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Role of Quantum Interference in Superradiant Decay.

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We have recently shown ([1]), by introducing a frequency renormalization and using the sequential decay theory ([2]) that the co-operative spontaneous emission of radiation by N two-level atoms enclosed in a region of linear size smaller than the wavelength (superradiance) displays the following features, not contained in the original paper by DICKE ([3]):

i) The line shape is asymmetric with respect to its geometrical center $\langle\omega\rangle$.

ii) Its center $\langle\omega\rangle$ is shifted with respect to the single-atom frequency ω_0 by an amount $\Delta\omega$, related to the co-operative decay rate $\gamma_c \sim N\gamma_0$ ($\gamma_0 =$ single-atom decay rate), the size l of the radiating region, and the wavelength $\lambda = 2\pi c/\omega_0$ by

$$\Delta\omega/\gamma_c \sim \lambda/l\,. \tag{1}$$

Equation (12) of ref. ([1]) contains much more information than the line shift as will be discussed elsewhere. Here we want to comment on the role played by interference among decay amplitudes in changing drastically the spectral profile of superradiance.

We consider for simplicity the case of two atoms a distance l apart both prepared in the excited state. The transition to the ground state implies a spontaneous emission of two photons, through the intermediate symmetric atomic state ([3]) of energy $\hbar\omega$. According to DICKE ([3]) we should measure a line width of order $\gamma_c \sim 2\gamma_0$. However, according to the usual theory of atomic decays ([4]) we should have a line width associated with the first decay given by the sum of the widths of the upper and intermediate

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([1]) F. T. ARECCHI and D. M. KIM: submitted for publication to *Phys. Rev. Lett.*
([2]) A. S. GOLDHABER and K. M. WATSON: *Phys. Rev.*, **160**, 1151 (1967).
([3]) R. H. DICKE: *Phys. Rev.*, **93**, 99 (1954).
([4]) V. WEISSKOPF and E. WIGNER: *Zeits. Phys.*, **63**, 54 (1930).

levels, *i.e.* $\gamma_c \sim 4\gamma_0 \neq 2\gamma_0$. If we *neglect* the line shift of ref. ([1]), then both the first and the second transitions occur at the same frequency and the solution of the paradox is like that for a harmonic oscillator ([5]). The excess line width is removed by the interference between the amplitudes of the two successive decays. When, however, the two atoms are so close that the line shift of eq. (1) greatly exceeds the damping, then the two successive photons are distinguishable and there is no longer interference between the two amplitudes. Hence we obtain two distinct lines of widths $4\gamma_0$ and $2\gamma_0$, rather than a single Dicke line of width $2\gamma_0$. This result can be easily demonstrated by specializing eq. (12) of ref. ([1]) for the case of two atoms. The amplitude for decay to a final state with two photons of frequencies ω_1 and ω_2 consists of two parts: an amplitude A_1 for the decay $|2; \text{vac}\rangle \to |1; \omega_1\rangle \to |0; \omega_1, \omega_2\rangle$ and an amplitude A_2 for the decay $|2; \text{vac}\rangle \to |1; \omega_2\rangle \to |0; \omega_1, \omega_2\rangle$, where the atomic states are $|2\rangle$ (upper), $|1\rangle$ (intermediate) and $|0\rangle$ (ground). The amplitudes are given by

$$(2)\qquad \begin{cases} A_1 = \dfrac{1}{x_1 + x_2 - \Delta\omega_2 + i\gamma_2}\,\dfrac{1}{x_2 - \Delta\omega_1 + i\gamma_1}\,, \\[2ex] A_2 = \dfrac{1}{x_1 + x_2 - \Delta\omega_2 + i\gamma_2}\,\dfrac{1}{x_1 - \Delta\omega_1 + i\gamma_1}\,. \end{cases}$$

Here $x_1 = \omega_1 - \omega_0$, $x_2 = \omega_2 - \omega_0$, $\Delta\omega_2 - i\gamma_2$ and $\Delta\omega_1 - i\gamma_1$ are the self-energies (level shift and damping) of $|2\rangle$ and $|1\rangle$ respectively, $\gamma_1 = \gamma_2 = 2\gamma_0$, and ω_0 is the single-atom transition frequency. Also, $\Delta\omega_1 = 2\gamma_0 \lambda/l$ and

$$(3)\qquad \Delta\omega_2 = (1 + \eta)\, 2\gamma_0 \lambda/l\,,$$

where $\eta = O(\gamma_0 \lambda/c) \ll 1$.

It is important to note that while the level shift $\Delta\omega_1$ of a two-atom system prepared in the first excited state $|1\rangle$ is given already by a Weisskopf-Wigner theory plus the renormalization of ref. ([1]), the level shift $\Delta\omega_2$ of the higher state $|2\rangle$ is affected by the peculiarities of the sequential decay process and cannot be described by the usual W.-W. theory. This is the origin of the correction term η. The emitted line shape is given by

$$(4)\qquad p(x_1) = \int \mathrm{d}x_2 |A_1 + A_2|^2\,.$$

Upon inserting eqs. (2) and (3) into this expression we obtain the following results:

i) If $l \sim \lambda$ (the shifts comparable with the widths) there is interference, *i.e.* $\mathrm{Re}\,(A_1^* A_2)$ is comparable with $|A_1|^2$ and $|A_2|^2$. In the limit when we neglect completely the shifts, we recover Dicke's result.

ii) If $l \ll \lambda$ (say $l = \lambda/100$) the shifts are much larger than the widths, the separation between the two resonances is $\Delta\omega_2 - \Delta\omega_1 \simeq \gamma_c \lambda/l \gg \gamma_c$. Hence the interference is negligible and eq. (4) yields two separated lines, one of width $4\gamma_0$ centered at $\omega_0 + \eta\,\Delta\omega_1 \approx \omega_0$ and one of width $2\gamma_0$ centered at $\omega_0 + \Delta\omega_1$.

We conclude that the solution to the harmonic-oscillator paradox can be applied to recover the usual results of superradiance theory only for large atomic separations l.

([5]) V. Weisskopf and E. Wigner: *Zeits. Phys.*, **65**, 18 (1930).