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DYNAMICS OF THE LASER RADIATION AT THRESHOLD*

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Received 31 July 1967

We report a spectral analysis of the intensity fluctuations in a laser at threshold. The experimental results confirm the time evolution of the intensity correlation function as described by the nonlinear theory of the oscillator in terms of a weighted sum of exponentials.

In a previous letter [1], we have measured the ensemble distribution of a single mode laser radiation in the threshold region, and shown that it fits the results of some recent theoretical works [2-4]. In this letter, we report the frequency spectra of the intensity fluctuations, and show that they are consistent with the transient solution of the same Fokker-Planck equation [5], whose stationary solution is fitted by the ensemble distribution reported in ref. 1.

The laser system plus feedback control is the same as the one described in ref. 1, the results are reported in fig. 1. The power spectra of the intensity fluctuations showed small deviations from a Lorentzian shape in the threshold region. We define the "effective" linewidth $\Delta\nu$ of an equivalent Lorentzian as the ratio $\pi^{-1} K(0)/S(0)$ between the total spectral power $K(0)$, measured with a r.m.s. voltmeter, and the zero frequency value $S(0)$ of the spectral density. For a fixed discharge current and different mode positions along the Doppler line we have plotted the quantities $\Delta\nu$, $S(0)$ and $K(0)$ **. The horizontal axis is calibrated in values I/I_0 of the ratio between the actual intensity I and the threshold intensity I_0, and in values of the pump parameter a introduced by Risken [2]. The average photon number $\langle n\rangle$ inside the cavity is also evaluated, by an absolute calibration of the output power and a measurement of the cavity losses.

The measured linewidth fits the numerical calculations of Risken and Vollmer [5] † who gave a dynamical solution for the intensity correlation function as a sum of several exponential terms (with decay constants λ_{0k}) occurring with different M_k. This leads to an equivalent decay constant

$$\lambda_{\mathrm{eff}} = \left[\sum \frac{M_k}{\lambda_{0k}}\right]^{-1} ; \quad \left(\sum_k M_k = 1\right). \tag{1}$$

In the same figure we have plotted the main decay constant λ_{01}, of the intensity correlation function. It is clear that a Lorentzian approximation with a single decay constant is not an adequate description.

* Work partially supported by the C.N.R. (Consiglio Nazionale delle Ricerche).

** Previously reported frequency measurements [6] referred to working conditions quite above and below threshold, leaving about 2 decades of intensity values around threshold not explored.

† Similar plots are reported by Lax and Louisell [3], however the scale of their figures is not sufficient for a quantitative comparison. We have therefore used the numerical values kindly given to us by the Stuttgart group.

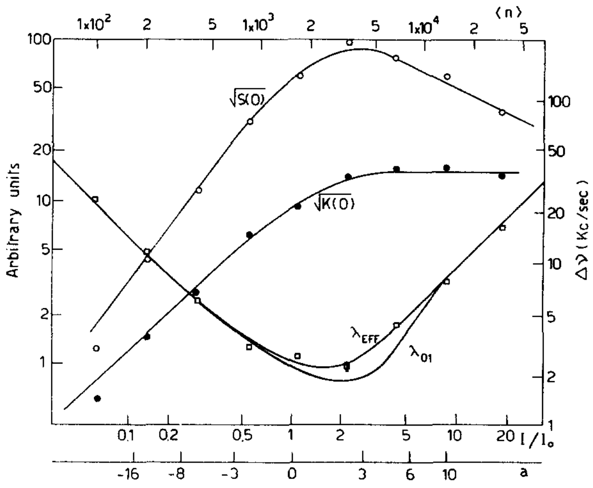

Fig. 1. Plot of "effective" linewidth $\Delta\nu$, square root of the spectral density $\sqrt{S(0)}$, and total power of the intensity fluctuations $\sqrt{K(0)}$ versus the laser intensity I normalized to the threshold value I_0. The horizontal axis is also calibrated in values of the pump parameter a and in average photon number $\langle n\rangle$ inside the cavity. The solid lines correspond to the numerical computations of ref. 5. Apart from one point, the dispersion in the experimental results over several runs is smaller than the size of a dot.

The total power $K(0)$ is connected to the correlation functions of the intensity fluctuations $\Delta I = I - \langle I\rangle$ by the relation $K(0) = \langle \Delta I^2\rangle$, which offers an alternative way to evaluate the second reduced moment H_2 of the statistical distribution of the photocounts. The results agree with those obtained by the photocount method [1].

For the discharge current used in taking the data, we have measured at threshold the value $\Delta\nu_s$ = 2500 c/sec and $\langle n_s\rangle$ = 1570 for the linewidth and the photon number in the cavity. Their product is a linear function of the population N_2 of the upper level of the laser transition (see last equation of ref. 1), and hence a linear function of the discharge current in the plasma tube. This linear dependence has been verified experimentally. Furthermore $\langle n_s\rangle$ and $\Delta\nu_s$ are related to the "friction" parameter β and the "diffusion" parameter q of the Fokker-Plancl equation for the laser field distribution as follows [7]

$$\langle n_s\rangle = \frac{2}{\sqrt{\pi}}\sqrt{\frac{q}{\beta}}, \qquad \Delta\nu_s = \frac{\sqrt{\beta q}}{2\pi}(\lambda_{\text{eff}})_{a=0} . \qquad (2)$$

From these one can obtain $q = 4\times10^6$ sec^{-1} and $\beta \approx 2$ sec^{-1}. The phenomenological parameters q and β can be related to the atomic parameters by some reasonable microscopic model. If one uses a two level scheme (with equal degeneracies $g_1 = g_2$) and an electric dipole transition, the reported q and β correspond to a value of the electric dipole matrix element $|r_{12}| \approx 0.16$ Å and to an upper level occupation number $N_2 \approx 3\times10^6$. These values are roughly in agreement with more "ad hoc" measurements [8]. A more detailed analysis is developed in a forthcoming paper.

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