2 lavori: p.225 e 345 del vol. Proc AIP Conf.Proc.742 (2004)-AIP-Melville New York

CONTROL AND SYNCHRONIZATION, OF HETEROCLINIC CHAOS : Implications for Neurodynamics

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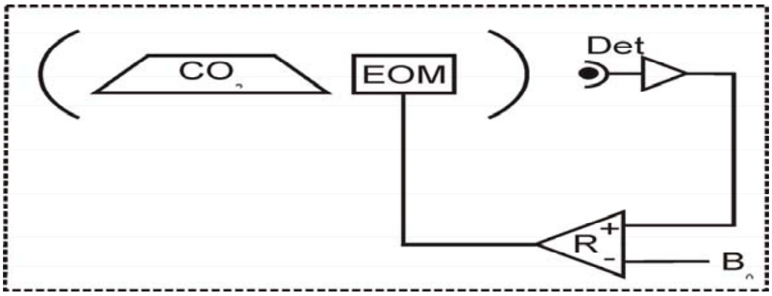
Abstract. Heteroclinic chaos (HC) implies the recurrent return of the dynamical trajectory to a saddle focus (SF) in whose neighborhood the system response to an external perturbation is very high and hence it is very easy to lock to an external stimulus. Thus HC appears as the easiest way to encode information in time by a train of equal spikes occurring at erratic times. Implementing such a dynamics with a single mode CO2 laser with feedback, we have a heteroclinic connection between the SF and a saddle node (SN) whose role it to regularize the phase space orbit away from SF. Due to these two different fixed points, the laser intensity displays identical spikes separated by erratic ISIs (interspike intervals). Such a dynamics is highly prone to spike-synchronization, either by an external signal or by mutual interaction in a network of identical systems. Applications to communication and noise induced synchronization will be reported. In experimental neuroscience a recent finding is that feature binding ,that is, combination of external stimuli with internal memories into new coherent patterns of meaning, implies the mutual synchronization of axonal spike trains in neurons which can be far away and yet share the same sequence. Several dynamical systems have been proposed to model such a behavior. We introduce a measurable parameter, namely, the synchronization “propensity”. Propensity is the amount of synchronization achieved in a chaotic system by a small sinusoidal perturbation of a control parameter. It is very low for coupled Lorenz or FitzHugh-Nagumo chains. It displays isolated peaks for the Hindmarsh-Rose model, showing that this is a convenient description of the bursting behavior typical of neurons in the CPG (central pattern generator) system. Instead, HC shows a high propensity over a wide input frequency range, demonstrating that it is the most convenient model for semantic neurons.

The starting point was (Fig.1) the study of a single mode CO2 laser with feedback, ruled by three coupled equations which provide chaos [1]. This chaos is of a novel type; in fact it consists of geometrically identical spikes which occur at erratic interspike intervals (ISI) (Fig.2).The attractor reconstructed by an embedding technique shows a confined chaotic tangle around a saddle focus (SF) plus a wide regular loop away from SF. The zero intensity point is another fixed point; it appears as a cusp in the attractor and as a flat baseline preceding the large spike in the time domain; in terms of local eigenvalues it corresponds to a saddle node (SN). Notice that the ISI fluctuations are due to the region around SF, whereas the approach to SN, as well as the development of the large spike, takes fixed amounts of time.

CO2 laser with feedback



Skeleton of the 3D model

$$\dot{x} = -k_0 x(1 - k_1 sin^2 z) + Gxy,$$
$$\dot{y} = -2Gxy - \gamma y + p_0,$$
$$\dot{z} = \beta(-z + b_0 - R \cdot x).$$

X laser intensity
Y population inversion
z feedback signal

FIGURE 1. Experimental set up of a CO_2 laser with electro optical loss modulator (EOM) fed by a voltage proportional to the detected output intensity.

To make the model more accurate, one should account that the resonant molecular levels interact with a rotational manifold; this provides three more kinetic equations , which act as linear filters; thus an accurate model implies 6 equations [2] and indeed this provides a good agreement with the experiment. However, around SF the extra variables, which are fast compared with x,y ,z, can be adiabatically eliminated and the phase space reduced to 3D as in Fig.2.

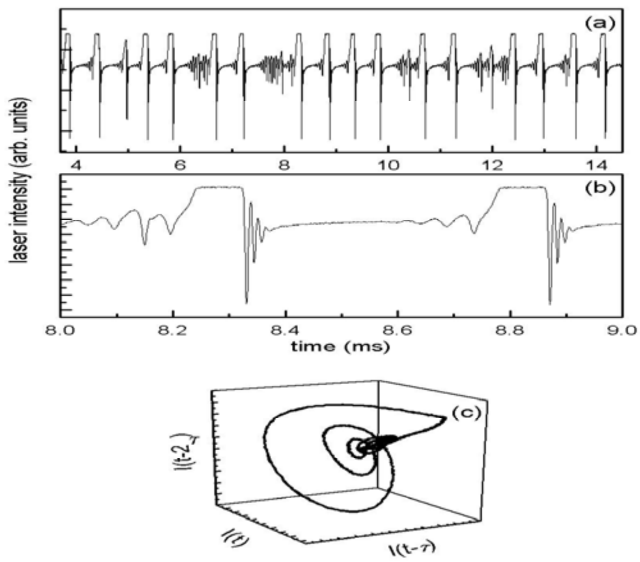


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FIGURE 2. (a) Experimental time series of the laser intensity for a CO2 laser with feedback in the regime of heteroclinic chaos. (b) Time expansion of a single orbit. (c) Phase space trajectory built by an embedding technique with appropriate delays

The heteroclinic chaos (HC) resulting from the interplay of SF and SF is common to other systems as e.g. Lorenz. In fact Lorenz has two SF and one SN, but an inversion operation around the origin maps the SF one onto the other, thus the two systems are formally similar. However, for the standard parameter values of Lorenz, SN is close to the chaotic tangle; instead in HC ,SN is far away from the chaotic tangle and hence it plays a stabilizing action on the orbit away from SF. This is the basis of the high propensity to synchronization of HC [3], which makes such a system an attractive model for neurons in the brain, as demonstrated later.

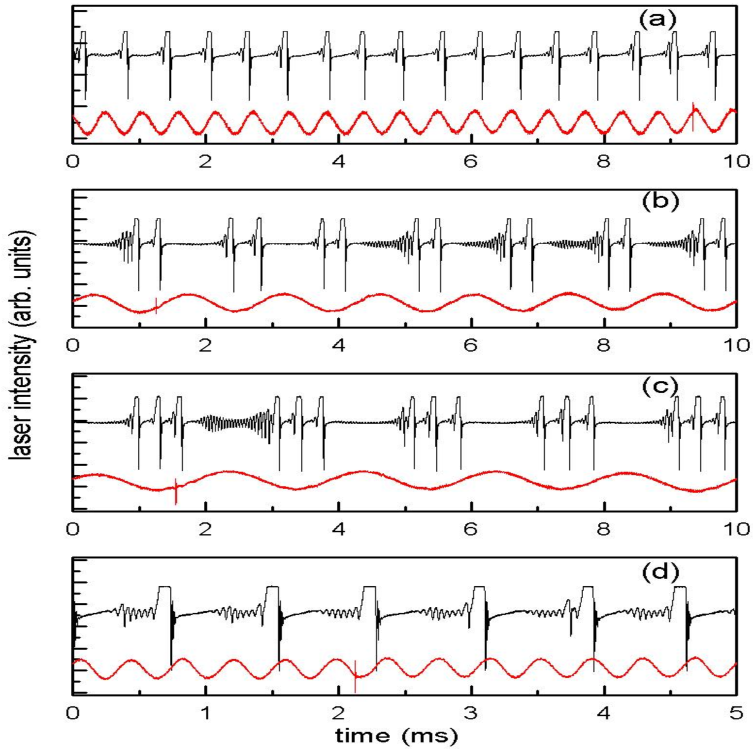


FIGURE 3. Experimental time series for different synchronization induced by periodic changes of the control parameter. (a) 1:1 locking, (b) 1:2, (c) 1:3, (d) 2:1

Fig.3 shows how the chaotic spike train locks to an external periodic perturbation, with a 1:1 locking if the applied period is close to the natural one, that is to the average repetition [4].

For shorter or longer forcing periods we observe 1:2,1:3 or 2:1 etc locking ratios. For frequencies away from the natural one or its multiples the HC systems looses perfect synchronization ,yielding extra spikes or missing spikes. If we normalize the ISI to ,then this looks like the occurrence of phase jumps, that we call phase slips. Fig. 4 shows the positive or negative phase slips for different perturbing frequencies.

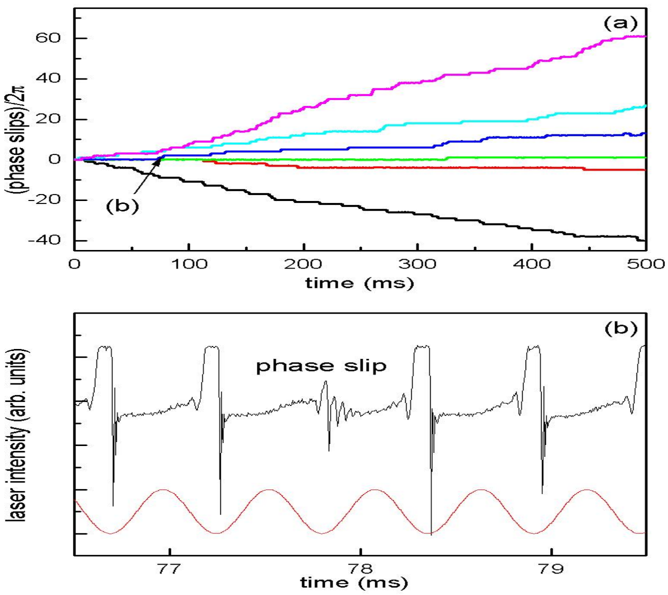


FIGURE 4. (a) Phase slips at different frequencies . The dynamical system monotonically lags or leads in phase depending on whether the modulation frequency is above or below the perfect synchronization (no slips) value. (b) Expanded view of the time in a phase slip. The phase slip is like a defect in a periodic one-dimensional pattern; in fact in this case it is a missed return to the surface of section.

The core of the spike synchronization here presented is the local variability of the maximum Lyapunov exponent, which is highly positive in the small chaotic tangle around SF and zero away from that region, yielding a global positive Lyapunov exponent. This local variability is exploited to induce a synchronization between to identical HC systems subjected to a noise sufficient to reduce the density of points in the chaotic tangle with respect to the regular region [5].

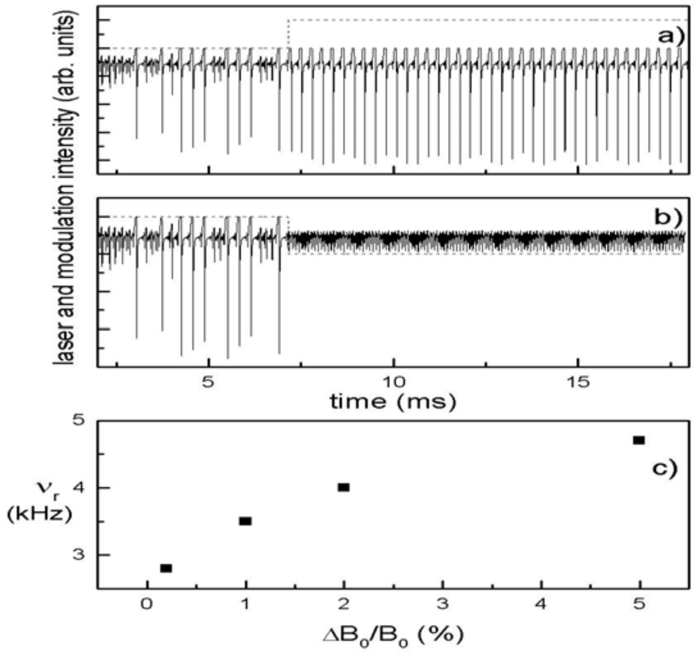


FIGURE 5. Stepwise increase (a) and decrease (b) of control parameter B_0 by ±1% brings the system from heteroclinic to periodic or excitable behavior. (c) In case (a) the frequency of the spikes increases monotonically with B_0

We have shown [2] that the ranges of control parameters for which HC occurs are intercalated by periodic or excitable intervals. Indeed a small jump of the bias voltage in the feedback amplifier takes the system in the periodic regime, with a frequency monotonically increasing with the voltage jump, or in the excitable regime (Fig.5). Applying to the laser cavity a second feedback loop where now the detected intensity is filtered by a low pass filter[6], we obtain a bursting behavior (Fig.6), which looks alike the action potential of neurons in CPG (central pattern generator) [7].

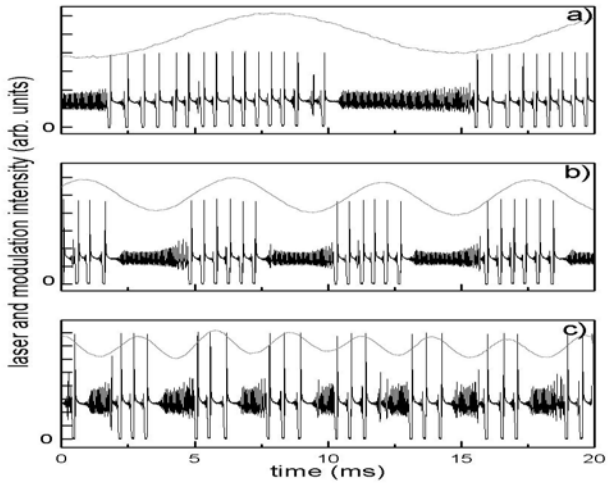


FIGURE 6. Bursts of different duration depending on the cutoff frequency of the low pass filter in the second feedback loop

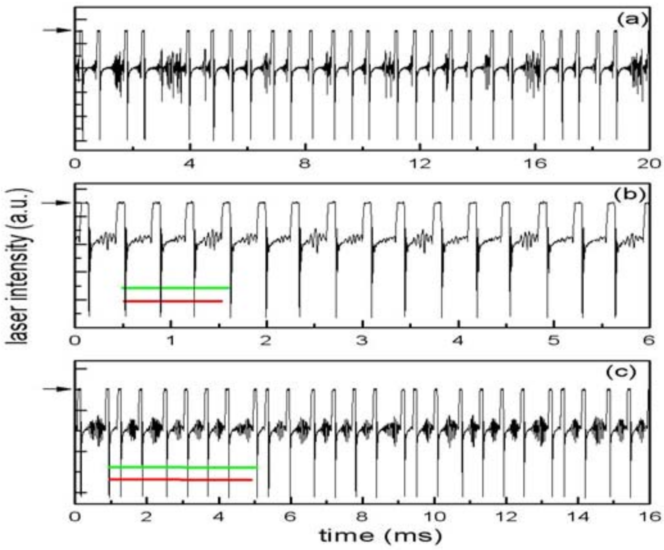


FIGURE 7. A sequence of homoclinic spikes in the output intensity of a CO_2 laser in the free-running regime and DSS for two different delays . A thick arrow denotes the zero intensity level. The two horizontal bars show the role of the refractory time . The shorter bar is the imposed delay , the larger one, with refractory time added, is the effective delay that characterizes DSS.

If now we apply a second feedback loop but with the signal delayed by a delay line [8],we observe a chaotic train over a delay interval ,followed by identical replicas of itself (Fig.7).We have called such a behavior as DSS (delayed self synchronization).Notice that the signal looks chaotic for times shorter than the delay, but for longer times, it is a periodic orbit with a period as long as the delay; it should rather be called pseudochaos. The autocorrelation function of the output intensity shows periodic revivals (Fig.8); thus DSS seems a plausible model for the short term memory phenomena in a brain.

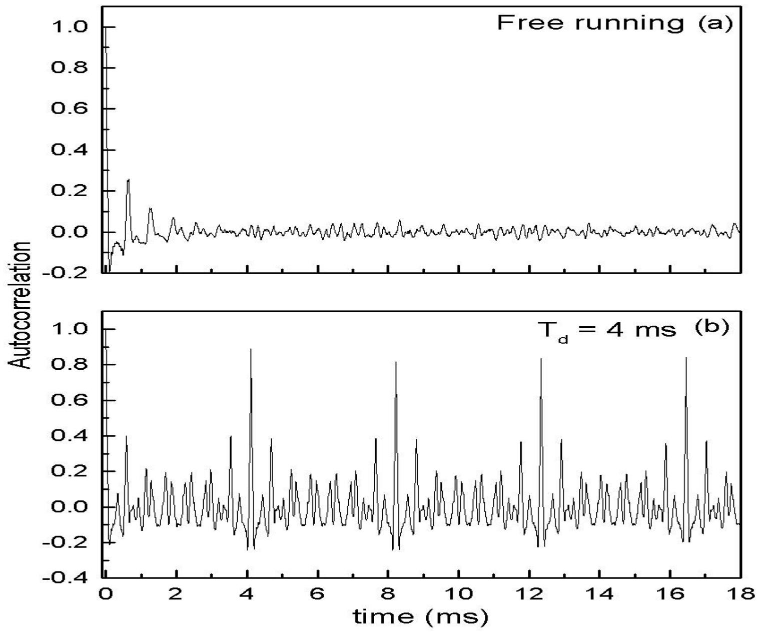


FIGURE 8. . Autocorrelation function of the laser intensity for the free running case and for DSS . The recurrence time for the revival of the correlation is the sum of the delay plus the refractory time .

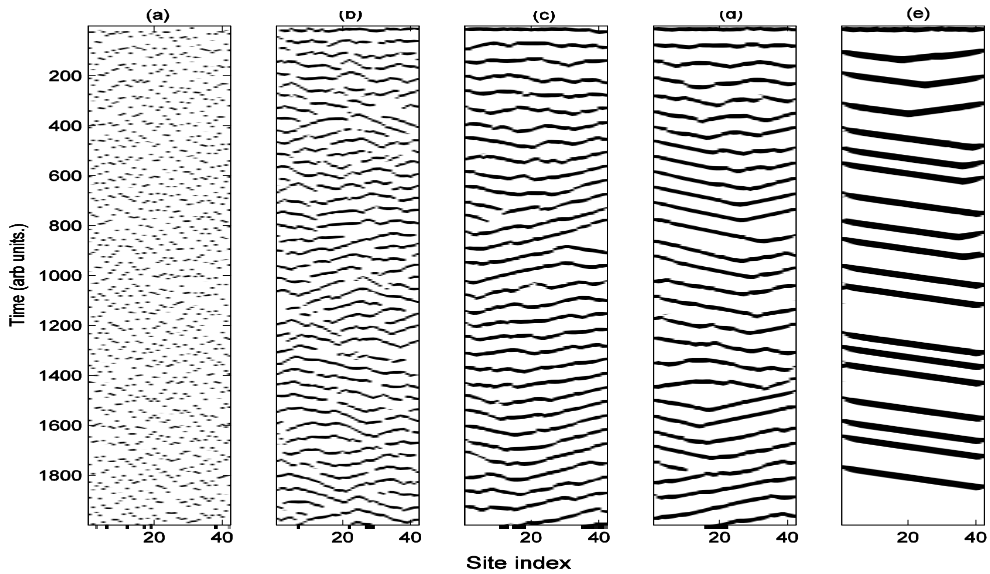
The reason why HC looks attractive to model neurodynamics is that it can contribute to understand feature binding . By this terminology, neuroscientists denote the fact that cortical neurons ,even very distant from each other and exposed to widely different external stimuli, end up by synchronizing their spikes when they have to contribute to an overall meaningful perception: This collective behavior requires a compromise between bottom –up stimuli, coming from the sensory detectors, and top-down perturbations provided by past memories .All this matter requires a scanning of a vast amount of specialized literature; in Ref [9] it is presented from the stand point of a physicist.

So far we dealt with a single HC system; by coupling many in a linear chain we can study the propagation of a synchronization state.

Since it would be very costly to have in the laboratory HC systems, we have tackled the problem in two ways.

In DSS case, we can record long time strings .Provided that the delay time is longer than the Liapunov time of the single HC, the delayed perturbation arrives when the single available HC system has lost memory of its initial conditions; it thus appears as an identical HC systems but with independent initial conditions .Thus the DSS spike sequence is as if we were coupling a chain of identical lasers with unidirectional coupling (from one to the next) [10].

In order to test bidirectional coupling, we perform computer simulations on coupled HC objects on a linear chain, with diffusive coupling [11].Some results are shown in Fig.9.



ε=(a) 0.0;(b)0.05;(c)0.1;(d)0.12;(e)0.25

FIGURE 9. Space time plots for spike trains (each spike is a single dot) for N = 40 size on a linear chain, with nearest neighbor coupling, for increasing values of mutual coupling. As we reduce the coupling below the value corresponding to complete synchronization, phase slips appear as defects in a fabric; for zero coupling there is no synchronization.

Since many models have been proposed and tested for the neuron, we must compare them on the basis of a quantitative indicator.

First of all ,we conjecture that a brain works on spike synchronization rather than phase synchronization [12].Indeed many perceptual tasks require an accurate timing ,and spike synchronization yields a time resolution as the temporal duration of the single spike, that is ,1 msec. On the contrary ,phase synchronization covers a sizable fraction of the interspike interval [12] and the associated time accuracy is of the order of 10 msec.

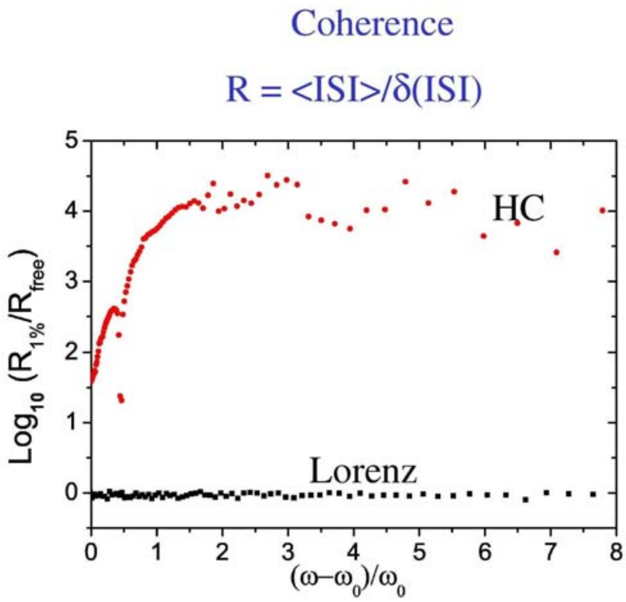


FIGURE 10. Coherence parameter R1% for a driving signal consisting of a 1 % periodic perturbation of a control parameter normalized to R $_{free}$ (the R value in the absence of perturbation), plotted versus the normalized distance of the perturbation frequency from the natural value .

We introduce as indicator the propensity to synchronization based on two considerations, namely,

i)let us consider as coherence parameter for HC the ratio between the mean ISI and its fluctuations, that is,

$$R = \frac{< ISI >}{\delta(ISI)};$$

as we introduce a periodic perturbation to a control parameter, we inspect how *R* changes with respect to the free system, and plot the ratio for many frequencies around the natural frequency corresponding to $< ISI >$; this ratio, that we call *propensity to synchronization* [3], is around 1 for Lorenz, is as high as 10^4 for HC (Fig.10);furthermore it reaches also high values for Hindmarsh –Rose, but only at a discrete set of frequencies, showing that such a dynamical system is unable to recognize broadband features [13]; finally it also around 1 for many popular systems as Roessler and FitzHugh-Nagumo.

ii) we inspect fir what values of the mutual coupling the propensity of the first system in the chain, which is the only one exposed to an external signal, propagates along the chain.For the case of HC, we have shown [3] that a reduction of the mutual coupling from 0.12 to 0.10 reduces the propagation by three orders of magnitude.

ACKNOWLEDGMENTS

This is the report of a long line of laboratory investigation started in 1987 with R.Meucci and W. Gadomski [1] and continued up to now with R.Meucci, E. Allaria, S.Boccaletti, A.Di Garbo, A.Pisarchik, L.Tsimring in Florence, plus collaborations with Potsdam (J. Kurths' group), and Madrid (I.Leyva and I.Marino) .

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