t')[\tilde{S}(t',t)\rho S(t) - S(t)\tilde{S}(t',t)\rho] + \text{H.c}\}. \quad (2)$$

Here $\Phi(t,t') = \langle U_B^\dagger(t-t')B(t)U_B(t-t')B(t')\rangle$ is the bath "memory" (correlation) function (CF), $\langle\ldots\rangle = \text{Tr}(\ldots\rho_B)$ and $\tilde{S}(t',t) = U_S(t,t')S(t')U_S^\dagger(t,t')$, written using the evolution operators $U_B(t) = \exp[(-i/\hbar)H_Bt]$ and

$$U_S(t,t') = T_+\exp\left[-\frac{i}{\hbar}\int_{t'}^t H_S(\tau)d\tau\right], \quad (3)$$

T_+ being the time-ordering operator. In the derivation of Eq. (2) we assumed that $\langle B(t)\rangle = 0$. It needs to be stressed that Eq. (2) *generalizes* previously known master equations [20] to *arbitrary* time-dependent hamiltonians, $H_S(t)$ for the system and $H_I(t)$ for system-bath coupling.

Henceforth, we explicitly consider a driven TLS undergoing decay, whose resonant frequency and *dipolar* coupling to the reservoir are dynamically modulated, so that

$$H_S(t) = \hbar[\omega_a + \delta_a(t) + \delta_r(t)]|e\rangle\langle e|,$$
$$H_I(t) = S(t)B = \tilde{\epsilon}(t)\sigma_x B. \quad (4)$$

Here $\delta_a(t)$ is the dynamically imposed Stark shift of the TLS resonance frequency, $\delta_r(t)$ is its *random* counterpart representing proper dephasing, $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$ is the dipole-transition operator, whose time-modulated form is given by $S(t)$, with the *real* amplitude $\tilde{\epsilon}(t)$. If the bath consists of oscillators, then

$$H_B = \sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger a_\lambda, \quad B = \hbar\sum_\lambda(\kappa_\lambda a_\lambda + \kappa_\lambda^* a_\lambda^\dagger), \quad (5)$$

where ω_λ and a_λ are the frequency and annihilation operator, respectively, of the mode λ and κ_λ is the coupling amplitude. Clearly, terms such as $|e\rangle\langle g|\kappa_\lambda^* a_\lambda^\dagger$ or $|g\rangle\langle e|\kappa_\lambda a_\lambda$ in the system-bath interaction $H_I(t)$ are *anti-resonant*, in violation of the RWA.

Upon using Eq. (4) in (2), we obtain our generalized Bloch equations for the components of the TLS density matrix (compare with [20])

$$\dot{\rho}_{ee} = -\dot{\rho}_{gg} = iV(t)(\rho_{eg} - \rho_{ge}) - R_e(t)\rho_{ee} + R_g(t)\rho_{gg}, \quad (6)$$

$$\dot{\rho}_{eg} = \dot{\rho}_{ge}^* = -\{R(t) + i[\tilde{\omega}_a(t) + \delta_a(t) + \delta_r(t)]\}\rho_{eg} + iV(t)(\rho_{ee} - \rho_{gg}) + [R(t) - i\Delta_a(t)]\rho_{ge}. \quad (7)$$

Equations (6) and (7) account for the presence of *upward transitions* $|g\rangle \to |e\rangle$ (caused by either temperature or anti-resonant effects – see below) at a rate $R_g(t)$, in addition to *downward* decay $|e\rangle \to |g\rangle$ at a rate $R_e(t)$. Their half-sum $R(t) = [R_e(t) + R_g(t)]/2$ contributes to the decoherence rate, which is further augmented by the random shift $\delta_r(t)$ (see below). The resonance frequency is dynamically shifted by $\tilde{\omega}_a(t) - \omega_a = \Delta_a(t) = \Delta_e(t) - \Delta_g(t)$, where $\hbar\Delta_{e(g)}(t)$ is the Lamb shift of $|e\rangle$ ($|g\rangle$), caused by the dynamically modified coupling to the bath. The last term on the right-hand side of Eq. (7) is known as "non-secular" [20]; though usually negligible, it can be important if the modulated resonant frequency $\omega_a + \delta_a(t)$ can vanish or be comparable to $R(t) + |\Delta_a(t)|$.

Here we consider situations wherein $R_{e(g)}(t)$ and $R(t)$, *the rates of decay and decoherence are dominant compared to the proper-dephasing rate* [determined by $\delta_r(t)$], so that the latter may be neglected in Eq. (7). The dynamically affected transition rates and shifts, obtained from Eqs. (4), (6) and (7), are then given by the real and imaginary parts of the expression

$$R_{e(g)}(t)/2 + i\Delta_{e(g)}(t) = \int_0^t dt' K_{e(g)}(t,t')\Phi_T(t-t') \quad (8)$$

Here $\Phi_T(t) = \langle\tilde{B}(t)\tilde{B}(0)\rangle$ is the bath CF at temperature T, where $\tilde{B}(t) = U_B^\dagger(t)BU_B(t)$ is the operator B in the interaction representation, and $K_i(t,t') = \langle i|S(t)\tilde{S}(t',t)|i\rangle$ ($i = e,g$) is the correlation function (CF) of the dipole moment in the state $|i\rangle$. One can show that

$$K_e(t,t') = K_g^*(t,t') = \epsilon(t)\epsilon^*(t'), \quad (9)$$

where $\epsilon(t) = \tilde{\epsilon}(t)\exp\left[i\int_0^t\delta_a(\tau)d\tau + i\omega_a t\right]$ is the coupling-modulation function, allowing for both amplitude and phase modulations.

Since we are interested here in dynamical control of relaxation, we shall concentrate on the transition rates $R_{e(g)}(t)$ rather than the level shifts. Using Eq. (8), one obtains that, for sufficiently short times, the probability of the $|e\rangle \to |g\rangle$ transition $\int_0^t dt' R_e(t')$ and its $|g\rangle \to |e\rangle$ counterpart $\int_0^t dt' R_g(t')$ are given

by

$$\int_0^t dt' R_{e(g)}(t') = 2\pi Q(t) \int_{-\infty}^{\infty} d\omega F_t(\omega) G_T(\pm\omega). \tag{10}$$

Here the upper (lower) sign corresponds to the subscript e (g), $Q(t) = \int_0^t d\tau |\bar{\epsilon}(\tau)|^2$, and

$$F_t(\omega) = [2\pi Q(t)]^{-1} \int_0^t \int_0^t dt' dt'' K_e(t', t'') e^{i\omega(t''-t')} \tag{11}$$

is the (normalized to unity) spectral density (SD) describing the modulation-induced splitting/shifting of the spectral line at the TLS transition frequency in the interval $(0, t)$. The SD of the bath CF

$$G_T(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} \Phi_T(t) e^{i\omega t} dt \tag{12}$$

can be shown [21] to be nonnegative, with $G_T(-\omega) = e^{-\beta\omega} G_T(\omega)$, and vanish for $\omega < 0$ at $T = 0$: $G_0(\omega) = 0$ $(\omega < 0)$. For the oscillator bath (5) one finds that

$$G_T(\omega) = [n(\omega) + 1] G_0(\omega) + n(-\omega) G_0(-\omega), \tag{13}$$

where $G_0(\omega) = \sum_\lambda |\kappa_\lambda|^2 \delta(\omega - \omega_\lambda)$ and $n(\omega) = (e^{\beta\omega} - 1)^{-1}$ is the average number of quanta in the oscillator (bath mode) with frequency ω.

We apply Eqs. (10) to the case of *coherent modulation of quasiperiodic* form, $\epsilon(t) = \sum_k \epsilon_k e^{i\omega_k t}$, where ω_k $(k = 0, \pm 1, \ldots)$ are arbitrary discrete frequencies with the minimal spectral distance Ω. Without a limitation of the generality, we can assume that $\sum_k |\epsilon_k|^2 = 1$. We then find, using Eq. (10), that the rates $R_{e(g)}(t)$ tend to the long-time limits

$$R_{e(g)} = 2\pi \int_{-\infty}^{\infty} d\omega F(\omega) G_T(\pm\omega), \tag{14}$$

where $F(\omega) = \lim_{t\to\infty} F_t(\omega) = \sum_k |\epsilon_k|^2 \delta(\omega - \omega_a - \omega_k)$, or

$$R_{e(g)} = 2\pi \sum_k |\epsilon_k|^2 G_T(\pm(\omega_a + \omega_k)). \tag{15}$$

Equation (14) is the *pivotal general expression* derived in this paper: it shows that R_e (R_g) *is given by the overlap of the modulation spectrum* $F(\omega)$ *with the bath-CF spectrum* $G_T(\omega)$ $[G_T(-\omega)]$. The limits (15) are approached when $\Omega t \gg 1$ and $t \gg t_c \equiv \max_k\{1/\xi(\pm(\omega_a + \omega_k))\}$. Here t_c is the bath memory (correlation) time, defined as the inverse of $\xi(\omega)$, the spectral interval over which $G_T(\omega)$ changes around the relevant frequencies.

Had we used the standard dipolar RWA hamiltonian in the case of an oscillator bath, dropping the antiresonant terms in $H_I(t)$ [Eqs. (4) and (5)], we would have arrived at the transition rates

$$R_{e(g)}^{\rm RWA} = 2\pi \int_0^{\infty} d\omega F(\omega) G_T(\pm\omega), \tag{16}$$

wherein the integration is performed from 0 to ∞, rather than from $-\infty$ to ∞, as in (14). This means that the RWA transition rates hold for a slow modulation, when $F(\omega) \simeq 0$ at $\omega < 0$, being peaked near ω_a. However, whenever the suppression of $R_{e(g)}$ requires modulation at a rate comparable to ω_a, the RWA is inadequate. For instance, Eqs. (13) and (16) imply that, at $T = 0$, the rate $R_g^{\rm RWA}$ vanishes identically, irrespective of $F(\omega)$, in contrast to the true upward-transition rate R_g in Eq. (14), which may be comparable to R_e for ultrafast modulation. The difference between the RWA and non-RWA decay rates stems from the fact that the RWA implies that a downward (upward) transition is accompanied by emission (absorption) of a bath quantum, whereas the non-RWA (negative-frequency) contribution to $R_{e(g)}$ in Eq. (14) allows for just the opposite: downward (upward) transitions that are accompanied by absorption (emission). The latter processes are possible since the modulation may cause *level* $|e\rangle$ *to be shifted below* $|g\rangle$.

The validity of the (decohering) TLS model in the presence of modulation at a rate $\gtrsim \omega_a$ is now elucidated: it requires that $R_{e(g)j} t \ll 1$, $R_{e(g)j}$ being the effective transition rate from level e (g) to *any other* level j, and, in particular, $R_{e(g)} t \ll 1$. If $R_{e(g)}$ are strongly suppressed by the modulation, the TLS model holds for long times.

3 Measurement-Modified Decay of a Single State Coupled to a Reservoir

Consider $|e\rangle$, the measured state in a system ruled by hamiltonian $H = H_0 + V$, where V causes the coupling (decay) of $|e\rangle$ to all other eigenstates of H_0, which we refer to as the "reservoir" (Fig. 1).

The probability amplitude $\alpha(t)$ to remain in $|e\rangle$, which has the energy $\hbar\omega_a$, obeys the following *exact* integro-differential equation

$$\dot{\alpha} = -\int_0^t dt' e^{i\omega_a(t-t')} \Phi(t - t') \alpha(t'). \tag{17}$$

Here $\alpha(t) = \langle e|\Psi(t)\rangle e^{i\omega_a t}$, $\hbar\omega_a$ is the energy of $|e\rangle$, and

$$\Phi(t) = \hbar^{-2} \langle e| V e^{-iH_0 t/\hbar} V |e\rangle = \hbar^{-2} \sum_j |V_{ej}|^2 e^{-i\omega_j t} \tag{18}$$

is the reservoir correlation function, expressed by $V_{ej} = \langle e|V|j\rangle$, where $|j\rangle$ $(\neq |e\rangle)$ are H_0 eigenvectors with eigenvalues $\hbar\omega_j$.

Equation (17) is *exactly* soluble, but it is enough to consider its short-time behavior by setting $\alpha(t) \approx \alpha(0) = 1$ in the integral of (1). This yields the expression

$$\alpha(t) = 1 - \int_0^t dt'(t-t')\Phi(t')e^{i\omega_a t'}, \qquad (19)$$

in which *all powers of* t (phase factors!) are included and interferences between various decay channels may occur. By contrast, the standard *quadratic* expansion in t for the population [3, 4] $\rho_{ee}(t) = |\alpha(t)|^2 \approx 1 - t^2/\tau_Z^2$, in which the Zeno time $\tau_Z = \hbar/(\langle e|H^2|e\rangle - \langle e|H|e\rangle^2)^{1/2}$ is the inverse variance of the energy in $|e\rangle$, may often fail, as discussed below. This is where we essentially differ from standard treatments. How does this difference show up?

Consider instantaneous measurements – projections on $|e\rangle$ interrupting its decay at intervals τ. We can use our result for $\alpha(t)$, Eq. (19), to express the population of $|e\rangle$ after n such measurements as exponentially decaying at a rate R,

$$\rho_{ee}(t = n\tau) = |\alpha(\tau)|^{2n} \approx \exp(-Rt). \qquad (20)$$

The *universal form* of R is (in the frequency domain)

$$R = 2\pi \int_0^\infty d\omega G(\omega)F(\omega). \qquad (21)$$

This expression is the overlap of the reservoir-coupling spectrum

$$G(\omega) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty dt\Phi(t)e^{i\omega t} = \hbar^{-2}\sum_j |V_{ej}|^2\delta(\omega-\omega_j) \qquad (22)$$

and the measurement-induced broadening of the measured energy level

$$F(\omega) = \frac{\tau}{2\pi}\mathrm{sinc}^2\left(\frac{(\omega-\omega_a)\tau}{2}\right). \qquad (23)$$

We can interpret the universal result (4) as an expression of the *time-energy uncertainty relation* for an unstable level with lifetime Δt, relating the energy broadening (uncertainty) of $|e\rangle$ to Δt, the interval between measurements (Fig. 1),

$$\text{Eq. (21)} \quad \Longrightarrow \quad \Delta E\Delta t \sim \hbar. \qquad (24)$$

More generally, in Eq. (24) $\Delta t = 1/\nu$, where ν is a characteristic rate of measurements. With this definition, Eq. (24) holds both for ideal and nonideal (realistic) measurements. Relation (24) comes about since measurements (projections) dephase level $|e\rangle$, analogously to phase randomization by collisions, which induce a linewidth that is equal to the collision rate ν.

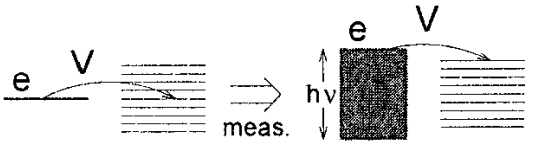

Figure 1: Left: The decay of a state into a "reservoir" via coupling. Right: Measurements broaden level $|e\rangle$, analogously to phase randomization by collisions at rate ν, drastically changing its decay into the reservoir.

A simple graphical analysis of the universal result Eq. (21) yields the main conclusions:

a) The QZE scaling (i.e., a *decrease of the decay rate* R *with an increase of* ν), is generally obtained when the measurement (dephasing) rate ν is much larger than the reservoir spectral width:

$$\nu \gg \Gamma_R, |\omega_a - \omega_M|. \qquad (25)$$

Here Γ_R is the reservoir width and ω_M is the center of gravity of $G(\omega)$. In the special case of a peak-shaped $G(\omega)$, in Eq. (25) ω_M can be replaced by the position ω_m of the maximum. In the limit (25), one can approximate the spectrum $G(\omega)$ by a δ-function, with a constant C being the integrated spectrum,

$$C = \int G(\omega)d\omega = \langle V^2\rangle. \qquad (26)$$

This approximation becomes *exact* in the case of resonant Rabi oscillations ($\Gamma_R = 0$, $\omega_a = \omega_M$), which explains why *the QZE is observable in Itano's experiment* [7] *for any* ν. More generally, this approximation holds for any $G(\omega)$ that falls off faster than $1/\omega$ on the wings. Then our universal expression yields the most general result for the QZE, namely that R decreases with ν:

$$R \approx 2C/\nu, \qquad (27)$$

where we defined generally

$$\nu = [\pi F(\omega_a)]^{-1} \qquad (28)$$

In particular, as follows from Eq. (23), $\nu = 2/\tau$ for instantaneous projections. The *flattening of the spectral peak* of $G(\omega)$ by the broad function $F(\omega)$ in the convolution is seen to be the origin of the QZE. To put it simply, if the system is probed frequently enough, the QZE arises since *the effective decay rate is averaged over all decay channels*, many of which are weak, due to the energy uncertainty incurred by the measurements.

This result contradicts the claim of QZE universality and demonstrates the failure of the standard

quadratic expansion: Eq. (8) shows that the QZE conditions can be much more stringent than the requirement to have $t \sim 1/\nu \ll \tau_Z$. The crucial point emphasized below is that Eq. (25) may be *principally impossible* to satisfy.

Under condition (25), the QZE scaling of R occurs for any $G(\omega)$ such that

$$G(\omega) \to 0 \quad \text{at} \quad \omega \to \infty. \tag{29}$$

Conditions (25) and (29) ensure the QZE scaling only for sufficiently large measurement rates and *do not always imply a monotonous decrease* of R as ν increases. The latter behavior, which is what one usually has in mind in discussions of the QZE, is obtained only in special situations.

b) The opposite to the QZE scaling is obtained whenever ω_a is significantly detuned from the nearest maximum of $G(\omega)$ at ω_m, so that $G(\omega_a) \ll G(\omega_m)$. In the limit (Fig. 2 - inset):

$$\nu \ll |\omega_m - \omega_a|. \tag{30}$$

the rate R *grows* with ν, since the dephasing function $F(\omega)$ is then probing more of the *rising* part of $G(\omega)$ in the convolution. This limit implies the *anti-Zeno effect* (AZE) of decay acceleration by frequent measurements. Physically, this means that, as the energy uncertainty increases with the measurement rate ν, *the state decays into more and more channels, whose weight $G(\omega)$ is progressively larger.*

Remarkably, we may impose condition (10) in *any reservoir* that is not spectrally flat. This reveals the *universality* of the AZE, which has been noted already for radiative decay in cavities [16].

As an example, consider the coupling spectrum of the form

$$G(\omega) = A\omega^\eta, \quad 0 < \omega < \omega_C,$$
$$G(\omega) = 0 \quad \text{otherwise}, \tag{31}$$

so that $\omega_a \ll \omega_C$. Using a Lorentzian $F(\omega)$, as obtained for realistic (continuous) measurements [5, 6, 7], one obtains from Eq. (21) that for $\nu \ll \omega_C$

$$R = \frac{2\pi A(\omega_a^2 + \nu^2)^{\eta/2} \sin \eta\chi}{\sin \eta\pi} \quad (0 < \eta < 1), \tag{32}$$

$$R = A\left[\nu \ln \frac{\omega_C^2}{\omega_a^2 + \nu^2} + \omega_a\left(\pi + 2\tan^{-1}\frac{\omega_a}{\nu}\right)\right] \quad (\eta = 1) \tag{33}$$

$$R = 2\pi A\omega_a^\eta + \frac{2A}{\eta - 1}\nu\omega_C^{\eta-1} \quad (\eta > 1), \tag{34}$$

where $\tan\chi = -\nu/\omega_a$ $(0 < \chi < \pi)$. Obviously, expressions (32)-(34) increase with ν.

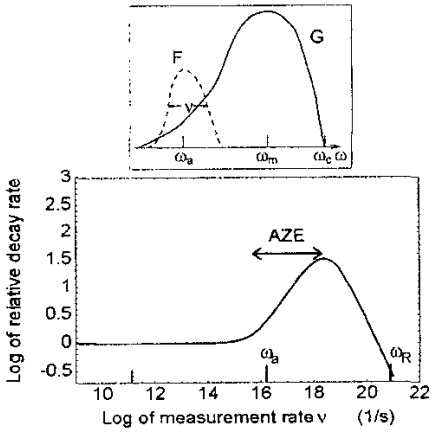

Figure 2: AZE dependence of the decay rate. Inset: Conditions for the AZE [Eq. (30)] Graph: the dependence of the logarithm of the normalized decay rate $\log_{10}(R/R_{RG})$ on $\log_{10}\nu$ for a spontaneously emitting hydrogenic state. The atomic transition frequency: $\omega_a = 1.55 \times 10^{16}\ s^{-1}$, whereas the relativistic cutoff $\omega_R = 7.76 \times 10^{20}\ s^{-1}$. The corresponding Bohr frequency is $\omega_B = 8.50 \times 10^{18}\ s^{-1}$. The AZE range is marked.

c) More subtle behavior occurs in the domain between the QZE and AZE limits. Assume, for simplicity, that $G(\omega)$ is single-peaked and satisfies condition (29). When ν increases from the limit (30) up to the range where the right-hand-side inequality is violated, then $\nu \gg |\omega_m - \omega_a|$, which is now equivalent to condition (25), implying the Zeno scaling of Eq. (27). But even in this QZE-scaling regime, R *remains larger than the Golden Rule rate* (i.e., the normal decay rate in the absence of measurements)

$$R_{GR} = 2\pi G(\omega_a), \tag{35}$$

up to much higher ν, as expressed by the following condition for *"genuine QZE"*

$$R < R_{GR} \quad \text{for} \quad \nu > \nu_{QZE}, \tag{36}$$

where

$$\nu_{QZE} = 2C/R_{GR} = C/\pi G(\omega_a) \tag{37}$$

in the case of a finite C.

The value of ν_{QZE} given by Eq. (37) was identified with the reciprocal "jump time", i.e., the maximal time interval between measurements for which the decay rate is appreciably changed [22]. However, the *correct value* of the reciprocal jump time may be smaller by many orders of magnitude than ν_{QZE}. In the special case of ideal instantaneous measurements and a Lorentzian or Lorentzian-like $G(\omega)$, the genuine-QZE condition (36) reduces to that of Ref. [23].

These considerations apply (with some limitations) to hydrogenic radiative decay (spontaneous emission), for which $G(\omega)$ can be calculated *exactly* [24]:

$$G(\omega) = \frac{\alpha\omega}{[1+(\omega/\omega_{\rm B})^2]^4}. \quad (38)$$

Here $\omega_{\rm B} \sim c/a_{\rm B}$, where c is the vacuum light speed and $a_{\rm B}$ is the radius of the electron orbit. Then Eqs. (21) and (23) may yield the AZE trend

$$R \approx \alpha\nu[\ln(\omega_{\rm B}/\nu) + C_1] \quad (\omega_{\rm a} \ll \nu \ll \omega_{\rm B}), \quad (39)$$

where $C_1 = 0.354$ and $\nu = 2/\tau$. The AZE trend should be observable (Fig. 2) for $\nu \gtrsim \omega_{\rm a}$, i.e., for microwave Rydberg transitions on a ps scale (provided we can isolate one transition) and for optical transitions on the sub-fs scale. The boundary between the AZE and QZE-scaling regions is now given by $\nu_1 \sim \omega_{\rm B}$ and the genuine-QZE condition (36) by $\nu > \nu_{\rm QZE} \sim \omega_{\rm B}^2/12\pi\omega_a \gg \nu_1$ [cf. Eq. (37)], rendering $R < R_{\rm GR} = 2\pi\alpha\omega_a$.

This analysis implies that the "genuine QZE" range $\nu > \nu_{\rm QZE}$ is *principally unattainable*, since it requires measurement rates above the relativistic cutoff $\omega_{\rm R}$, which are *detrimental* to the system, leading to the production of new particles. A similar principal obstacle occurs for radioactive decay. By contrast, the AZE is *accessible* in decay processes, such as spontaneous emission or the nuclear β-decay, and can essentially always be imposed.

4 Decay Modification by Measurements in Multilevel Systems

Here we discuss in detail a model for measurement-induced decay modification in a multilevel system. The system with energies $\hbar\omega_n$, $1 \le n \le N$, is coupled to a zero-temperature bath of harmonic oscillators with frequencies ω_λ. The corresponding Hamiltonian, in the rotating wave approximation, is

$$H = \hbar\sum_n \omega_n |n\rangle\langle n| + \hbar\sum_\lambda \omega_\lambda a_\lambda^\dagger a_\lambda + \hbar\sum_{n,\lambda}(\kappa_{n,\lambda} a_\lambda^\dagger |g\rangle\langle n| + \kappa_{n,\lambda}^* a_\lambda |n\rangle\langle g|). \quad (40)$$

We note that the same Hamiltonian decribes an n-level system coupled to a continuum, as occurs, e.g., in autoionization. The both cases are described by the same equations.

The wavefunction is

$$|\Psi(t)\rangle = \sum_n \alpha_n(t)|n,\{0_\lambda\}\rangle + \sum_\lambda \beta_\lambda(t)|g,1_\lambda\rangle, \quad (41)$$

where $\{0_\lambda\}$ and 1_λ denote, respectively, the bath ground state and one quantum in the mode λ. Inserting Eqs. (40) and (41) into the Schrödinger equation yields the set of equations

$$\dot\alpha_n = -i\omega_n\alpha_n - i\sum_\lambda \kappa_{n,\lambda}^*\beta_\lambda, \quad (42)$$

$$\dot\beta_\lambda = -i\omega_\lambda\beta_\lambda - i\sum_n \kappa_{n,\lambda}\alpha_n. \quad (43)$$

The solution of Eq. (43) with the initial condition $\beta_\lambda(0) = 0$ is

$$\beta_\lambda(t) = -i\int_0^t dt' e^{-i\omega_\lambda(t-t')} i\sum_n \kappa_{n,\lambda}\alpha_n(t'). \quad (44)$$

Inserting Eq. (44) into (42) yields the set of integro-differential equations

$$\dot\alpha_n = -i\omega_n\alpha_n - \sum_{n'}\int_0^t dt'\Phi_{nn'}(t-t')\alpha_{n'}(t'). \quad (45)$$

Here

$$\Phi_{nn'}(t) = \int d\omega G_{nn'}(\omega)e^{-i\omega t}, \quad (46)$$

where

$$G_{nn'}(\omega) = \sum_\lambda \kappa_{n\lambda}^*\kappa_{n'\lambda}\delta(\omega-\omega_\lambda). \quad (47)$$

The nondiagonal terms here result in *multilevel quantum interference*. One can show that

$$|G_{nn'}(\omega)|^2 \le G_{nn}(\omega)G_{n'n'}(\omega). \quad (48)$$

Hence maximum quantum interference is achieved when Eq. (48) is the equality. For example, if all transition dipoles are parallel, $\vec d_{ng} \parallel \hat x$, where $\hat x$ is the unit vector along the x axis, then Eq. (49) holds, with

$$G_{nn'}(\omega) = d_n^* d_{n'} g(\omega), \quad (49)$$

$$d_n = \vec d_{ng}\cdot\hat x, \quad g(\omega) = \sum_\lambda |\vec q_\lambda\cdot\hat x|^2\delta(\omega-\omega_\lambda). \quad (50)$$

Let the reservoir be specifically Lorentzian,

$$g(\omega) = \frac{1}{\pi}\frac{A\Gamma}{(\omega-\omega_R)^2+\Gamma^2}, \quad (51)$$

where ω_R and Γ are the reservoir center frequency and width, respectively. Equations (46), (49), and (51) yield that

$$\Phi_{nn'}(t) = d_n^* d_{n'} A e^{-(i\omega_R+\Gamma)t}. \quad (52)$$

In this case the reservoir can be substituted by one level $|0\rangle$ with the energy $\hbar\omega_R$ and the decay rate 2Γ, the effective Hamiltonian being

$$H_{\rm eff} = \hbar\sum_{n=1}^N \omega_n|n\rangle\langle n| + \hbar(\omega_R - i\Gamma)|0\rangle\langle 0| + \quad (53)$$

$$+ h\sum_{n=1}^{N}(\kappa_n|0\rangle\langle n| + \kappa_n^* n\rangle\langle 0|),$$

where κ_n is the coupling matrix element for the $|n\rangle$-$|g\rangle$ transition. The wavefunction is now

$$|\Psi(t)\rangle = \sum_{n=1}^{N}\alpha_n(t)|n\rangle + \beta(t)|0\rangle, \tag{54}$$

with the initial codition $\beta(0) = 0$. The Schrödinger equation now yields

$$\dot{\alpha_n} = -i\omega_n\alpha_n - i\kappa_n^*\beta, \tag{55}$$

$$\dot{\beta} = -(i\omega_R + \Gamma)\beta - i\sum_{n}^{N}\kappa_n\alpha_n. \tag{56}$$

The equivalence of the effective and exact Hamiltonians for the relaxation of the system follows from the fact that the exclusion of $\beta(t)$ from Eqs. (55) and (56) yields Eq. (45) with definition (52), if one sets

$$\kappa_n = d_n\sqrt{A}. \tag{57}$$

Thus, one can simulate relaxation of an n-level sytem, at least, in case (49), (51), by solving the set of equations (55) and (56). This is simpler than considering a more general case, when one should solve differential equations (42) and (43) or, alternatively, the integro-differential equations (45).

Consider now how one can modify the decoherence of the system. In order to imitate repeated instantaneous measurements, one can use a train of short, *random*, nonresonant pulses (see also [25, 26]). Let the pulses be sufficiently strong, so that the phase shift of the amplitude of state $|n\rangle$ (relative to that of $|g\rangle$) due to each pulse is large,

$$\int_{t_k}^{t_k'} dt(\chi_n - \chi_g)|E_k(t)|^2 \gg 2\pi. \tag{58}$$

Here χ_n is the polarizability of level $|n\rangle$, whereas t_k and t_k' are the beginning and end moments of the kth pulse. If these pulses slightly differ one from another (i.e., in their integrated intensity), the phase shifts (47) can be considered *completely random*. For simplicity, we assume that χ_n's are the same for all levels $|n\rangle$. We also assume that the pulses are randomly distributed in time with the average interval T, take equal and real $\kappa_n = k$ and equidistant ω_n. Then, for the above case where (53) applies one obtains the equation for the density matrix

$$\dot{\rho} = -(i/\hbar)[H_{\text{eff}}, \rho] - R\rho. \tag{59}$$

Here R is the superoperator of measurement-dependent relaxation, whose only nonzero elements are $R_{ng,ng} = R_{gn,gn} = 1/T$, the measurement rate. With the increase of $1/T$, the relaxation dynamics should change.

Quantum interference creates a *decoherence-free subspace* and thus slows down the relaxation. Therefore, one may expect that nonzero Γ and $1/T$ can in this case *accelerate* decay. Quantum interference will be significant only for sufficiently large k, so that $\pi G(\omega_n) \gtrsim \omega_{n+1,n}$, for some n. To observe *reduced decay* with $1/T$ (the analog of QZE), one should take $\Gamma \gtrsim (\omega_n - \omega_1)/2$ and $1/T \gtrsim \Gamma$.

This expectation was indeed confirmed by our numerical studies. The system was acted upon by impulsive kicks which had the property of randomly changing all phases of the complex coefficients α_n, $n = 1, \ldots, N$. Kicks were effected at random times, with a Poisson distribution of parameter T. We have assumed equally spaced ω_n in an interval of unit width, ω_R equal to the center of this interval, all coupling constants $\kappa_n = \kappa_n^* = k$, $N = 16$ (number of levels) and set the initial state $\Psi(0) = |4\rangle$ or $|8\rangle$.

In Fig. 3 we show the time evolution of the system *without* external forcing, *i.e.* when $T = \infty$. We can observe how the oscillations in the occupation probability diminish in time, due to coupling to the $|0\rangle$ state, which has a finite life-time.

By comparison, Fig. 4 depicts the same evolution, with external forcing, $T = 100$. A *much faster decay* of the wave-packet is observed. This happens in coincidence with the decoherence of the wave-packet, induced by phase shuffling.

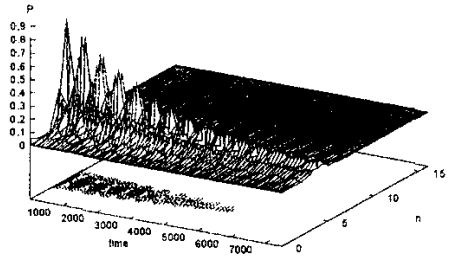

Figure 3: Wave function probability coefficients $|\alpha_n(t)|^2$ versus n and time t. Here, $k = 0.4$, $\Gamma = 0.05$, and $T = \infty$. The initial state is $|4\rangle$. Decay happens via coupling to the $|0\rangle$ state.

We concentrate our attention on the *survival probability* $P(t) = \sum_{n=1}^{N}|\alpha_n(t)|^2$, which measures the population on the N states of the system. This quantity is plotted in Fig. 5 versus time for the case of the two previous figures, with and without external

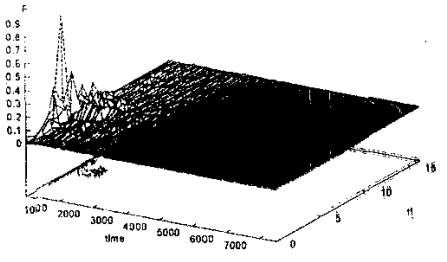

Figure 4: Wave function probability coefficients $|\alpha_n(t)|^2$ versus n and time t. Here, $k = 0.4$ and $\Gamma = 0.05$, $T = 100$. The first decohering kick takes place at $t = 53$. The initial state is $|4\rangle$.

forcing. This figure shows the data of a single realization, as well as those of an average over a large number of realizations. For the former, also reported in figure are the times at which external kicks have taken place. We see in Fig. 5 that a strongly enhanced decay of probability is clearly associated with decoherence.

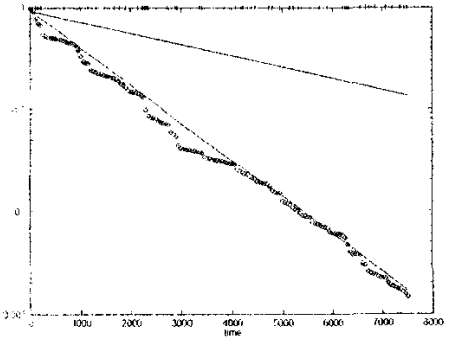

Figure 5: Survival probability P versus time for $k = 0.4$ and $\Gamma = 0.05$. Different curves are for $T = \infty$ (full line), corresponding to fig. 3, $T = 100$, one realization of the stochastic process (diamonds-fig. 4), and average over 200 realizations (dotted line). Crosses on the axis at $P = 1$ mark the times of external kicks in the single realization data.

In Fig. 6 we observe that the decay is always augmented by increasing either Γ or $1/T$ or both. Two limiting behaviors emerge: (i) For large T (compared with the relaxation time $1/R$ of the unperturbed system), the decay is obviously the one dictated by the Hamiltonian (53). (ii) As T tends to zero, a limit curve of P versus Γ is obtained: it is roughly parallel to the $T \to \infty$ curve, but represents stronger decay for a given value of Γ.

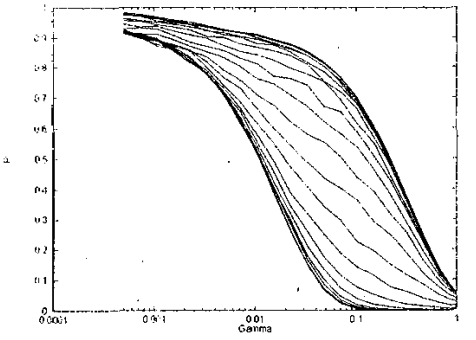

Figure 6: Final survival probability at $t = 503$ for $k = .4$ versus Γ. Different curves (from top to bottom) correspond to the (increasing) values of $\frac{1}{T}$.

5 Conclusions

To conclude, we have presented a unified, comprehensive anlysis of dynamically suppressed decay and decoherence in driven TLS coupled to finite-temperature baths and undergoing random frequency fluctuations. This treatment has resulted in both principal and practical general conclusions: (a) Anti-resonant (non-RWA) effects that cause dynamically modified "upward" ($|g\rangle \to |e\rangle$) transitions may dominate over their temperature-activated counterparts. (b) Control (qubit-flipping) fields can yield resonantly suppressed dephasing [Eq.(31)] as effectively as the "bang-bang" technique. (c) The dependence of dynamically suppressed decay on ω_a and T [Eqs. (11) and (13)] allows us to design the optimal phase jumps or Stark shifts Δ, i.e., the largest τ or the smallest Δ that can effect the suppression.

The universal formulae derived in Secs. 3 and 4 result in the following general criteria:
(i) The QZE can only occur in systems with spectral width below the resonance energy.
(ii) It is *principally unattainable* in open-space radiative or nuclear β-decay, because the required measurement rates would cause the creation of new particles.
(iii) Contrary to the widespread view, frequent measurements can be chosen to *accelerate* essentially any decay process. Hence, the *anti-Zeno effect (AZE)* should be *far more ubiquitous than the QZE*.
(iv) Acceleration of decay by frequent measurements can occur in a multilevel system, where interference effects (in the absence of phase-randomizing measurements) can inhibit the decay.

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