Complexity , Information Loss and Model Building: from neuro- to cognitive dynamics

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A scientific problem described within a given code is mapped by a corresponding computational problem, We call *complexity (algorithmic)* the bit length of the shortest instruction which solves the problem. Deterministic chaos in general affects a dynamical systems making the corresponding problem experimentally and computationally heavy, since one must reset the initial conditions at a rate higher than that of information loss (Kolmogorov entropy). One can control chaos by adding to the system new degrees of freedom (information swapping: information lost by chaos is replaced by that arising from the new degrees of freedom). This implies a change of code, or a new augmented model.

Within a single code, changing hypotheses is equivalent to fixing different sets of control parameters, each with a different a-priori probability, to be then confirmed and transformed to an a-posteriori probability via Bayes theorem. Sequential application of Bayes rule is nothing else than the Darwinian strategy in evolutionary biology. The sequence is a steepest ascent algorithm ,which stops once maximum probability has been reached. At this point the hypothesis exploration stops.
By changing code (and hence the set of relevant variables) one can start again to formulate new classes of hypotheses .
We call *semantic complexity* the number of accessible scientific codes, or models, that describe a situation. It is however a fuzzy concept, in so far as this number changes due to interaction of the operator with the system under investigation.
These considerations are illustrated with reference to a cognitive task, starting from synchronization of neuron arrays in a perceptual area and tracing the putative path toward a model building.

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1. Introduction: Complexity and Chaos

A large amount of papers have appeared at the interface of neuroscience, computational science and physics and many computational models of perceptual tasks have been introduced.

On the other hand, cognition, defined as a world view that can be formulated in a given language and shared with other agents, is considered as a higher level endeavour with respect to perception

A cognitive agent is not susceptible of a closed physical description, since it changes its amount of information by exposure to an environment; this fact has been called *bio-semiosis* and it has been taken as the distinctive feature of a living system, from a single cell to humans(Sebeok). For this reason , *cognitive science* has been considered thus far as a territory extraneous to physics. Here we explore intersections of this area of investigation with physics, having in mind the recent progress in nonlinear dynamics and complex systems.

Since the problem area we are facing is new, it is my aim to provide a heuristic introduction which may be instrumental for further insight .I do not pretend to offer a theory ,but just a preliminary discussion; thus many statements are just qualitative hints which should be reconsidered in a more formal way.

In computer science, we call "*complexity" C* of a problem the number of bits of the computer program which solves the problem. In physics, the most common approach to a problem consists of a description of the system in terms of its elementary components and their mutual interactions *(reductionistic code).* In general, this entails a chaotic dynamics with a non zero information loss rate *K.* Since these concepts have been circulating for quite a time, I just summarize some qualitative points, with the help of heuristic pictures.

In Fig 1 the thick line with an arrow means that, for a given dynamical law, the trajectory emerging from a precise initial condition (the star) is unique. The space coordinates of the initial point are in general assigned by real numbers, that we truncate to a finite number of digits. Thus the initial condition is not a Euclidean point, but a whole segment. Initial conditions to the left or right of the ideal one converge toward the ideal trajectory or diverge away depending on whether the transverse stability analysis yields a valley–like (left) or hill-like (right) landscape. In the second case, we loose information of the initial preparation at a rate which depends on the steepness of the down-hill. This information loss does not require interaction with a disturbing environment as in noise problems; it is just a sensitive dependence on the initial conditions nowadays called *deterministic chaos*. *K* denotes the rate of information loss, after Kolmogorov. Newton restricted his dynamics to 2-body interactions which are regular as the in the left figure. In 1890 Henry Poincaré showed that the gravitational problem with 3 or more interacting bodies displays generically the transverse instability depicted on the right.

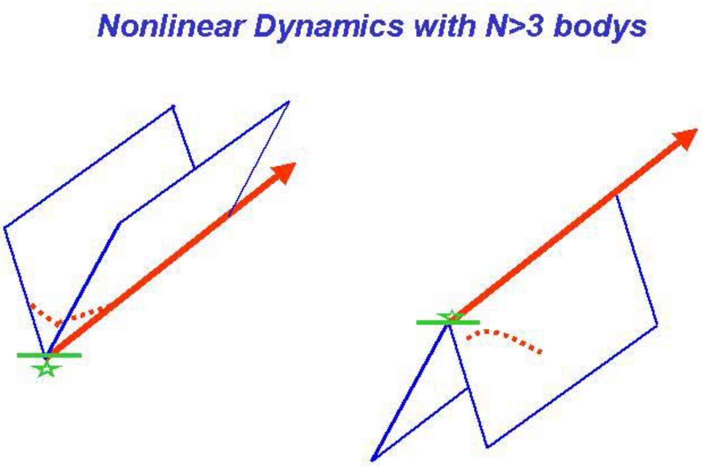


Fig.1- *Deterministic Chaos. The thick line with an arrow represents the unique trajectory emerging as solution of the equations of motion from the initial condition denoted by a star. The transverse stability (left) or instability (right) means that trajectories starting from nearby initial points (dotted lines) converge to (left) or diverge from(right) the ideal trajectory. The right case is a regular motion; the left one is chaotic, with information loss..*

Chaos can be controlled by additional external degrees of freedom, which change the slope of the transverse potential without perturbing the longitudinal trajectory (Fig 2). Changing the number of degrees of freedom amounts to changing the *descriptive code*. In the perceptual case, we will see in Sec. 2, that a collective neuron dynamics is in general chaotic. In presence of a specific sensory input, different top-down perturbations due to different stored memories modify the transverse stability by different amounts. There will be a competition among different interpretations, that is, different perturbations of the sensorial stimulus by past memories. The winning result is that which assures the highest stability during the perception time (usually, a perceptual window is of the order of a few hundred milliseconds). In fig.2 we depict the role of two different control perturbations. Thus, we anticipate already from Sec.2 that any coherent perception is an *interpretation*, that is, *a change of code* with respect to that imposed by the sheer sensorial stimulus.

If we do not introduce control, information on the initial setting has been lost after a time K^{-1} and one must re-assign the initial condition in order to predict the future: think e.g. of meteorological forecast. This may be very information consuming; that's why a novel descriptive code, which reduces K, may be more effective than the old one.

Within the reductionistic code, problems have a monotonic *C-K* behavior. For $K=0$ we have $C=0$; thus it has been straightforward to design an algorithm, BACON, (Simon) which infers Kepler's laws from the regularities of the planets' motions.

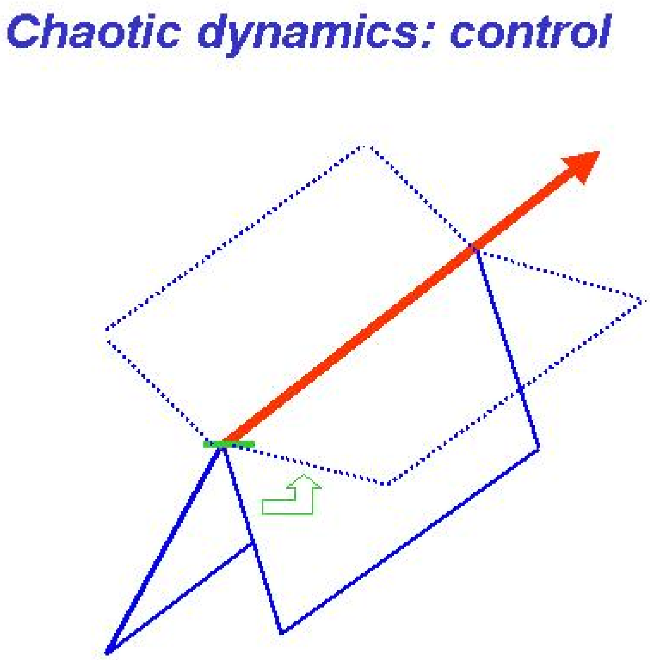


Fig. 2- *Control of Chaos - Adding new degrees of freedom in a suitable way, the transverse instability can be reduced (right) or wholly eliminated (left) ,while conserving the longitudinal trajectory The addition of extra degrees of freedom implies a change of code, thus it can be seen as a new level of description of the same physical system.*

For $K \to \infty$ (Boltzmann gas) a dynamical description requires $N \to \infty$ variables hence $C \to \infty$. However, for most experimental situations a thermodynamical description is sufficient, and *thermodynamics* has a very low C.

This means that re-coding a problem in terms of new indicators suggested by a specific experience (*semiosis*) replaces the old code of the microscopic description with a new one having lower complexity. This reduction is by no means a loss of resolution, since the lost bits are inaccessible and hence they could not be dubbed as *hidden variables.*

We generalize saying that any code transformation implying a complexity reduction –as it occurs in most human endeavours, as e.g. translation of a text to another language- requires a mechanism of information loss and replacement with new information not included in the primitive code *(information swapping).*

On the contrary, working within a fixed code, any complexity change is due to a different organization of the system under scrutiny, as it occurs in Renormalization Group applications called *Multi-Grid* (Solomon).

The fixed code means that the analysis can be carried by a computer; this automatic version of complexity does not match our pre-scientific expectations. We rather call it *complication,* leaving the concept of complexity to the different points of view under which we grasp the same system under different codes (Sec 3).

In Fig 3 we compare two different definitions of *complexity-complication*, namely the *algorithmic* one *CA*, already defined above and introduced by G. Chaitin in 1965 (Chaitin), and the logical depth *D, introduced* in 1985 by C.Bennett (Bennett) as the time necessary to execute a program starting from the shortest instruction. For low *K* the two definitions are equivalent, but, while *CA* increases monotonically with *K, D* goes to zero for high *K.* Indeed think of a random number. *CA*, and *K* as well, will increase with the number of digits, whereas *D* is very short: once the number of digits has been collected in the long instruction, then the execution is just: "print it".

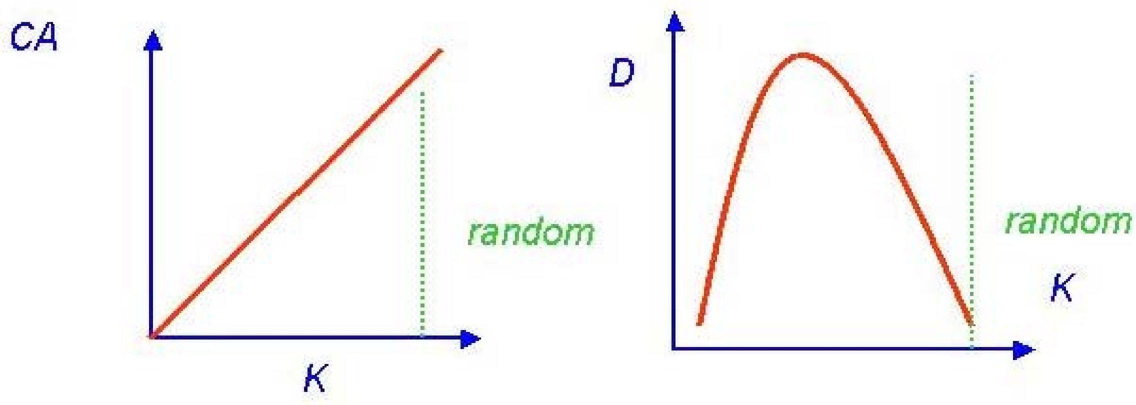


Fig.3 *Two definitions of complexity-complication-Left: CA=algorithmic complexity (Chaitin); it increases monotonically with Kolmogorov entropy K and is maximal for a random bit sequence. Right: D= logical depth (Bennett);it has a maximum for intermediate K and is very small for a random sequence.*

As an example, let us consider the Ising model. It consists of a set of coupled spin ½ particles in a thermostat at fixed temperature. At low temperature (left), the mutual interactions prevail over the thermal disturbance and all spins will be aligned parallel in an ordered collective state with low complexity. At high temperature (right), the thermal perturbation exceeds the mutual coupling and the spins are randomly distributed, hence both *CA* and *D* will be like the right extreme of Fig.3. In the middle, at a critical temperature Tc, the two forces acting on each spin will be comparable and the whole system organizes in large clusters of parallel spins. The clusters undergo large size fluctuations; hence *D* reaches a maximum which scales with a power z of the system size L. In 3D, $z = 0.2$.

Many further measures of complexity have been introduced and a list can be found in a review volume (Arecchi and Farini).They have a common feature, namely, since they rely on the information content of the problem under consideration, they can be considered as varieties of what we called *complication.*

We call ***creativity*** any code change, that takes place from a high *C* model to a lower *C* model. Some well-known examples are collected in the Table 1.

The rest of this paper is organised as follows: In Sec 2 we apply dynamical considerations to explain coherent perceptions, and how they are categorized as cognitions. In Sec 3 we discuss the procedures whereby cognitions are organized into models and hint at how *creativity* emerges.

Table 1- Reduction of complexity by code change

1. Electricity; magnetism; optics	*Maxwell electromagnetic equations*
2. Mendeleev table	*Quantum atom (Bohr,Pauli)*
3. Zoo of more than 100 elementary particles	*SU (3)- quarks (M Gell Mann)*
4. Scaling laws in phase transitions	*Renormalization Group (K.Wilson)*

2-Dynamics of a cognitive task

As well known, a neuron is a brain cell with mainly chemical exchange (neurotransmitters) at both input and output. The body (soma) performs the usual cell operations, namely, cell metabolism and processing of input signals to be coupled as output signals. However, at variance with the other cells, the signal travels over long distances within the neuron as an electric signal (train of stereotyped *spikes*, of about 70-millivolt height and *1 ms* duration) through a transmission line (*axon*) with a propagation speed around 1m/sec. The input information is coded as variable *inter-spike separation (ISI)* and then re-coded into an amount of output neurotransmitter (Rieke et al.). Since the axons have lengths between some micrometers in the brain cortex and one meter in the spinal chord, only the electrical propagation assures transmission times of a fraction of a second. The alternative would be a transport by flow as with hormones in the blood, or by molecular diffusion between two cell membranes. The former may require tens or hundreds of seconds; the latter is convenient only for very small separations d, since the diffusion time T scales as $T = d^2/D$, where the diffusion constant D for bio-molecules in water is around $10^{-6}\ cm^2/sec$. Thus for $d = 1mm$, T would be $10^4\ sec$ (about 3 hours) against an electric transport time $d/v=1ms$.

A neuron behaves as a threshold circuit which fires whenever the algebraic sum of the inputs (taking as positive the excitatory signals, and as negative the inhibitory ones) overcomes a given value.

Neural integration consists of a correlation between neurons, even far away from each other, when their receptive fields extract different features of the same object. This correlation (*feature binding;* see: Singer, Gray, Chawla, Duret) is a collective state with neurons having their spikes synchronized.

Psychophysical studies have shown that the analysis of visual scenes occurs in two phases. First, the elementary features of the objects, as color, motion, contour orientation, are locally detected in parallel. Next, these components are connected to provide a coherent object representation (*Gestalt*)

More precisely, *feature binding* denotes how coupled neurons combine external signals with internal memories into new coherent patterns of meaning. An external stimulus spreads over an assembly of coupled neurons, building up a corresponding collective state by *synchronization* of the spike trains of individual neurons. In presence of different external stimuli, different *clusters* of synchronized neurons are present within the same cortical area. The microscopic dynamics of N coupled neurons is thus replaced by the interplay of $n<<N$ clusters .The n objects are the *attractors* of a chaotic dynamics.

The crucial fact has been the dissipation of information. This means that a perception based on the n collective clusters has lost the detailed information of the $N>>n$ elementary components. Information loss and subsequent change of relevant variables means that coding at a higher hierarchical level is not just a computational task, but it violates the statute of a Turing machine.

Let us explore feature binding in detail with reference to vision. In vision, each fiber connecting the retina with the primary visual cortex has a limited receptive field. An extended input image is dissected over many channels, like a mosaic. A holistic perception emerges combining stimuli on different receptive fields by synchronization of the corresponding spike trains. Neural communication is based on a code whereby different regions, which must contribute to the same perception, synchronize their spikes.

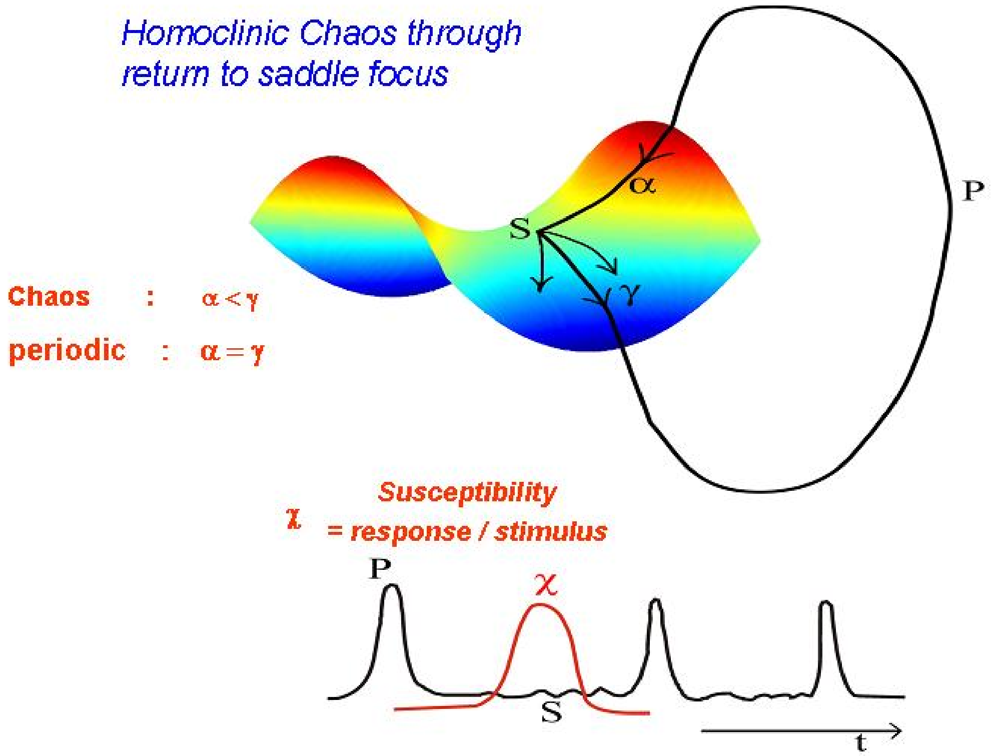
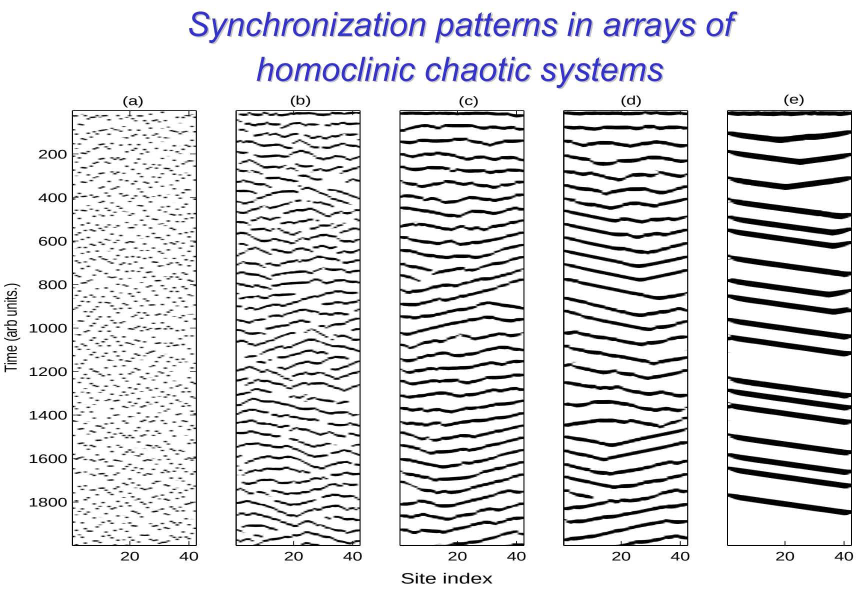


Fig. 4. *Homoclinic chaos through a saddle focus bifurcation (HC). The phase space trajectory escapes from S through the unstable manifold and returns to S through the stable one .HC has been found in laser equations with feedback as well as in several neuron models as e.g. Hodgkin –Huxley and Hindmarsh-Rose.*

The spike emission from a nonlinear dynamical system is a matching between *bottom-up* (input) stimuli and resetting of the control parameters by *top-down* controls. The most plausible mechanism for it is the chaos due to a saddle focus S bifurcation (Shilnikov chaos) (Shilnikov). Let us see how it occurs (Fig.5). The trajectory in phase space is a closed one *(HC =homoclinic chaos)* with a return to S through a stable manifold with contraction rate α and escape through the unstable manifold with expansion rate γ. Chaos requires $\alpha<\gamma$. Away from S the motion is regular and gives rise to an identical peak P per turn. The inter-peak interval (ISI) is chaotic due to the variable amount of time spent around S. The HC dynamics has been studied in detail for a CO2 laser (Arecchi et al, 1987,1988); for convenient control parameters, neuron models as Hodgkin-Huxley or Hindmarsh-Rose present HC .(Feudel et al).

The qualitative dynamics sketched in Fig. 4 shows that HC occurs under very general assumptions; thus we set aside specific physiological mechanisms and model the individual neuron as an HC system. The susceptibility χ (sensitivity to an external perturbation) is high around S and low everywhere else [] (Arecchi 2004); thus the system is very resilient to uniformly distributed noise. The high χ allows a response correlated in time with an external perturbation. This has been proved by synchronization to a periodic applied signal (Allaria et al) or by mutual synchronization of an array of coupled identical HC systems, modeling the neurons an interacting cortical area (Leyva et al.).



ε=(a) 0.0;(b)0.05;(c)0.1;(d)0.12;(e)0.25

Fig. 5 *Space-time representations of spike positions for different coupling strengths. Connected lines mean that adjacent sites have spike separation much smaller than the ISI (interspike interval); disconnection denote loci of local defects (one spike more or less with respect to the previous site).*

In fig. 5 I plot the space –time locations of spikes (each one represented as a point) in an array of HC systems with nearest neighbor coupling. They represent a plausible model for a set of coupled neurons. From left to right, we have different degrees of coupling. Full synchronization is reached above a critical coupling strength. By synchronization we do not mean isochronism, which would imply horizontal lines in the plot of fig.5. We have to account for lags in the dynamical coupling; furthermore, in real neuron arrays one should also account for delays in the axonal propagation. Thus, synchronism means that the spikes in neighboring sites are separated by much less than the average ISI; otherwise there would be a spike missing (as the “holes” or defects in the patterns at intermediate couplings). Of course, in a long chain the single spike time separation between the first and the last can be much large than the average ISI; yet the system has a coherent pattern without defects.

So far we referred to spontaneous synchronization for a sufficient mutual coupling. In the presence of an n input signal at one (or a few) neurons, the whole array can undergo synchronization even for coupling below the critical one. These considerations have been developed in (Leyva et al.).

In the case of two competing inputs (Fig. 6), the respective responses are two synchronized clusters of different size, depending on the input feature (the frequency of a periodic signal). It is plausible to assume that, if an equilibrium configuration is reached after a transient (Fig.7 a and b), then a majority rule will select the perceived pattern. If no equilibrium is reached and strong fluctuations persist (Fig. 7 c) then we are in presence of ambiguous patterns as investigated by Gestalt psychologists.

Thus, in this dynamical model of feature binding, we take neurons as coupled HC systems; a feature is recognized it the corresponding input induces a collective synchronized state; competing features will induce different coherent domains and the winner should be the largest coherent domain. At a higher perceptual level, a “reader” of these collective states classifies as categories the collective states, not just the single neuron states.

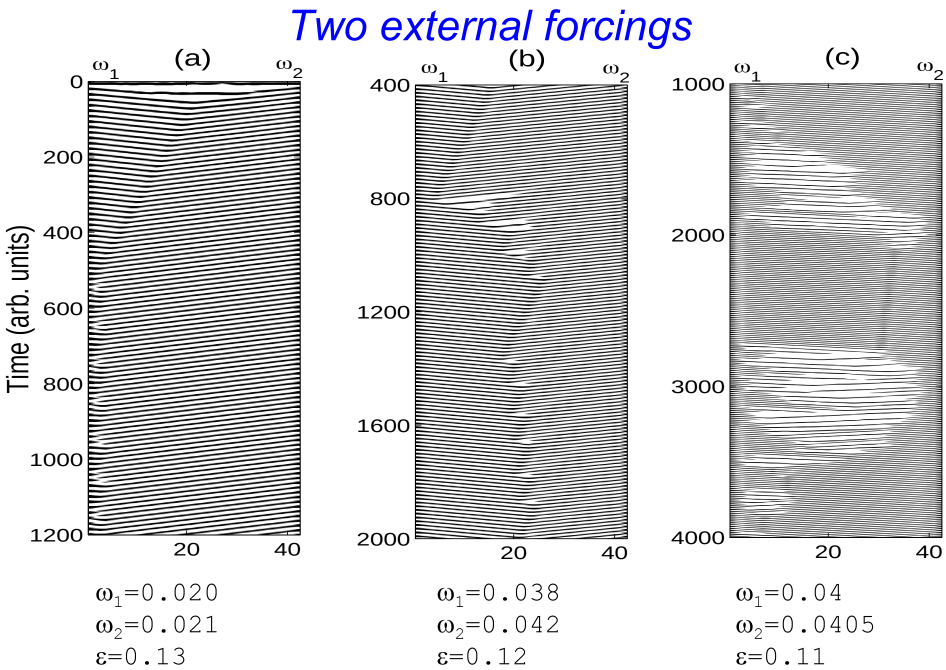


Fig. 6 *Competition between synchronisation regimes imposed by two external periodic stimuli applied respectively at the first and last site of a linear array of coupled systems, for different frequencies* ω_1, ω_2 *of the two input signals and for different coupling strengths* ε *of nearest neighbour neurons. The vertical scales in the three frames are different since we wish to select different time slots (from I.Leyva et al)*

Spike emission from a neuron is a matching between a *bottom-up* input signal arriving from the sensory detectors (retina in the visual case) and control parameter setting due to a *top-down* signal sent by the memory and corresponding to a possible categorical interpretation of the input. The bottom-up signal arriving from the early visual stages codes an elementary feature, e.g. a horizontal bar, independently on where it comes from (it can be apriori associated with any of different objects, say a dog and a cat, included in the visual field). The top-down perturbation originates from an "interpretation" provided by the semantic memory, where the categories "dog" and "cat" are stored. A focal attention mechanism keeps trying different categories until a matching is reached.

This model has been called *ART (adaptive resonance theory)* by S Grossberg (Grossberg). It is plausible to conjecture that only for resonant states (that is, those in which top-down and bottom-up match reciprocally) awareness is reached, and this seems to require a time around 200 ms.

ART is very successful in explaining the perception formation. There is however a strong limitation in it, namely, since it is the basis of a computational program, the stored categories are classified as fixed objects, Whereas in cognitive science they are considered as modifiable entities, molded by the ongoing increase of knowledge; we will return on this in the next Section.

3. Two types of cognitive task. Semantic complexity

We distinguish two types of cognitive task. In *type I*, we work within a prefixed framework and readjust the hypotheses at each new cognitive session, by a Bayes strategy. Bayes theorem (Bayes) consists of the relation:

$$P(h \mid data) = P(data \mid h)\, P(h)/P(data) \qquad (1)$$

That is: the probability *P(h | data)* of an hypothesis *h*, conditioned by the observed *data* (this is the meaning of the bar |) and called *a-posteriori probability of h*, is the product of the probability *P(data |h)* that *data* is generated by an hypothesis *h*, times the a-priori probability *P(h)* of that hypothesis (we assume to have a package of convenient hypotheses with different probabilities) and divided the probabilty *P(data)* of the effectively occurred data. As shown in Fig. 7, starting from an initial observation and formulating a large number of different hypotheses, the one supported by the experiment suggests the most appropriate dynamical explanation. Going a step forward and repeating the Bayes procedure amounts to climbing a probability mountain along a steepest gradient line.

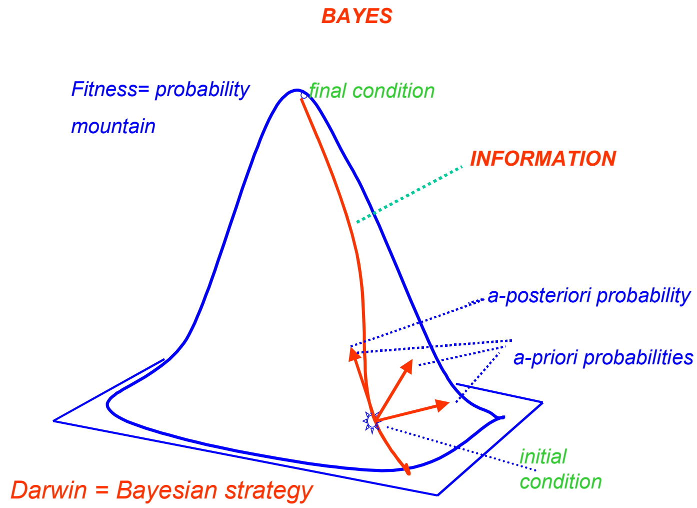


Fig. 7. *Successive applications of the Bayes theorem to the experiments. The procedure is an ascent of the Probability Mountain through a steepest gradient line. Each point of the line carries an information related to the local probability by Shannon formula*.

The evolutionary strategy postulated by Darwin, as sequences of *mutations* followed by *selection* of that mutant which best fits the environment (that is, which maximizes the fitness) is in fact an application of Bayes theorem, once we call *fitness* the probability mountain. Such an endeavor can be performed by a computer, since it has been performed within one code.

A complex problem is characterized by a probability landscape with many peaks (Fig. 8). Jumping from a probability hill to another is not Bayesian; I call it *type II* cognition. A deterministic computer can not do it.

In human cognition, *type II* is driven by hints suggested by the context (*semiosis*) yet not included in the model. *Type II* task is a *creativity* act because it goes beyond it implies a change of code, at variance with *type I,* which operates within a fixed code. . The ascent to a single peak can be automatized in a steepest gradient program; once the peak has been reached, the program stops, any further step would be a downfall. A non-deterministic computer can not perform the jumps of *type II*, since it intrinsically lacks semiotic abilities. In order to do that, the computer must be assisted by a human operator. We call "meaning" the multi-peak landscape and "semantic complexity" the number of peaks. However, this is a fuzzy concept, which varies as our comprehension evolves.

Let us discuss in detail the difference between *type I cognitive task,* which implies changing hypothesis *h within a model,* that is, climbing a single mountain, and type *II cognitive task* which implies *changing model,* that is, jumping over to another mountain.

We formalize a model as a set of dynamical variables xi (i=1,2,…N) ,N being the number of degrees of freedom, with the equations of motion

$$\dot{x}_i = F_i(x_1 ..., x_N; \mu_1 ..., \mu_M) \qquad (2)$$

Where F_i are the force laws and the M numbers $\mu_{i\,j}$ represent the *control parameters*; .The set {F,x,μ} is the model.

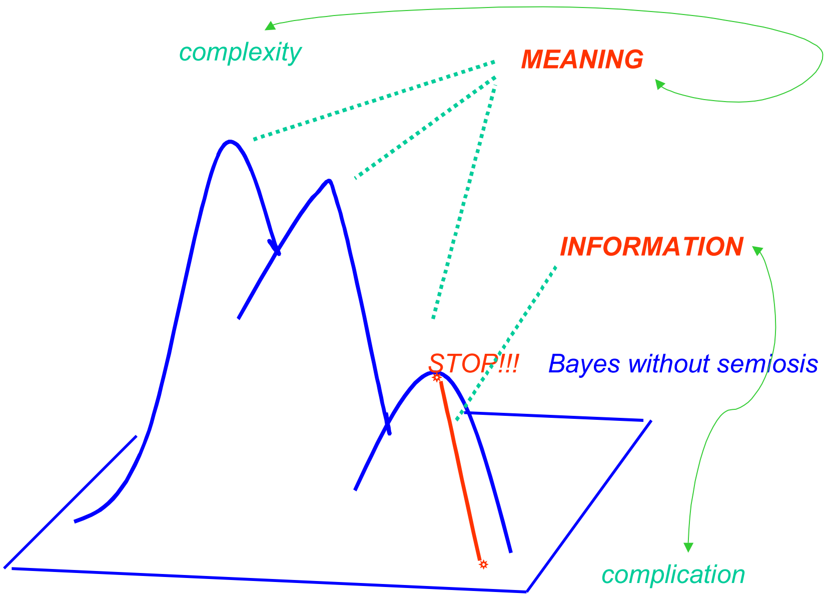


Fig.8- *Semantic complexity- A complex system is one with a many-peak probability landscape. The ascent to a single peak can be automatized in a steepest gradient program; to record the other peaks, and thus continue the Bayes strategy elsewhere, is a creativity act, implying a holistic comprehension of the surrounding world (semiosis) .We call "meaning" the multi-peak landscape and "semantic complexity" the number of peaks.*

Changing hypotheses within a model means varying the control parameters, as we do when exploring the transition from regular to chaotic motion in some model dynamics.

Instead, *changing code, or model,* means selecting different sets of degrees of freedom, control parameters and equations of motion as follows:

$$\dot{x}_i = G_i(x_1 ..., x_R ; \mu_1 ..., \mu_L) \qquad (3)$$

where G_i, R and L are different respectively from F_i, N and M. The set {G,x,μ} is the new model

While changing hypotheses within a model is an a-semiotic procedure that can be automatized in an computerized *expert system,* changing model implies catching the meaning of the observed world, and this requires what has been called *embodied cognition* [5] (Varela). Embodied cognition has been developed over thousands of generations of evolutionary adaptation, and we are unable so far to formalize it as an algorithm.

This no-go statement seems to be violated by a class of complex systems, which has been dealt with successfully by recursive algorithms. Let us consider a space lattice of spins, with couplings that can be Ferro or anti-ferromagnetic in a disordered, but frozen way (spin glass at zero temperature, with quenched disorder). It will be impossible to find a unique ground state. For instance having three spins A, B, and C in a triangular lattice, if all have ferromagnetic interaction, then the ground state will consist of parallel spins, but if instead one (and only one) of the mutual coupling is anti-ferromagnetic, then there will be no satisfactory spin orientation compatible with the coupling (try with: A-up, B-up, C-up; it does not work; then try to reverse a single spin, but it does not work either).

This model has a cognitive flavor, since a brain region can be modeled as a lattice of coupled neurons with coupling either excitatory or inhibitory, thus resembling a spin glass, (Hopfield, Amit, Toulouse).

We have a large number of possible ground states, all including some frustration. Trying to classify all possible configurations is a task whose computational difficulty (either, program length or execution time) diverges exponentially with the size of the system. Sequentially related changes of code have been successfully introduced to arrive at finite-time solutions. (Mezard et al, Solomon).

Can we say that the mentioned solutions realize the reductionistic dream of finding a suitable computer program that not only climbs the single probability peak, but also is able to chose the highest peak? If so, the optimization problem would correspond to understanding the *meanin*g of the object under scrutiny.

We should realize however that spin glasses are frozen objects, given once for ever. A clever search of symmetries has produced a spin glass theory (Mezard et al) that, like the Renormalization Group (RG) for critical phenomena (Wilson) discovers a recursive procedure for changing codes in an optimized way. Even though the problem has a large number of potential minima, and hence of probability peaks, a suitable insight in the topology of the abstract space embedding the dynamical system has led to an optimized trajectory across the peaks. In other words, the correlated clusters can be ordered in a hierarchical way and a formalism analogous to RG applied.

It must be stressed that this has been possible because the system under scrutiny has a structure assigned once for ever. In everyday tasks, we face a system embedded in an environment, which induces a-priori unpredictable changes in course of time. This rules out the nice symmetries of hierarchical approaches, and rather requires an adaptive approach. Furthermore, a real life context sensitive system has to be understood within a reasonably short time, in order to take vital decisions about it.

4. Conclusion

Recently, I have discussed the time code in neural information processing (Arecchi, 2004). If we further inquire on the intrinsically human semiotic ability, a peculiar feature emerges .In fact, concepts are not static representations that can be stored in a memory (either, human or computer) but they are continuously molded by interaction between mental *states* and *context.*

With reference to Fig. 8, we can conclude that the algorithmic procedure within a specific formalized model (we say: within a single code) is the ascent of a single probability mountain, that is, an a-semiotic task that can be carried on by a computer.

The ability to “read” the signs of the environment in which the cognitive agent is embedded (semiosis) stimulates tentative jumps to other models. The tentative explanation here presented in terms of nonlinear dynamics is the following. The extra degrees of freedom introduced by the environment would induce deterministic chaos; the cognitive agent tends to stabilize its worldview (fig2) by a suitable mechanism of chaos control based upon its available resources (previous memories).The combination of the inner resources and of the novel elements provided by the environment implies a new dynamical model (a different mountain in the picture of fig.8) . Creativity is such a model change, which is not arbitrary, whimsical, but is guided by the requirement of a small *K (*maximal stability ,or reliability, of the new model).
To refer to the current debate among epistemologists, the model building here presented is neither *solipsistic* (in fact ,it includes information from the environment) nor *passive* (in fact, the chaos control is done by the agent).

References

- E. Allaria, F. T. Arecchi, A. Di Garbo and R. Meucci,Phys. Rev. Lett 86, 791 (2001).
- DJ Amit, H Gutfreund ,and H Sompolinski , Phys. Rev A32, 1007 (1985)
- F. T. Arecchi, R. Meucci and W. Gadomski, Phys.Rev. Lett. 58,2205(1987);
- F. T. Arecchi, A. Lapucci, R.Meucci, J. A. Roversi and P. Coullet, Europhys. Lett. 6,77 (1988);
- F.T. Arecchi and A. Farini. *Lexicon of Complexity (EC Contract n. PSS*0813),* Studio Editoriale Fiorentino,Firenze, 1996
- FT Arecchi- Physica A 338, 218-237 (2004).

-T. Bayes, Phil. Trans. Royal Soc. London 53, 370-418 (1763)

- CH Bennett and G Grinstein ,Phys Rev Lett, 55, 657 (1985)
- GJ Chaitin “Algorithmic information theory”, Cambridge University Press 1987
- D.Chawla, E. D. Lumer, and K. J. Friston, Neural Computation 12, 2805 (2000);
- F.Duret, F.Shumikhina, S.Molotchnikoff, BMC Neuroscience 7, 72 (2006).
- U. Feudel et al., Chaos 10, 231 (2000).-
- W.J. Freeman, *How Brains Make Up Their Minds*, Weidenfeld and Nicolson, London, 1999.
- C. M. Gray, Neuron, 24, 31 (1999);
- S Grossberg, The American Scientist, 83,439 (1995)
- JJ Hopfield, Proc.Nat.Aca.Sci.USA 79,2554(1982)
- I. Leyva, E. Allaria, S. Boccaletti and F. T. Arecchi, Phys. Rev. E 68, 066209 (2003).
- M Mezard, G Parisi, MA Virasoro, *Spin glass theory and beyond,* World Scientific, Singapore,1987;
- F. Rieke et al., ”Spikes: Exploring the Neuronal Code”, MIT Press, Cambridge MA, 1997.
- TA Sebeok,*Biosemiotics:its roots, proliferation,and prospects* Semiotica,1341/4,61-78 (2001);
- A. Shilnikov, L. Shilnikov and D. Turaev, Int. J. Bif. And Chaos 14, 2143 (2004).
- H.A. Simon (1980). *Cognitive science: The newest science of the artificial*. Cognitive Science, 4, 33-46.
- W. Singer and E. C. M. Gray, Annu. Rev. Neurosci. 18 555 (1995);
-- S Solomon, *The microscopic representation of complex macroscopic phenomena,* Annual Reviews of Computational Physics II,pp:243-294 (World Scientific,1995).
- G Toulouse,S Dehaene,and JP Changeux, Proc.Nat.Aca.Sci.USA 83, 1695 (1986)
- F. Varela, E. Thompson, E. Rosch, *The Embodied Mind*, MIT Press, Cambridge, MA, 1991.
- K.G Wilson , Rev. Mod. Phys. **47**, 773 (1975)