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**Neuron Dynamics and Chaotic  
Synchronization**

F. T. Arecchi

## NEURON DYNAMICS AND CHAOTIC SYNCHRONIZATION

F. TITO ARECCHI

*Department of Physics University of Firenze and Istituto Nazionale di Ottica Applicata, Italy  
L.go E. Fermi, 6-50125 Firenze (Italy)  
arecchi@ino.it*

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At the borderline between neuroscience and physics of complex phenomena, a new paradigm is under investigation, namely *feature binding*. This terminology denotes how a large collection of coupled neurons combines external signals with internal memories into new coherent patterns of meaning. An external stimulus spreads over an assembly of coupled neurons, building up a corresponding collective state. Thus, the *synchronization* of spike trains of many individual neurons is the basis of a *coherent perception*. Based on recent investigations, a novel conjecture for the dynamics of single neurons and, consequently, for neuron assemblies has been formulated. *Homoclinic chaos* is proposed as the most suitable way to code information in time by trains of equal spikes occurring at apparently erratic times; a new quantitative indicator, called *propensity*, is introduced to select the most appropriate neuron model. In order to classify the set of different perceptions, the percept space is given a metric structure by introducing a distance measure between distinct percepts. The distance in percept space is conjugate to the duration of the perception in the sense that an uncertainty relation in percept space is associated with time limited perceptions. (Thus coding of different percepts by synchronized spike trains entails fundamental *quantum* features. It is conjectured that they are related to the details of the perceptual chain rather than depending on Planck's action.)

*Keywords:* Neuron dynamics; synchronization.

### 1. Feature Binding

#### 1.1. Neuron synchronization

It is by now established that a holistic perception emerges, out of separate stimuli entering different receptive fields, by synchronizing the corresponding spike trains of neural action potentials [1, 2]. Action potentials play a crucial role for communication between neurons [3, 4]. They are steep variations in the electric potential across a cell's membrane, and they propagate in essentially constant shape from the soma (neuron's body) along axons toward synaptic connections with other neurons. At the synapses they release an amount of neurotransmitter molecules depending

upon the temporal sequences of spikes, thus transforming the electrical into a chemical carrier. As a fact, neural communication is based on a temporal code whereby different cortical areas which have to contribute to the same percept P synchronize their spikes. Limiting for convenience the discussion to the visual system, spike emission in a single neuron of the higher cortical regions results as a trade off between bottom-up stimuli arriving through the LGN (lateral geniculate nucleus) from the retinal detectors and threshold modulation due to top-down signals sent as conjectures by the semantic memory. This is the core of ART (adaptive resonance theory [5]) or other computational models of perception [6] which assume that a stable cortical pattern is the result of a Darwinian competition among different percepts with different strength. The winning pattern must be confirmed by some matching procedure between bottom-up and top-down signals.

### 1.2. Perceptions, feature binding and qualia

The role of elementary feature detectors has been extensively studied in the past decades [7]. By now we know that some neurons are specialized in detecting exclusively vertical or horizontal bars, or a specific luminance contrast, etc. However the problem arises: how elementary detectors contribute to a holistic (Gestalt) perception? A hint is provided by [2]. Suppose we are exposed to a visual field containing two separate objects. Both objects are made of the same visual elements, horizontal and vertical contour bars, different degrees of luminance, etc. What are then the neural correlates of the identification of the two objects? We have one million fibers connecting the retina to the visual cortex, through the LGN. Each fiber results from the merging of approximately 100 retinal detectors (rods and cones) and as a result it has its own receptive field. Each receptive field isolates a specific detail of an object (e.g. a vertical bar). We thus split an image into a mosaic of adjacent receptive fields. Now the "feature" hypothesis consists of assuming that all the cortical neurons whose receptive fields are pointing to a specific object synchronize the corresponding spikes, and as a consequence the visual cortex organizes into separate neuron groups oscillating on two distinct spike trains for the two objects (Fig. 1). Direct experimental evidence of this synchronization is obtained by insertion of microelectrodes in the cortical tissue of animals just sensing the single neuron [2]. Indirect evidence of synchronization has been reached for human beings as well, by processing the EEG (electro-encephalo-gram) data [8]. Based on the neurodynamical facts reported above, we can understand how this occurs [5]. The higher cortical stages where synchronization takes place have two inputs. One (bottom-up) comes from the sensory detectors via the early stages which classify elementary features. This single input is insufficient, because it would provide the same signal for e.g. horizontal bars belonging indifferently to either one of the two objects. However each neuron is a nonlinear system passing close to a saddle point, and the application of a suitable perturbation can stretch or shrink the interval of time spent around the saddle, and thus lengthen or shorten the interspike interval. The perturbation consists of top-down signals corresponding to conjectures made by the semantic memory (Fig. 2). In other words, the perception process is not like the passive imprinting of a camera film, but it is an active process whereby the external stimuli are interpreted in terms of past memories. A focal attention mechanism

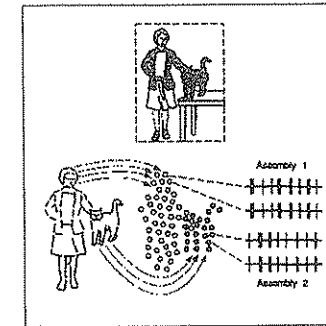


Fig. 1. Feature binding: the lady and the cat are respectively represented by the mosaic of empty and filled circles, each one representing the receptive field of a neuron group in the visual cortex. Within each circle the processing refers to a specific detail (e.g. contour orientation). The relations between details are coded by the temporal correlation among neurons, as shown by the same sequences of electrical pulses for two filled circles or two empty circles. Neurons referring to the same individual (e.g. the cat) have synchronous discharges, whereas their spikes are uncorrelated with those referring to another individual (the lady) (see Ref. [2]).

assures that a matching is eventually reached. This matching consists of resonant or coherent behavior between bottom-up and top-down signals. If matching does not occur, different memories are tried, until the matching is realized. In presence of a fully new image without memorized correlates, then the brain has to accept the fact that it is exposed to a new experience. Notice the advantage of this time dependent use of neurons, which become available to be active in different perceptions at different times, as compared to the computer paradigm of fixed memory elements which store a specific object and are not available for others (the so called "grandmother neuron" hypothesis). We have above presented qualitative reasons why the degree of synchronization represents the perceptual salience of an object. Synchronization of neurons located even far away from each other yields a space pattern on the sensory cortex, which can be as wide as a few square millimeters, involving millions of neurons. The winning pattern is determined by dynamic competition (the so-called "winner takes all" dynamics). This model has an early formulation in ART and has been later substantiated by the synchronization mechanisms. Perceptual knowledge appears as a complex self-organizing process. The physicist's goal is to model such phenomena in terms of synchronization processes of networks of chaotic dynamical systems. Chaotic synchronization is a hot issue nowadays and comprehensive reviews are available [9,10]. Naively, one might expect that a given "qualia", that is, a private sensation as e.g. the red of a Titian painting, is always coded by the same sequence of spikes. If so, in a near future the corresponding information could be retrieved by a high resolution detector, and hence a Rosetta stone could be established between the spike sequences and the qualia. Such a naive expectation which would lead to a world without privacy, is altogether wrong for the following reasons. After the initial experience of that qualia, the first time one has seen that Titian, any further repetition of that experience, either by memory recollection or by re-watching the painting occurs in presence of new experiential

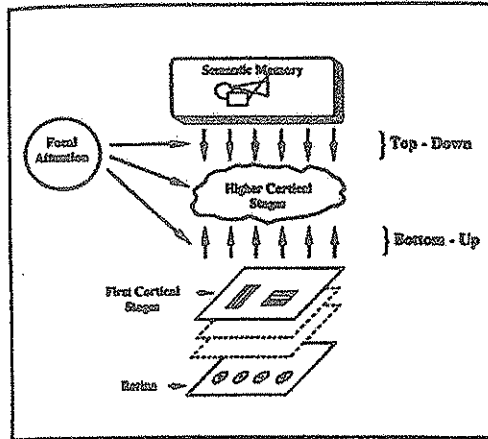


Fig. 2. ART = Adaptive Resonance Theory. Role of bottom-up stimuli from the early visual stages and top-down signals due to expectations formulated by the semantic memory. The focal attention assures the matching (resonance) between the two streams (see Ref. [11]).

elements (one has become older, his/her store of memories has drastically mutated) and these novelties contribute to feature binding by a modified synchronization pattern. Evidence of such a fact has been established by Freeman [12] reporting the synchronization pattern of the olfactory bulb of a rabbit, recorded by a large number of electrodes; as the same odor is presented twice, with an intermediate odor in between, the two patterns are all together different, even though the animal behavior hints at the same reaction. Freeman's experiment is contrasted by the fact that some olfactory neurons of the locust yield the same bursts of spikes for the same odor [13]. Presumably, lower animals as locusts have a much smaller semantic repertoire than rabbits or humans, and hence for them the dream of the Rosetta stone has some validity.

## 2. Homoclinic Chaos, Synchronization and Propensity

Let us model the neurodynamics of spike formation. As for the dynamics of the single neuron, a saddle point instability separates in parameter space an *excitable* region, where axons are silent, from a *periodic* region, where the spike train is periodic (equal interspike intervals). If a control parameter is tuned at the saddle point, the corresponding dynamical behavior (homoclinic chaos) consists of a frequent return to the instability [14]. This manifests as a train of geometrically identical spikes, which however occur at erratic times (chaotic interspike intervals). Around the saddle point the system displays a large susceptibility to an external stimulus, hence it is easily adjustable and prone to respond to an input, provided this is at sufficiently low frequencies; this means that such a system is robust against high frequency noise as discussed later. That a saddle point instability is crucial for neuron coding is well accepted on the following facts: (i) in order to code an analog

signal as yielded by a sensory detector (visual, audio, etc.) into a spike, or a spike train, the coding neuron has to start from an excitable regime, and go to a periodic one with a frequency depending on the input signal [7]; (ii) a periodic bursting, even though efficient for motor control, is not economic timewise whenever information can be carried by a single spike and by its time of occurrence; whence chaotic spike trains and associated network synchronization appear a better strategy, and many dynamical systems can fulfill these criteria, see e.g. [4]. Such a type of dynamics has been recently dealt with by my research group in a series of reports that here I recapitulate as the following chain of linked facts.

1. A single spike in a 3D dynamics corresponds to a quasi-homoclinic trajectory around a saddle focus SF (fixed point with 1 (2) stable direction and 2 (1) unstable ones); the trajectory leaves the saddle and returns to it (Fig. 3). We say "quasi-homoclinic" because, in order to stabilize the trajectory away from SF, a second fixed point, namely a saddle node SN, is necessary to assure a *heteroclinic* connection. The experiment on a  $CO_2$  laser confirms this behavior (Fig. 4).

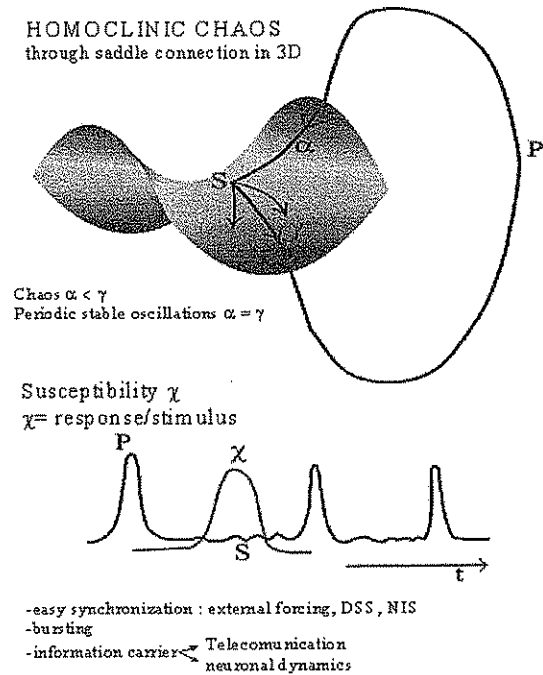


Fig. 3. Schematic view of the phase space trajectory approaching the saddle S and escaping from it. Chaos is due to the shorter or longer permanence around S; from a geometrical point of view most of the orbit P provides a regular spike.

2. A train of spikes corresponds to the sequential return to, and escape from, the SF. A control parameter can be set at a value  $B_C$  for which this return is erratic

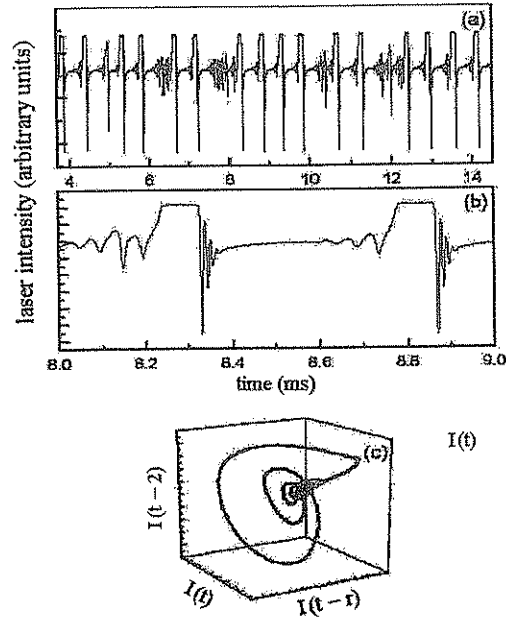


Fig. 4. Experimental time series of the laser intensity for a  $CO_2$  laser with feedback in the regime of homoclinic chaos. (b) Time expansion of a single orbit. (c) Phase space trajectory built by an embedding technique with appropriate delays (see Ref. [14]).

(chaotic interspike interval). As the control parameter is set above or below  $B_C$ , the system moves from excitable (single spike triggered by an input signal) to *periodic* (yielding a regular sequence of spikes without need for an input), with a frequency monotonically increasing with the separation  $\Delta B$  from  $B_C$  [15]. 3. Around SF, any tiny disturbance provides a large response. Thus the homoclinic spike trains can be synchronized by a periodic sequence of small disturbances (Fig. 5). However each disturbance has to be applied for a minimal time, below which it is no longer effective; this means that the system is insensitive to broadband noise, which is a random collection of fast positive and negative signals [16].

4. The above considerations lay the floor for the use of mutual synchronization as the most convenient way to let different neurons respond coherently to the same stimulus, organizing as a space pattern. In the case of a single dynamical system, it can be fed back by its own delayed signal. As the delay is long enough the system is decorrelated with itself and this is equivalent to feeding an independent system. This process allows to store meaningful sequences of spikes as necessary for a short term memory [17].

5. Several neuron models (integrate-and-fire, Hodgkin-Huxley, Hindmarsh-Rose, FitzHugh-Nagumo) have been used by different investigators. We have introduced the *propensity to synchronization* as a quantitative indicator of how easy is for a chaotic system to recognize an external input (Fig. 6) [18].

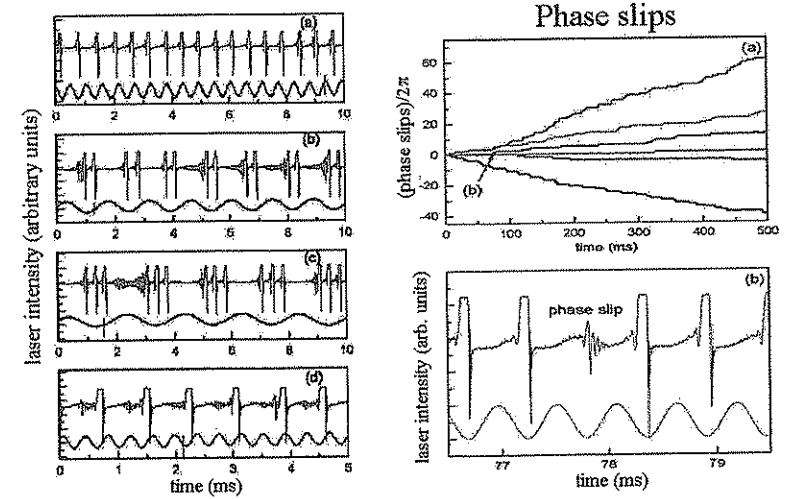


Fig. 5. Left: experimental time series for different synchronization ratios induced by periodic changes of the control parameter. (a) 1:1 locking, (b) 1:2, (c) 1:3, (d) 2:1. Right: when the system is not able to spike for each period of the driver, a phase slip (one spike less or more) occurs, it is a jump of  $\pm 2\pi$  if the interspike interval is normalized to  $2\pi$ . The rate of  $\pm$  phase slips increases with the offset of the driving frequency from the natural frequency (associated with the average interspike interval of the free system).

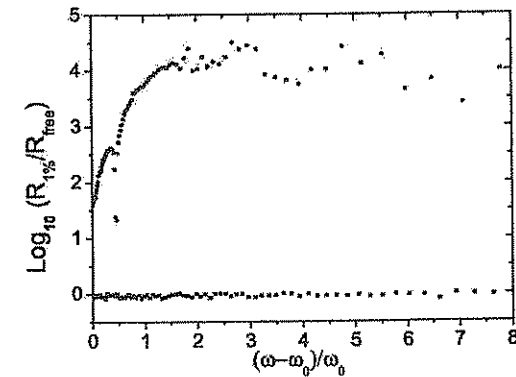


Fig. 6. The coherence parameter  $R$  is defined as the ratio between the average ISI (interspike interval) and its r.m.s. fluctuation.  $R$  is unity for a fully chaotic system and tends to infinity for a periodic system. Here we plot in log scale the ratio between  $R$  for a driving periodic disturbance of 1% to a control parameter and  $R_{free}$  for the free system, at different frequencies  $\omega$  away from the natural one  $\omega_0$  (average of the chaotic spiking in the unperturbed system). For HC (circles) the ratio is 30 at  $\omega_0$  and it goes up to 104 for higher frequencies; for the Lorenz system (squares) the ratio stays flat to 1. We thus take this ratio as a quantitative indicator of the *propensity to synchronization*.

6. In the presence of localized stimuli over a few neurons, the corresponding disturbances propagate by inter-neuron coupling (either excitatory or inhibitory); a synchronized pattern is uniquely associated with each stimulus; the degree of mutual synchronization is measured by the disappearance of phase slips, or defects in a space-time fabric [19].

These facts have been established experimentally and confirmed by a convenient model in the case of a class B laser with a feedback loop which readjusts the amount of losses depending on the value of the light intensity output [20]. I here recall the classification widely accepted in laser physics. Class A lasers are ruled by a single order parameter, the amplitude of the laser field, which obeys a closed dynamical equation; all the other variables having much faster decay rate, thus adjusting almost instantly to the local field value. Class B lasers are ruled by two order parameters, the laser field and the material energy storage providing gain; the two degrees of freedom having comparable characteristic times and behaving as activator and inhibitor in chemical dynamics [21]. The above listed facts hold in general for any dynamical system which has a 3-dimensional sub-manifold separating a region of excitability from a region of periodic oscillations: indeed, this separatrix has to be a saddle focus.

### 3. Time Code in Neural Information Exchange

How does a synchronized pattern of neuronal action potentials become a relevant perception? Not only the different receptive fields of the visual system, but also other sensory channels as auditory, olfactory, etc. integrate via feature binding into a holistic perception. Its meaning is "decided" in the PFC (pre-frontal cortex) which is a kind of arrival station from the sensory areas and departure for signals going to the motor areas. On the basis of the perceived information, motor actions are started, including linguistic utterances [8]. Sticking to the neurodynamical level, and leaving to psychophysics the investigation of what goes on at higher levels of organization, we stress here a fundamental temporal limitation. Taking into account that each spike lasts about 1 msec, that the minimal interspike separation is 3 msec, and that the decision time at the PCF level is estimated to be  $\bar{T} \approx 200$  ms, we can split  $\bar{T}$  into  $200/3 \approx 66$  bins of 3 msec duration, which are designated by 1 or 0 depending on whether they have a spike or not. Thus the *a priori* total number of different messages which can be transmitted is  $2^{66} \approx 6 \cdot 10^{19}$ . However we must account also for the average rate at which spikes proceed in our brain, which is  $r = 40$  Hz (so called  $\gamma$  band,  $\langle ISI \rangle$  (average ISI) = 25 ms). When we account for this rate we can evaluate a reduction factor  $\alpha = S/\bar{T} = 0.54$  where S is an entropy [22], thus there are roughly  $2^S \approx 10^{11}$  words with significant probability. Even though this number is large, we are still within a finite realm. Provided we have enough time to ascertain which one of the different messages we are dealing with, we can classify it with the accuracy of a digital processor, without residual error. But suppose we expose the cognitive agent to fast changing scenes, for instance by presenting in sequence unrelated video frames with a time separation less than 200 msec. While small gradual changes induce the sense of motion as in movies, big differences imply completely different subsequent spike trains. Here any spike train gets *interrupted* after a duration  $\Delta T$  less than the canonical  $\bar{T}$ . This means that

the brain cannot decide among *all* coded perceptions having the same structure up to  $\Delta T$ , but different afterwards. Whenever we stop the perceptual task at  $\Delta T$  shorter than the total time  $\bar{T}$ , then the bin stretch  $T - \Delta T$  (we measure the times in bin units) is not explored. This means that all stimuli which provide equal spike sequences up to  $\Delta T$ , and differ afterwards by *at least one* spike will cover an uncertainty region  $\Delta P$  whose size is given by

$$\Delta P = 2^{\alpha T} 2^{-\alpha \Delta T} = P_M e^{-\alpha \Delta T \ln 2} \quad (1)$$

where  $P_M \approx 10^{11}$  is the maximum perceptual size available with the chosen  $\bar{T} \approx 66.6$  bins per perceptual session and rate  $r = 40$  Hz. Relation (1) is very different from the standard uncertainty relation

$$\Delta P \cdot \Delta T = C \quad (2)$$

that we would expect in a word-bin space ruled by Fourier transform relations. Indeed, the transcendental equation (1) is converging more rapidly at short and long  $\Delta T$  than the hyperbola (2). We fit (1) by (2) in the neighborhood of a small uncertainty  $\Delta P = 10$  words, which corresponds to  $\Delta T = 62$  bins. Around  $\Delta T = 62$  bins the local uncertainty (2) yields a quantum constant  $C = 10 \cdot 62 = 620$  words  $\times$  bins. To convert  $C$  into  $J_s$  as Planck's  $h$ , consider that: (i) 1 bin = 3 ms; (ii) in order to jump from an attractor corresponding to one perception to a nearby one, a *minimal amount* of energy is needed, corresponding to one spike; but one spike requires the energy corresponding to about  $10^7$  transitions  $ATP \rightarrow ADP + P$  [23] each one taking 0.3 eV; thus the total energy quantum is about  $10^{-14}$  Joules. The conversion factor is then:  $C \approx 10^{-14} J_s \approx 10^{20} \eta$ .

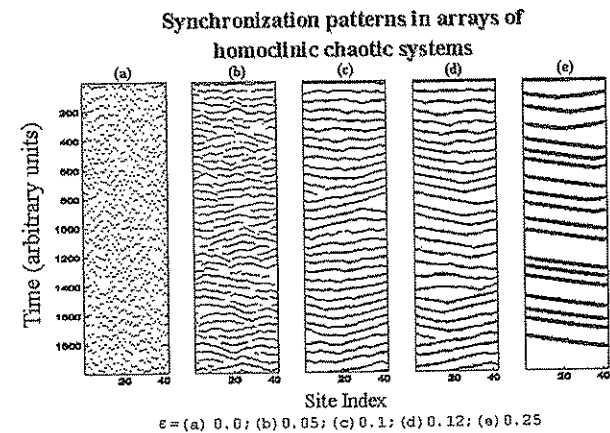


Fig. 7. Space-time representation of spike positions for a linear array of 40 neurons and for different amounts of nearest neighbor coupling. At  $\epsilon = 0.25$  percolation has been reached, in the sense that spikes at all sites are connected (besides a mutual time lag to the transfer operation); at  $\epsilon = 0$ , no correlation at all among different sites; in between, a partial synchronization with the evidence of defects as phase slips (one spike more, or less, with respect to the neighbor site).

Quantum limitations were also put forward by Penrose [24] but on a completely different basis. In his proposal, the quantum character was attributed to the physical behavior of the "microtubules" which are microscopic components of the neurons playing a central role in the synaptic activity. However, speaking of quantum coherence at the  $\hbar$ -level in biological processes is not plausible, if one accounts for the extreme vulnerability of any quantum system to *decoherence* processes, which make quantum superposition effects observable only in extremely controlled laboratory situations, and at sub-picosecond time ranges, not relevant for synchronization purposes in the 10 – 100 msec range.

Our tenet is that the quantum C-level in a living being emerges from the limited time available in order to take vital decisions; it is logically based on a non-commutative set of relevant variables and hence it requires the logical machinery built for the  $\hbar$ -bar quantum description of the microscopic world where non-commutativity emerges from use of variables coming from macroscopic experience, as coordinate and momenta, to account for new facts.

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