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With reference to other works on timing with scintillation counters, results on variances in time measurements are extended to the case of slow scintillators as NaI(Tl), for two different methods of timing. The systematic dependence of machine time on the amplitude of the scintillation (the so called « machine time walk ») has also been given.

Statistical limits in timing with slow scintillators

F. T. Arecchi *, A. De Matteis *, E. Gatti * *

* *Laboratori CISE - Segrate (Milano)*
* *Ingegneria Reattore, Centro Studi Nucleari - Ispra*
* *Istituto di Fisica del Politecnico, Cattedra di Elettronica Applicata - Milano*

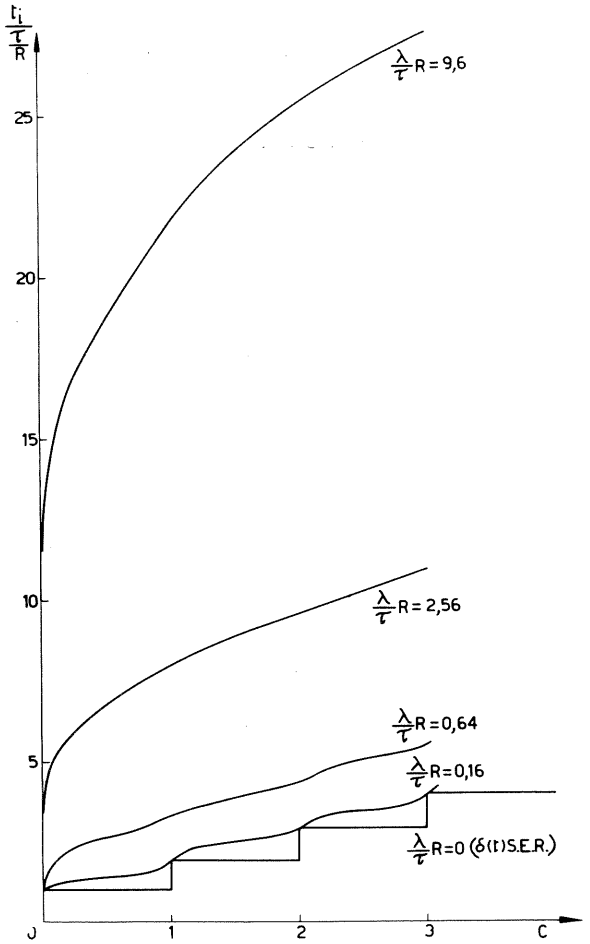
In two works on time resolution in scintillator counters [1, 2] results has been given for variances of « machine time » due to statistics of emission of photoelectrons from the photocathode. More precisely formulas (11) and first term of (18) of ref. 2 give the variances ε_c^2 and $\varepsilon_{t_i}^2$ of the centroid t_c of the used pulse fraction and of the time t_i at which a definite charge C is collected. It was also remarked in section 4 of ref. 2 that these formulas are not expected to be correct, because of the approximation made in deducing them when the charge C collected to the time t_i is caused by the contribution of a few photoelectron pulses: a rigorous solution was available only for ideal phototube with infinitely narrow single electron response ($\lambda = 0$) and constant illumination function. (App. I of ref. 1).

The region not covered by the quoted formulas is of no interest for fast scintillators but becomes the most interesting one when working with slow scintillators like NaI (Tl) and attempts are made to achieve high resolutions.

As the solution of the problem seemed too much involved for a direct analytical approach, a Montecarlo method was used in connection with the IBM 650 electronic computer. Constant illumination has been assumed as we are interested in the first part of the scintillation process, moreover no fluctuation ε_{ph}^2 in the mean transit time has been taken into account as this contribution to the variance can be calculated separately and in the case of slow scintillators is generally a small one. No spread in phototube multiplier amplification factor ε_A^2 has been introduced in order to simplify the program for the computer (its influence should be, apart from the variance of the first photoelectron, approximatively obtained by multiplying our results by $(1 + \varepsilon_A^2)$ as in the « continuous theory » developed in refs. 1 and 2. A S.E.R. (single electron response) of second moment λ referred to its centroid has been assumed.

The experiments on the computer where the exact simulation of the physical process. Times of occurrence of photoelectrons were drawn from the expected time distribution and centroid t_c of the set of S.E.R.s until a definited charge C has been reached was calculated and memorized as well as the time t_i at which the charge C was reached. Variances ε_c^2 and $\varepsilon_{t_i}^2$ of these two variables were estimated from a set of experiments. The experiments were pursued until a sufficient stabilization in t_c and t_i values, as well as their variances, was reached.

For the sake of simplicity, the time of occurrence of the first photoelectron was not drawn. Its mean



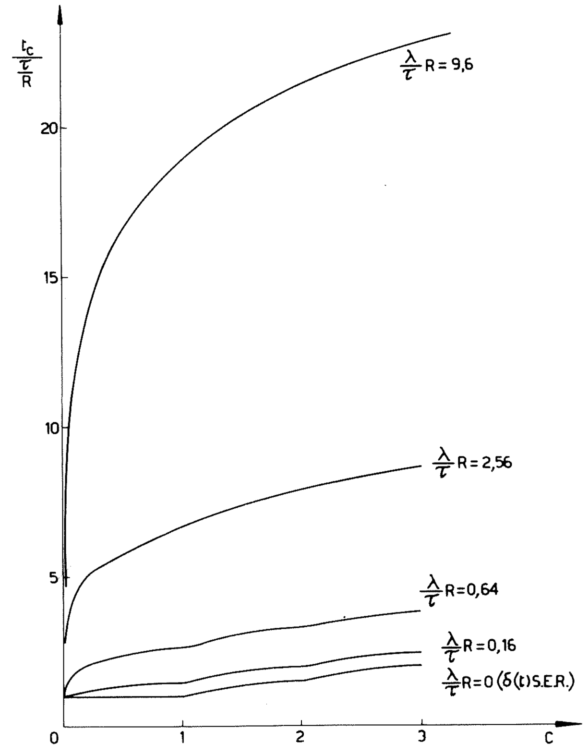
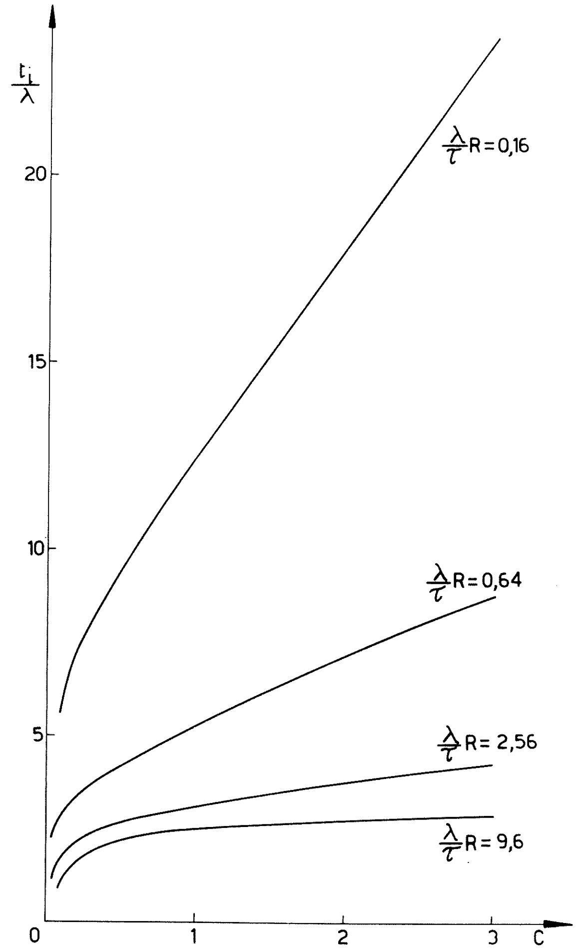


Fig. 1 and 2 - Plot of $\frac{t_i}{\tau / R}$ and $\frac{t_c}{\tau / R}$ vs. C for different $\frac{\lambda R}{\tau}$

value τ/R and the corresponding variance τ^2/R^2 are simply added up. This is a rigorous procedure because the occurrences of photoelectrons are independent events.
Results are shown in the figures for the case A) and B) (see ref. 2) that is for machine time corresponding A): to the centroid t_c of the used part of the input current pulse until the charge C is collected, and B): to the time t_i at which the charge C is collected.

In figs. 1 and 2 it is given the plot of t_i and t_c values versus C, for different widths λ of the S.E.R. The graphs are plotted again in figs. 3 and 4 with different variables more convenient to estimate the systematic « machine time walk » due to different input energies R. The diagrams are to be used in the range in which the hypothesis of constant illumination is true, that is for low C/R.
In figs. 5 and 6 we report the variances $\varepsilon_{t_i}^2$ and ε_c^2, normalized to the variance τ^2/R^2 of the first



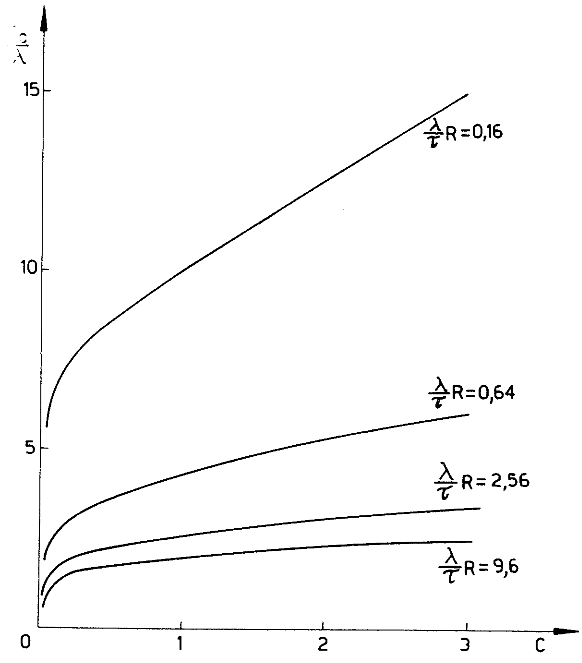
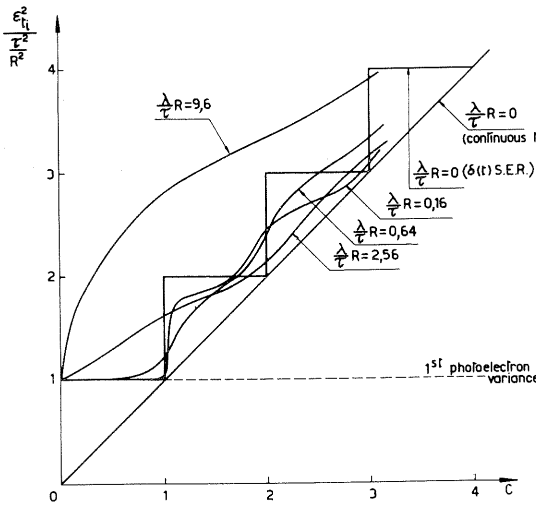


Fig. 3 and 4 - Plot of $\frac{t_i}{\lambda}$ and $\frac{t_c}{\lambda}$ vs. C for different $\frac{\lambda R}{\tau}$



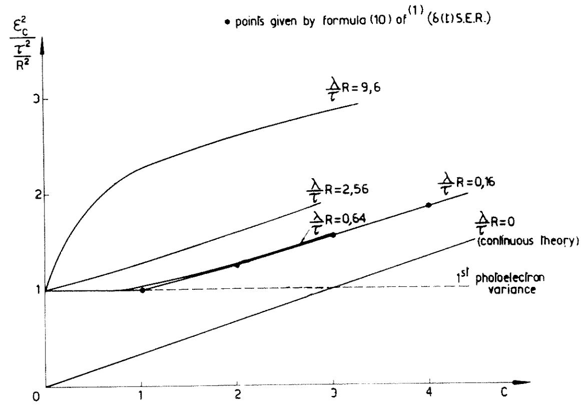


Fig. 5 and 6 - Variances $\varepsilon_{t_i}{}^2$ and $\varepsilon_c{}^2$ vs. C for different $\frac{\lambda R}{\tau}$

photoelectron. For comparison, in the same plots we have reported the variances for the case $\lambda = 0$, calculated from the « continuous theory », which is a good approximation for high C (*), and the ones calculated with the hypothesis of $\delta\,(t)$ S.E.R.s (See App. I of ref. 1) which are the rigorous solution for $\lambda = 0$. The steps of the curves for $(\lambda/\tau) \cdot R = 0$ in figures 1 and 5 are due to the fact that when a charge C is considered within m and $m + 1$, we observe always the time of arrival and the variance of the $(m+1)^{th}$ photoelectron and therefore in this range these variables are independent on C. Considering the centroid, on the contrary (figs. 2 and 6), contributions of pulses are averaged and no discontinuities appear.

These discontinuities and the corresponding oscillatory behaviour of t_i and $\varepsilon_{t_i}{}^2$ versus C for $\lambda \neq 0$ as shown in figs. 1 and 5 are of merely academical interest as the variance $\varepsilon_A{}^2$ in phototube multiplier gain smears them out.

CONCLUSIONS

Comparing figures 3 with 4 and 5 with 6 a slight superiority of the centroid method A) in comparison with the collection time method B) is observed both in variances and in systematic timing dependence on pulse amplitude R (« machine time walk »). As expected all methods becomes equivalent for $C \to 0$ as only the statistical properties of first photoelectron are involved in this case.

From the diagrams the influence of the S.E.R. width λ on timing variances and on « machine time walk » can also be evaluated. ■

Acknowledgments. We are glad to thank prof. Albertoni for discussing the convenience of the Montecarlo method for solving our problem.

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(*) The curve for $\frac{\lambda}{\tau} R = 9.6$ of the continuous theory has not been plotted, but we have checked that it is in good agreement with the Montecarlo one for $C > 1$.

bibliography

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[2] E. GATTI, V. SVELTO: Nucl. Instr., *4*, 189 (1959).

sommario

LIMITI STATISTICI DELLE MISURE TEMPORALI CON SCINTILLATORI LENTI

Si estendono i risultati di lavori precedenti sulla precisione di misure temporali con contatori a scintillazione al caso in cui si adoperino scintillatori lenti, quali NaI(Tl). Si riportano le varianze e la dipendenza sistematica del tempo di macchina dall'ampiezza dell'impulso di scintillazione in corrispondenza a due diversi metodi di utilizzazione dell'impulso di uscita dal rivelatore.