STATISTICS OF THE LASER RADIATION AT THRESHOLD *

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The ensemble distribution of the intensity fluctuations of a laser at threshold has been measured. Furthermore, the simultaneous measurement of linewidth and photon number at threshold gives a parameter connected to the atomic dipole noise.

The statistical properties of the radiation emitted from a single-mode c.w. gas laser measured by us in the threshold region fit the results of some recent theoretical works [1-4].

Preliminary measurements on laser fluctuations above threshold and rather far from the threshold point could be explained by an approximate model in which the field is a linear superposition of a Gaussian field plus an amplitude stabilized field [5-7]. Smith and Armstrong [8] were able to go closer to threshold, and their experimental points fitted the results of the non linear oscillator theories [1-4] but they left unexplored a decade of intensities around threshold.

Our measurements have been obtained by careful control of the environmental conditions to avoid noise, high stabilization of the pump power (1 part in 10^4) and a feedback system described below. We have used a 6328 Å He-Ne laser, d.c. discharge, in a confocal cavity 20 cm long and with a Fresnel number ~ 1, supporting a single TEM_{00} mode, with one mirror supported by a piezoceramic disc.

To reach a high accuracy we "prepared the initial state", i.e. forced the dynamical system in a wanted condition before measuring each sample, as follows: Before each measurement a periodic 1msec monitor pulse, applied to the piezoceramic, drives the mode well above threshold up to a point where the laser intensity is sampled and compared with a standard, any error signal giving a d.c. correction to the piezoceramic tuner. Thus, we separate the operating point (which can be at low intensity, and with high fluctuations) from the control point (which must be at high intensity, and with low fluctuations) but keeping a fixed frequency difference between the two positions of the cavity mode, so that any correction is active also on the operating point ***. After each monitor pulse, the laser intensity relaxes to a steady state condition with a relaxation time of maximum value $\tau = 140$ µsec near threshold, and photocount distributions $p(n)$ are measured at the end of a time interval $T > 10\tau$. The repetition rate of the monitor pulses is adjusted to take measuring samples for away from the region perturbed by the control operation.

The technique for measuring $p(n)$ has been reported elsewhere [7]. Each sample is taken over a measurement time of 1 µsec (shorter than the shortest coherence time measured in the spanned range $0.05 < M_1/M_{10} < 15$). For comparing experiments and theory we used the second reduced factorial moment of the photocount distribution, $H_2 = (\langle n(n-1)\rangle - \langle n\rangle^2)/\langle n\rangle^2$, which goes from 1 (Gaussian field distribution; well below threshold) to 0 for an amplitude stabilized field (well above threshold) (fig. 1).

From the staionary solution of the Fokker-Plank equation [1] for the statistical distribution of the laser field one can derive the distribution of photocounts from a photodetector, within a coherence time of the field [9]. The associated k-th factorial moment is

$$\langle n(n-1)\dots(n-k+1)\rangle = \\ = k!\left(\sqrt{\frac{\pi}{a}}\,M_{10}\right)^k \sqrt{\frac{2}{\pi}}\,\frac{\exp{-a^2}}{1+\operatorname{erf} a}\,D_{-(k+1)}(\sqrt{2}a) \qquad (1)$$

where M_{10} is the average number of photoelectron counts at threshold, a is a quantity proportional

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*** Obviously this way of control adds a phase perturbation, but this is immaterial when measuring the intensity fluctuations through photocount statistics.

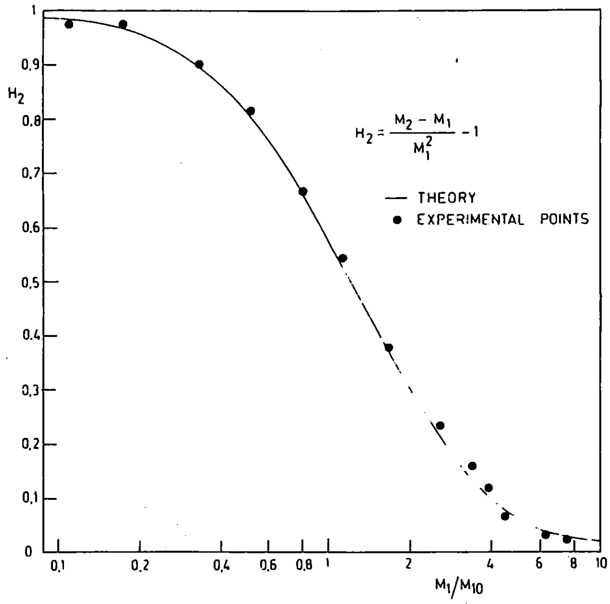

Fig. 1. Measured and theoretical values (from (1) of the reduced second order factorial moment H_2 as a function of the normalized intensity M_1/M_{10} in the threshold region.

to the difference between the actual population inversion and the threshold population inversion [1] and $D_{-(k+1)}$ is the parabolic cylinder function [10].

Our method of stabilization has allowed an accurate measurement of the relaxation times of the intensity fluctuations and a calibration of the photon number inside the cavity. For the laser and the pump power used we have measured, at threshold, a bandwidth $\Delta\nu = 1.4$ kHz and a photon number $\langle n_s \rangle \approx 4000$. The product of these two quantities is related to the correlation function of the noise source, i.e. calling κ the cavity damping constant, $\gamma_\perp = \frac{1}{2}(\gamma_a + \gamma_b)$ the relaxation time of the induced atomic dipoles, N_2 the population of the upper level of the laser transition, and $(N_2 - N_1)_s$ the population inversion at threshold, one finds [11]

$$\langle n_s \rangle \Delta\nu_s = A\, \frac{\kappa}{2} \left(\frac{\gamma_\perp}{\kappa + \gamma_\perp}\right)^2 \left(n_{th} + \frac{N_2}{(N_2 - N_1)_s}\right)$$

where $A \approx 1$ and n_{th} is the blackbody photon number, which can be neglected. Using the reported value [12] $\gamma_\perp = 1.2 \times 10^8 (\text{sec}^{-1})$ and the measured values $\kappa = 2.7 \times 10^7 (\text{sec}^{-1})$ and $\Delta\nu_s$ and $\langle n_s \rangle$ as above, we find $N_2/(N_2 - N_1)_s \approx 1$, i.e. the atomic system behaves as a four-level system *.

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* This is by no means obvious, since it is easy to have a He-Ne laser working in a three-level (or intermediate) region by increasing the pump power until the terminal state of the laser transition is appreciably populated [13].