Cognitive dynamics: complexity and creativity

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Abstract. A scientific problem described within a given code is mapped by a corresponding computational problem. We call *complexity* (*algorithmic*) the bit length of the shortest instruction which solves the problem. Deterministic chaos in general affects a dynamical systems making the corresponding problem experimentally and computationally heavy, since one must reset the initial conditions at a rate higher than that of information loss (Kolmogorov entropy). One can control chaos by adding to the system new degrees of freedom (information swapping: information lost by chaos is replaced by that arising from the new degrees of freedom). This implies a change of code, or a new augmented model. Within a single code, changing hypotheses is equivalent to fixing different sets of control parameters, each with a different a-priori probability, to be then confirmed and transformed to an a-posteriori probability via Bayes theorem. Sequential application of Bayes rule is nothing else than the Darwinian strategy in evolutionary biology. The sequence is a steepest ascent algorithm ,which stops once maximum probability has been reached. At this point the hypothesis exploration stops.

By changing code (and hence the set of relevant variables) one can start again to formulate new classes of hypotheses. We call *creativity* the action of code changing, which is guided by hints not formalized within the previous code, whence not accessible to a computer. We call *semantic complexity* the number of different scientific codes, or *models*, that describe a situation. It is however a fuzzy concept, in so far as this number changes due to interaction of the operator with the context.

These considerations are illustrated with reference to a cognitive task, starting from synchronization of neuron arrays in a perceptual area and tracing the putative path toward a model building. Since this is a report on work in progress, we skip technicalities in order to stress the gist of the question, and provide references to more detailed work.

1. Introduction: Complexity and chaos

A large amount of papers have appeared at the interface of neuroscience, computational science and physics. On the contrary, cognitive science has been considered thus far as territory of epistemology, psychology and computer science. We investigate for useful intersections of this area of investigation with physics, having in mind the recent progress in nonlinear dynamics and complex systems.

In computer science, we call problem *complexity* C the number of bits of the computer program which solves the problem. In physics , the most common approach to a problem consists of a description of the physical system in terms of its elementary components and their mutual interactions (*reductionistic code*). In general, this entails a chaotic dynamics with a non zero information loss rate K. Since these concepts have been circulating for quite a time, I just summarize some qualitative points, with the help of heuristic pictures.

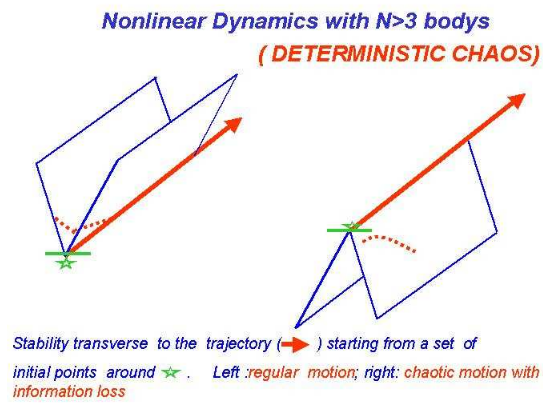

Figure 1. Deterministic Chaos

In Fig 1 the thick line with an arrow means that , for a given dynamical law , the trajectory emerging from a precise initial condition (the star) is unique . The space coordinates of the initial point are in general assigned by real numbers, that we truncate to a finite number of digits. Thus the initial condition is not a Euclidean point , but a whole segment. Initial conditions to the left or right of the ideal one converge toward the ideal trajectory or diverge away depending on whether the transverse stability analysis yields a valley-like (left) or hill-like (right) landscape. In the second case, we loose information of the initial preparation at a rate which depends on the steepness of the down-hill. This information loss does not require interaction with a disturbing environment as in noise problems; it is just a sensitive dependence on the initial conditions nowadays called *deterministic chaos*. The rate of information loss is denoted by K after Kolmogorov. Newton restricted his dynamics to 2-body interactions which are regular as the left figure. In 1890 Henry Poincaré showed that the gravitational problem with 3 or more interacting bodies displays generically the transverse instability depicted on the right.

Chaos can be controlled by additional external degrees of freedom ,which change the slope of the transverse potential without perturbing the longitudinal trajectory (Fig 2).Changing the number of degrees of freedom amounts to changing the *descriptive* code. In the perceptual case, we will see in Sec2 that a collective neuron dynamics is in general chaotic. In presence of a specific sensory input, different top-down perturbations due to different stored memories modify the transverse stability by different amounts. There will be a competition among different interpretations, that is, different perturbations of the sensorial stimulus by past memories. The winning result is that which assures the highest stability during the perception time (usually, a perceptual window is of the order of a few hundred milliseconds). In fig.2 we depict the role of two different control perturbations.

Thus, we anticipate already from Sec.2 that any coherent perception is an *interpretation*, that is, a *change of code* with respect to that imposed by the sheer sensorial stimulus.

If we do not introduce control, information on the initial setting has been lost after a time K^{-1} and one must re-assign the initial condition in order to predict the future: think e.g. of meteorological forecast. This may be very information-consuming; thats why a novel descriptive code which reduces K may be more effective than the old one.

Within the reductionistic code, problems have a monotonic $C-K$ behavior. For $K=0$ we have $C=0$; thus it has been straightforward to design an algorithm, $BACON$, (Simon) which infers Kepler laws from the regularities of the planet motions.

For $K \to \infty$ (Boltzmann gas) a dynamical description requires $N \to \infty$ variables hence

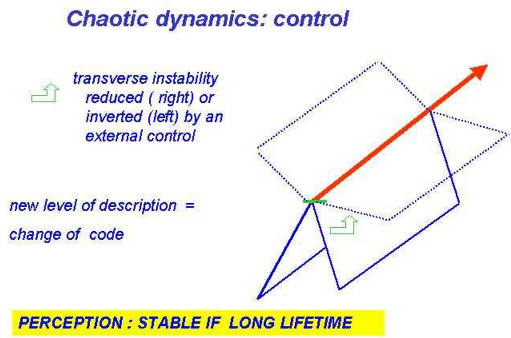

Figure 2. Control of Chaos

$C \to \infty$. However, for most experimental situations a thermodynamical description is sufficient, and *thermodynamics* has a very low C. This means that re-coding a problem in terms of new indicators suggested by a specific experience (*semiosis*) replaces the old code of the microscopic description with a new one having lower complexity . This reduction is by no means a loss of resolution, since the lost bits are inaccessible and hence they could not be dubbed as *hidden variables*.

We generalize saying that any code transformation implying a complexity reduction as it occurs in most human endeavours, as e.g. translation of a text to another language - requires a mechanism of information loss and replacement with new information not included in the primitive code (*information swapping*).

On the contrary, working within a fixed code, any complexity change is due to a different organization of the system under scrutiny,as it occurs in Renormalization Group applications called *Multi-Grid* (Solomon). The fixed code means that the analysis can be carried by a computer; this automatic version of complexity does not match our pre-scientific expectations. We rather call it *complication*, leaving the concept of complexity to the different points of view under which we grasp the same system under different codes (Sec 3).

In Fig. 3 we compare two different definitions of *complexity-complication*, namely the *algorithmic* one CA, already defined above and introduced by G. Chaitin in 1965 (Chaitin), and the logical depth D, introduced in 1985 by C. Bennett (Bennett) as the time necessary to execute a program starting from the shortest instruction. For low K the two definitions are equivalent, but, while CA increases monotonically with K, D goes to zero for high K. Indeed think of a random number. CA ,and K as well, will increase with the number of digits, whereas D is very short; once the number of digits has been collected in the long instruction, then the execution is just : "print it".

As an example, let us consider the Ising model. It consists of a set of coupled spin 1/2 particles in a thermostat at fixed temperature. At low temperature, the mutual interactions prevail over the thermal disturbance and all spins will be aligned parallel in an ordered collective state with low complexity. At high temperature, the thermal perturbation exceeds the mutual coupling and the spins are randomly distributed, hence both CA and D will be like the right extreme of Fig. 3. In the middle, at a critical temperature T_c , the two forces acting on each spin will be comparable and the whole system organizes in large clusters of parallel spins. The clusters undergo large size fluctuations, hence D reaches a maximum which scales with a power z of the system size L .In 3D, $z = 0.2$.

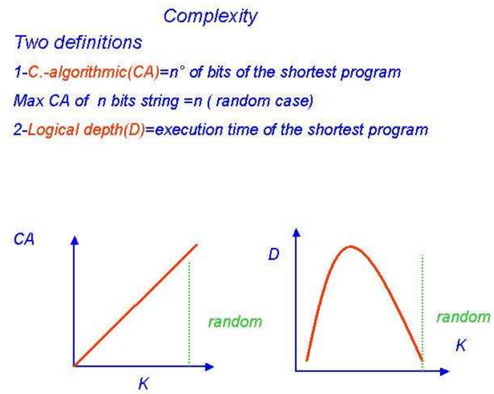

Figure 3. Two definitions of complexity as complication: 1- CA: algorithmic complexity(Chaitin); 2- D: logical depth (Bennett).

2. Dynamics of a cognitive task

As well known, a neuron is a brain cell with mainly chemical exchange (neurotransmitters) at both input and output. The body (soma) performs the usual cell operations, namely, cell metabolism and processing of input signals to be coupled as output signals. However, at variance with the other cells, the signal travels over long distances within the neuron as an electric signal (train of stereotyped spikes, of about 70 millivolt height and 1 ms duration) through a transmission line (axon) with a propagation speed around 10^2 cm/sec. The input information is coded as variable *inter-spike separation* (ISI) and then re-coded as an output neurotransmitter (Rieke et al). Since the axons have lengths between some micrometers in the brain cortex and one meter in the spinal chord, only the electrical propagation assures transmission times of a fraction of a second. The alternative would be a transportation in flow as with hormones in the blood, or by molecular diffusion between two cell membranes. The former may require tens or hundreds of seconds; the latter is convenient only for very small separations d, since the diffusion time T scales as

$$T = d^2/D, \tag{1}$$

where the diffusion constant D for bio-molecules in water is around 10^{-6} cm2/sec.Thus for $d = 1$ mm, T would be 10^4 sec (about 3 hours) against an electric transport time $d/v = 1$ms.

A neuron behaves as a threshold circuit which fires whenever the algebraic sum of the inputs (taking as $+$ the excitatory signals, and as $-$ the inhibitory ones) overcomes a given value.

Neural integration consists of a correlation between neurons, even far away from each other, when their receptive fields extract different features of the same object. This correlation (*feature binding*; see: Singer, Gray, Chawla, Duret) is a collective state with neurons having their spikes synchronized. Psychophysical studies have shown that the analysis of visual scenes occurs in two phases. First, the elementary features of the objects, as colour, motion, contour orientation, are locally detected in parallel. Next, these components are connected to provide a *Gestalt* or coherent object representation.

More precisely, *feature binding* denotes how coupled neurons combine external signals with internal memories into new coherent patterns of meaning. An external stimulus spreads over an assembly of coupled neurons, building up a corresponding collective state by *synchronization* of the spike trains of individual neurons. In presence of different external stimuli, different *clusters* of synchronized neurons are present within the same cortical area . The microscopic dynamics of N coupled neurons is thus replaced by the interplay of $n \ll N$ *clusters*. The n objects are

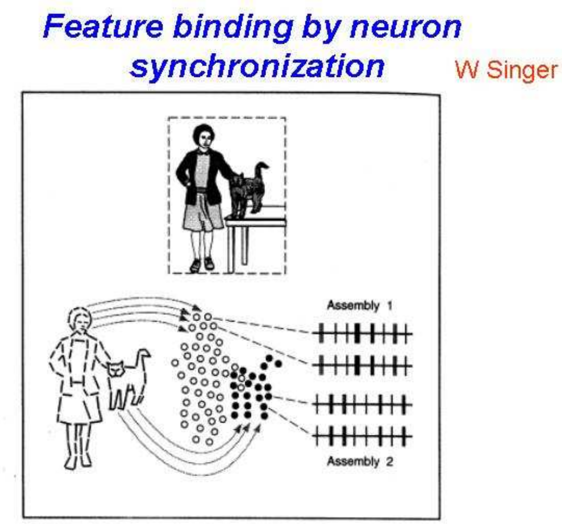

Figure 4. Feature binding.

the *attractors* of a chaotic dynamics.

The crucial fact has been the dissipation of information. This means that a perception based on the n collective clusters has lost the detailed information of the $N \gg n$ elementary components. Information loss and subsequent change of relevant variables means that coding at a higher hierarchical level is not just a computational task, but it violates the statute of a Turing machine.

Let us explore feature binding in detail, with reference to vision (Fig. 4). In vision, each fiber connecting the retina with the primary visual cortex has a limited receptive field. An extended figure is dissected over many channels, like a mosaic. A holistic perception emerges combining stimuli on different receptive fields by synchronization of the corresponding spike trains. Neural communication is based on a code whereby different regions which must contribute to the same perception synchronize their spikes.

The spike emission from a nonlinear dynamical system is a matching between *bottom-up* (input) stimuli and resetting of the control parameters by *top-down* controls. The most plausible mechanism for it is the chaos due to a saddle focus S bifurcation (Shilnikov chaos) (Shilnikov). Let us see how it occurs (Fig. 5). The trajectory in phase space is a closed one (HC =*homoclinic chaos*) having the return to S through a stable manifold with contraction rate α and the escape from S through the unstable manifold with expansion rate γ. Chaos requires $\alpha \ll \gamma$. Away from S the motion is regular and gives rise to an identical peak P per turn. The inter-peak interval (ISI) is chaotic due to the variable amount of time spent around S . The HC dynamics has been studied in detail for a $CO2$ laser (Arecchi et al, 1987, 1988); for convenient control parameters, neuron models as Hodgkin-Huxley or Hindmarsh-Rose present HC(Feudel et al). The qualitative dynamics sketched in Fig. 5 shows that HC occurs under very general assumptions; thus we set aside specific physiological mechanisms and model the individual neuron as an HC system.

The susceptibility χ (sensitivity to an external perturbation) is high around S and low everywhere else (Arecchi 2004); thus the system is very resilient to uniformly distributed noise. The high χ allows a response correlated in time with an external perturbation. This has been proved by synchronization to a periodic applied signal (Allaria et al) or by mutual synchronization of an array of coupled identical HC systems, modeling the neurons an

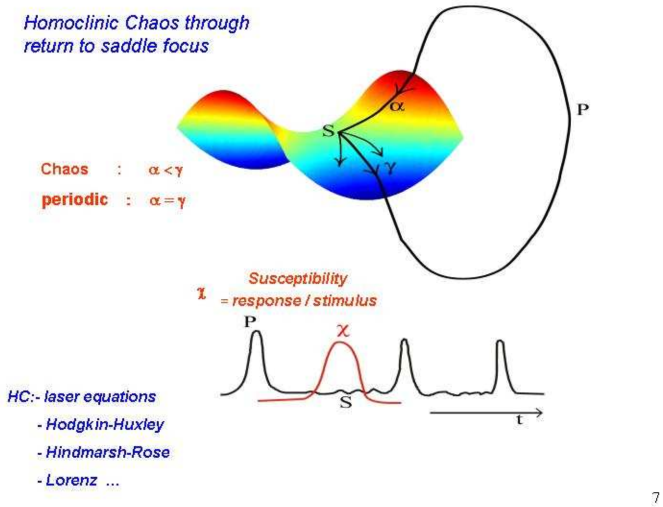

Figure 5. Homoclinic chaos through a saddle focus bifurcation.

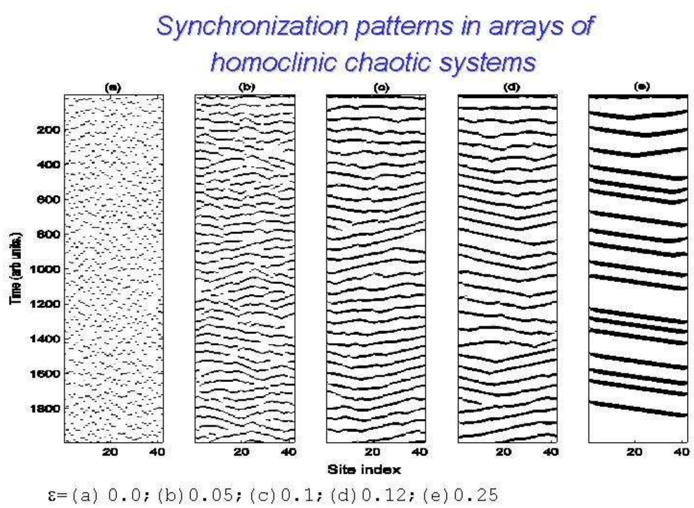

Figure 6. Space-time representations of spike positions for different coupling strengths. Connected lines mean that adjacent sites have spike separation much smaller than the ISI (interspike interval); Disconnections denote loci of local defects (one spike more or less with respect to the previous site).

interacting cortical area (Fig. 6) (Leyva et al).

In the case of two competing inputs (Fig, 7) (Leyva et al), the respective responses are two synchronized clusters of different size, depending on the input feature (the frequency of a periodic signal, in the model of Fig. 7. It is plausible to assume that, if an equilibrium configuration is reached after a transient (Fig. 7a-b)), then a majority rule will select the perceived pattern. If no equilibrium is reached and strong fluctuations persist (Fig. 7c) then we are in presence of ambiguous patterns as investigated by Gestalt psychologists.

Spike emission from a neuron is a matching between a bottom-up input signal arriving from the sensory detectors (retina in the visual case) and control parameter setting due to a top-down

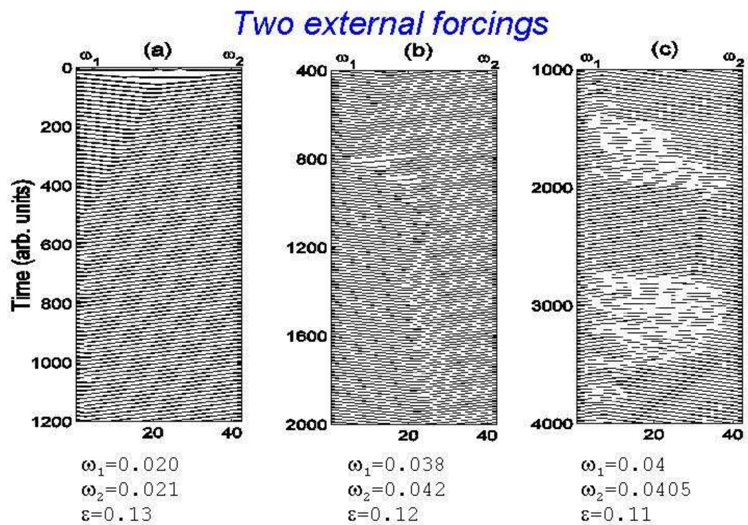

Figure 7. Competition between synchronization regimes imposed by two external periodic stimuli applied respectively at the first and last site of a linear array of coupled systems, for different frequencies and coupling strengths.

signal sent by the memory and corresponding to a possible categorical interpretation of the input (Fig. 9). The bottom-up signal arriving from the early visual stages codes an elementary feature, e.g. a horizontal bar, independently on whether the receptive fields points at the cat or at the lady of Fig. 4. The top-down perturbation originates from an "interpretation" provided by the semantic memory, where the categories "lady" and "cat" are stored. A focal attention mechanism keeps trying different categories until a matching is reached .

The model of Fig. 8 has been called *ART* (*adaptive resonance theory*) by S Grossberg (Grossberg). It is plausible to conjecture that only for resonant states (that is, those in which top-down and bottom-up match reciprocally) awareness is reached, and this seems to require a time around 200 ms. *ART* is very successful in explaining the perception formation. There is however a strong limitation in it, namely, since it is the basis of a computational program, the stored categories are classified as fixed objects, whereas in cognitive science they are considered as modifiable entities, molded by the ongoing increase of knowledge; we will return on this in the next Section.

3. Two types of cognitive task - Semantic complexity

We distinguish two types of cognitive task. In type I, we work within a prefixed framework and readjust the hypotheses at each new cognitive session, by a Bayes strategy. Bayes theorem (Bayes) consists of the relation:

$$P(h|data) = P(data|h)P(h)/P(data) \tag{2}$$

That is: the probability $P(h|data)$ of an hypothesis h, conditioned by the observed *data* (this is the meaning of the bar $|$) and called *a-posteriori probability* of h, is the product of the probability $P(data|h)$ that *data* is generated by an hypothesis h , times the a-priori probability $P(h)$ of that hypothesis (we assume to have a package of convenient hypotheses with different probabilities) and divided the probabilty $P(data)$ of the effectively occurred *data*. As shown in Fig. 9, starting from an initial observation and formulating a large number of different hypotheses, the one

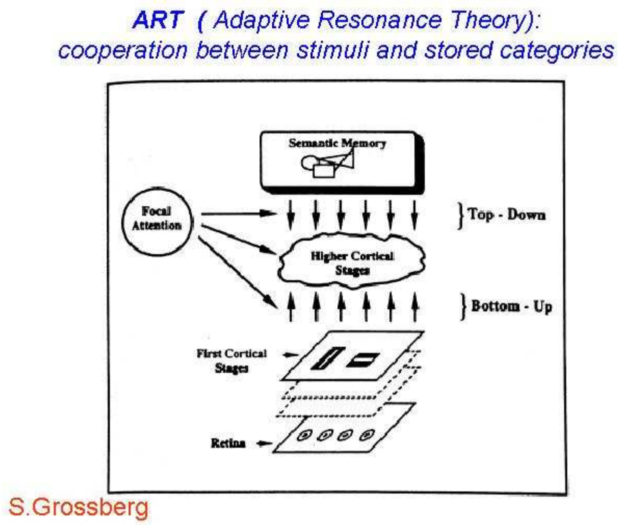

Figure 8. ART(adaptive resonance theory): cooperation between stimuli (bottom-up) and stored categories providing top-down modifications of control parameters . The bottom-up signal codes an elementary feature, e.g. a horizontal bar, independently on whether the receptive field points at the cat or at the lady of fig.4. The top-down perturbation originates from an "interpretation" by the semantic memory, where the categories "lady" and "cat" are stored. Focal attention keeps trying different categories until a matching is reached.

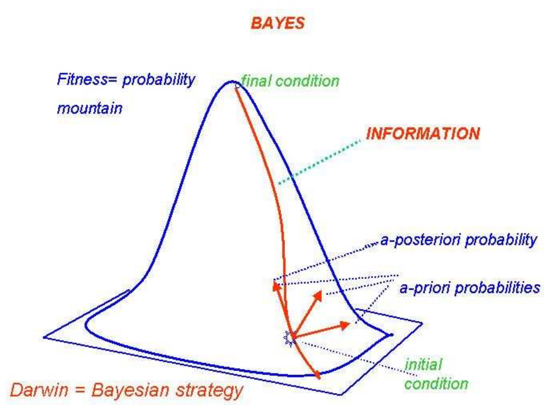

Figure 9. How successive applications of the Bayes theorem provide a scientific description more and more adequate to the experiments. The procedure is an ascent of the probability mountain through a steepest gradient line. Each point of the line carries an information related to the local probability by Shannon formula. Notice that the Darwinian mutation and successive selection of the mutant with maximum fitness is a recursive application of Bayes theorem.

supported by the experiment suggests the most appropriate dynamical explanation. Going a step forward and repeating the Bayes procedure amounts to climbing a probability mountain along a steepest gradient line.

The evolutionary strategy postulated by Darwin, as sequences of *mutations* followed by

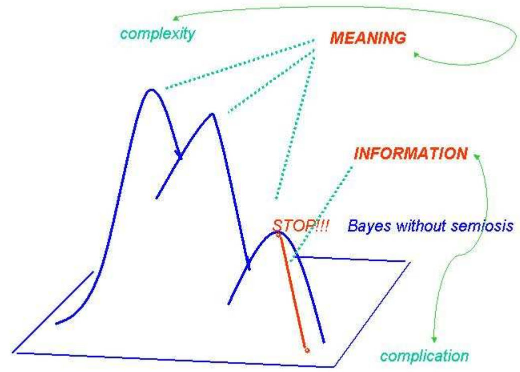

Figure 10. *Semantic complexity.* A complex system could be defined as one with a many-peak probability landscape. The ascent to a single peak can be automatized in a steepest gradient program. Once the peak has been reached, the program stops, any further step would be a down fall. To realize the other peaks, and thus continue the Bayes strategy elsewhere, is a creativity act, implying a holistic comprehension of the surrounding world (semiosis) .We call meaning the multi-peak landscape, whereas information was bound to a probability branch. We might associate the semantic complexity with the number of peaks. However, this is a fuzzy concept, which varies as our comprehension evolves.

selection of that mutant which best fits the environment (that is, which maximizes the fitness) is in fact an application of Bayes theorem, once we call *fitness* the probability mountain. Such an endeavour can be done by a computer, since it has been performed within one code.

A complex problem is characterized by a probability landscape with many peaks (Fig. 10). Jumping from a probability hill to another is not Bayesian ; I call it *type II* cognition. It can not be done by a deterministic computer. In human cognition, *type II* is driven by hints suggested by the context (semiosis) yet not included in the model. *Type II* task is a *creativity* act because it goes beyond it implies a change of code, at variance with *type I*, which operates within a fixed code. A non-deterministic computer can not perform the jumps of *type II*, since it intrinsically lacks semiotic abilities. In order to do that, the computer must be assisted by a human operator.

Let us elaborate in detail the difference between *type I cognitive task*, which implies changing hypothesis h within a model, that is, climbing a single mountain, and *type II cognitive task* which implies *changing model*, that is, jumping over to another mountain. We formalize a model as a set of dynamical variables x_i $(i = 1, ..., N)$, N being the number of degrees of freedom, with the equations of motion

$$\dot{x}_i = F_i(x_1, ..., x_N; \mu_1, ..., \mu_M), \tag{3}$$

where the M numbers μ_j represent the *control parameters.*

Changing hypotheses within a model means varying the control parameters, as we do when exploring the transition from regular to chaotic motion in some model dynamics. Instead, *changing code*, or *model*, means selecting different sets of degrees of freedom, control parameters and equations of motion as follows:

$$\dot{x}_i = G_i(x_1, ..., x_R; \mu_1, ..., \mu_L), \tag{4}$$

where G_i, R and L are different respectively from F_i, N and M.

While changing hypotheses within a model is an a-semiotic procedure that can be automatized in an computerized *expert system*, changing model implies catching the meaning of the observed world, and this requires what has been called *embodied cognition* (Varela). Embodied cognition has been developed over thousands of generations of evolutionary adaptation, and we are unable so far to formalize it as an algorithm.

This no-go statement seems to be violated by a class of complex systems which has been dealt with successfully by recursive algorithms. Let us consider a space lattice of spins, with couplings that can be ferro-or antiferro-magnetic in a disordered, but frozen way (spin glass at zero temperature, with quenched disorder). It will be impossible to find a unique ground state. For instance having three spins A, B, and C in a triangular lattice, if all have ferromagnetic interaction, then the ground state will consist of parallel spins, but if instead one (and only one) of the mutual coupling is antiferromagnetic, then there will be no satisfactory spin orientation compatible with the coupling (try with: $A - up$, $B - up$, $C - up$; it does not work; then try to reverse a single spin, but it does not work either). This model has a cognitive flavor, since a brain region can be modeled as a lattice of coupled neurons with coupling either excitatory or inhibitory, thus resembling a spin glass (Hopfield, Amit, Toulouse). We have a large number of possible ground states ,all including some frustration .Trying to classify all possible configurations is a task whose computational difficulty (either, program length or execution time) diverges exponentially with the size of the system. Sequentially related changes of code have been successfully introduced to arrive at finite-time solutions (Mezard et al, Solomon). Can we say that the mentioned solutions realize the reductionistic dream of finding a suitable computer program which not only climbs the single probability peak, but also is able to chose the highest peak? If so, the optimization problem would correspond to understanding the *meaning* of the object under scrutiny.

We should realize however that spin glasses are frozen objects, given once for ever. A clever search of symmetries has produced a spin glass theory (Mezard et al) that, like the Renormalization Group (RG) for critical phenomena (Wilson) discovers a recursive procedure for changing codes in an optimized way. Even though the problem has a large number of potential minima ,and hence of probability peaks, a suitable insight in the topology of the abstract space embedding the dynamical system has led to an optimized trajectory across the peaks. In other words, the correlated clusters can be ordered in a hierarchical way and a formalism analogous to RG applied.

This has been possible because the system under scrutiny has a structure assigned once for ever. In reality, a system is embedded in an environment, which induces a-priori unpredictable changes in course of time, and thus requires an adaptive approach to it. Furthermore, a real life, context-sensitive, system has to be understood within a reasonably short time, in order to take vital decisions about it.

We symbolize the computer versus human comparison by introducing the *hermeneutic circle* and the *hermeneutic spiral* (Fig. 11). Consider a semantic space, where each point represents a meaning. Assume that each word A, B,... is assigned a finite number of different meanings, or connotations. Once chosen a connotation $A1$ for A, linguistic connectors map it to $B1$ in B. The inverse operation takes back to $A1$, hence the *hermeneutic circle*.

If the meaning set is not frozen once for ever, but evolves in the interaction of the cognitive agent with the context, then the inverse mapping leads in general to $A2$ different from $A1$, since in between the two linguistic mappings the agent has evolved under the influence of the context. The meaning set is no longer finite, but it evolves in time, and the sequence of cognitive acts in semantic space is a *hermeneutic spiral*.

From the point of view of experimental neuroscience, repeated odor exposures of a locust probed by electrodes put in the olfactory neurons yield identical spike trains (Laurent et al). Thus the locust acts as a computer, since its previous learning holds un-modified in course of time. On the contrary, in a rabbit exposed to the same odor at different times, the neural patterns of electrical activity display modifications, even though the rabbit reacts in the same

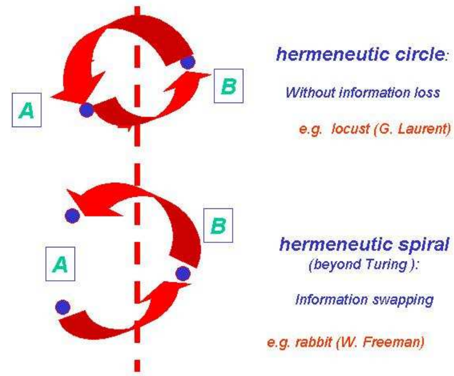

Figure 11. Hermeneutic circle and hermeneutic spiral.

way (Freeman); the second time the rabbit does not just feel the sensory input, but it combines the sensorial stimulus with the memory of a past experience, in a bottom-up/top-down interplay; we may dare to say that the rabbit has a creative knowledge.

4. Future agenda

At DICE 2004 , I discussed the time code in neural information exchange (Arecchi,2005).There, I sketched a possible quantum formalism for cognitive processes. Since then, I came across two independent research lines.

The first one (Aerts) treats the concepts not as fixed representational units but as potentialities actualized by different contexts in different ways, thus suggesting a close parallelism with the measurement process in quantum mechanics.

The second one (chaotic quantization in cosmology,'t Hooft, Biro et al) tries to harmonize quantum field theories and general relativity. With reference to primordial black holes at the Plancks length, it considers Planck's quantum of action as the product of a time over which information is lost (this is the ratio of the Planck's length to the light speed) and the energy of the primordial black hole.

Both approaches have promises and shortcomings as well, that I'll try to summarize. Since this is work in progress not yet accomplished, whatever I say must be taken as future agenda.

If we further inquire on the intrinsically human semiotic ability, a peculiar feature emerges which requires a quantum formalism. In fact, concepts are not static representations that can be stored in a memory (either, human or computer) but they are continuously molded by interaction between mental states and *context*. This research line has been carried for many years by D. Aerts and collaborators. However, even though they exploit a Hilbert space representation, no quantum of action has been assigned. This somewhat qualitative approach does not attribute a dynamical nature to the context or measurement system.

On the other hand, as we look at microscopic treatments, we realize that their peculiarity consists of the fact that states and context are made by the same *objects*; thus any measurement act can be described as identical particle-particle scattering. This led 't Hooft and Biro et al. to express $\hbar$ as the product of a fundamental time and a fundamental energy, respectively, the information loss time and the energy of primordial black holes at the Planck scale. However their argument is circular, as the numbers for Planck's length and energy rely on $\hbar$.

In brain dynamics, a neuron state is read by other neurons; thus we have a macroscopic situation analogous to the primordial one (system and measurement device physically identical).

Drawing a qualitative analogy with the above approach, we introduce in brain dynamics a new quantum of action a as the product of the energy of a single neuronal spike (about 10^{-13} joule) times the information loss time (about 10msec). It follows that

$$a = 10^{19}\hbar. \quad (5)$$

If the approach is sound, such a large value for the quantum of action should allow for long decoherence times of superposition states, thus hinting at a possible quantum computation at room temperature which may introduce a massive parallelism in some brain computations. Is there a way to combine the virtues of the two approaches, avoiding the respective shortcomings?

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