PHOTOCOUNT VERSUS PHOTON DISTRIBUTIONS *

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Photocount distributions and different treatments of laser statistics. Experimental results are given.

Different treatments of the laser statistics seem to assign conflicting criteria to estimate the degree of coherence of a laser radiation. Treatments based on a Fokker-Planck equation for the probability distribution of the field [1,2] take as a critical parameter the second reduced factorial moment H_2 of the photon distribution $p(n)$, that is

$$H_2 = F_2/F_1^2 - 1 \qquad (F_k = \langle n!/(n-k)!\rangle) \; . \tag{1}$$

This becomes very close to zero (as one would expect for a coherent state) for a mean photon number only 10 times the threshold value, as has been shown by experimental results [3].

A treatment based on a Master equation for the density operator of the field [4] characterizes the shape of $p(n)$ by its variance $\langle \Delta n^2 \rangle$. By this criterion, the difference from a coherent state (Poisson distribution) can be very large even for a mean photon number 10^3 times the threshold value. This fact can be easily explained remembering that a laser reasonably far above threshold is adequately described by the linear superposition of a coherent field with an average photon number S and a Gaussian field with average photon number $N = \frac{1}{4}\pi\langle n\rangle_0^2/S$ (n_0 being the photon number at threshold). Therefore the variance is given by [5-7]

$$\langle \Delta n^2 \rangle = S + N(1+N) + 2SN = \\ = S + \tfrac{1}{4}\pi \frac{\langle n\rangle_0^2}{S}\left(1 + \tfrac{1}{4}\pi \frac{\langle n\rangle_0^2}{S}\right) + \tfrac{1}{2}\pi \langle n\rangle_0^2 \tag{2}$$

that is, besides the contribution due to the Gaussian field, which becomes negligible as soon as $S > \langle n\rangle_0^{4/3}$, the variance is heavily affected by a constant interference term. Hence to have a

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nearly Poissonian variance requires a mean photon number much larger than the square of the mean photon number at threshold (this being of the order of 10^3 to 10^4, depending on the mode volume).

Let us see the question from the view point of the measuring procedure. It is known that an attenuation by a factor $\sqrt{\eta}$ of a field described by a $P(\alpha)$ distribution in the coherent state representation, leads to an excitation described by the weight function [6]

$$P(\alpha') = \eta P(\alpha) \ . \qquad (3)$$

Hence the k-th factorial moment

$$F_k = \int |\alpha|^{2k} P(\alpha) \, d^2\alpha$$

transforms as

$$F'_k = \eta^k F_k \qquad (4)$$

and H_2 remains independent of the attenuation. Using these properties it is easily seen that while the average photon number changes as $\langle m \rangle = \eta \langle n \rangle$, the variance changes as

$$\langle \Delta m^2 \rangle = \eta^2 \langle \Delta n^2 \rangle + \eta(1-\eta)\langle n \rangle \qquad (5)$$

and for $\eta \ll 1$, it reduces to $\langle \Delta m^2 \rangle = \eta \langle n \rangle$ as would be obtained if the statistics were Poissonian.

However in the case of photocount measurements η *is not* an attenuation factor, and it can take values even larger than 1. If the whole beam cross-section is collected on a photosurface (without diaphragms) the conversion ratio from photons to photoelectrons is given by [8]

$$\eta = \frac{cT}{L} \theta_1 \theta_2 \theta_3 \qquad (6)$$

where c is the velocity of light, θ_1 the mirror transmittance, θ_2 the filters transmittance, θ_3 the photocathode quantum efficiency, L the cavity length, and T the measuring time of a single sample. In a typical case, $\theta_1 = 0.02$, $\theta_2 = 0.2$, $\theta_3 = 0.05$ and $L = 20$ cm, therefore for $T \approx 3$ μsec one has $\eta \approx 1$*. By standard photocount techniques this would be an impossible experiment, because with such a low attenuation the interval T would be overcrowded by 10^3 to 10^4 photoelectrons, giving enormous dead-time problems.

* In order to have meaningful photocount distributions T must be much smaller than the relaxation time τ_c for intensity fluctuations. In the experiment reported here τ_c is 16 μsec (see fig. 1 of ref. 9).

Table 1

	Measured (photocount distributions)			Calculated (photons in the cavity)
η	1.6×10^{-2}	6.6×10^{-2}	0.6	1
$\langle m \rangle$	160	670	6100	10150
$\langle \Delta m^2 \rangle$	430	4500	310000	940000

By use of the so called "linear" method applied by us in several experiments [8], it becomes possible to verify the above considerations. We have used a single-mode He-Ne laser with $\langle n \rangle_0 \approx 10^3$, amplitude stabilized above threshold at a photon number $S \approx 10^4$. Starting with the above reported values for $\theta_{1,2,3}$ and L, and with $T = 2$ μsec, and introducing different degrees of extra-attenuation, we have measured the values reported in table 1. They result in complete agreement with eqs. (2) and (5).

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