# Chaos in optics

**From Scholarpedia**

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**Chaos in optics** refers to the onset of deterministic chaos in optical systems. In a continuous dynamical system chaos may occur only for a coupled nonlinear system of three degrees of freedom. Since Maxwell equations are linear, provided the optical polarization of a medium is linear in the applied field, the overall field-medium equations are linear and consequently chaos would be ruled out from optics with natural light sources whose weak field is unable to elicit sizeable nonlinearities. In order to observe chaos, one has to recur to fields of high enough intesity to induce a nonlinear medium response; thus chaos in optics has become observable in conjunction with laser sources. Due to the fact that just three coupled degrees of freedom are sufficient for one positive Liapunov exponent, preliminary studies of chaos in optics had addressed single mode lasers (see Chaos in lasers). On the other hand, some breakthroughs have signed optical science and technology in recent years, namely, i) many mode lasers, emitting over many frequencies and directions, and providing high light intensities, sufficient to elicit nonlinear optical responses either within the same laser medium or in other media shined upon by the lasers; ii) introducing a delay in the laser dynamics implies a large number of degrees of freedom, even without redesigning the laser cavity for housing many optical modes; iii) if many chaotic lasers are coupled in a network, even though the single one is still chaotic (that is, without attempting some of the control operations discussed in Chaos in lasers), the individual chaotic behaviors can be synchronized; the onset of such a collective synchronization is relevant for communication technologies as well as for modeling neuron behavior in a brain during a perceptual task. In view of these considerations, the subject matter is organized as listed in the following outline with many emphasis on the case studies i),ii) and iii). As for generic chaos in nonlinear optical media due to incident fields, we present two examples, namely, active media providing gain and thus oscillation in a cavity, and passive media, transforming a plane wave incident field in a chaotic pattern.

## Contents

## The many mode laser and space-time chaos (patterning)

**longitudinal case**

In Fig.1, i) and ii) refer to laser cavities designed to house only longitudinal modes,that is, fields travelling along the cavity axis. In fact, if $L$ is the mirror separation and $d$ the mirror diameter, the number of diffraction angles $\lambda/d$ that can be accepted by the aperture angle $d/L$ is given by the Fresnel number:

$$F = \frac{d^2}{\lambda L}\,.$$

For example in a gas laser $L = 1m$, $\lambda \approx 10^{-6}m$ and $d \approx 10^{-3}m$, so that $F \approx 1$, and the laser cavity hosts a single transverse mode. If however the gain line is much larger than the longitudinal mode separation (see Fig.1 (ii)),

$$\Delta V_{gain} \gg c/2L\,,$$

then many longitudinal modes can be simultaneously above threshold. In such a case the nonlinear mode-mode coupling, due to the medium interaction, gives an overall high dimensional dynamical system which may undergo chaos. This explains the apparently random spiking behaviour of long lasers. The regular spiking in time associated with mode locking is an example of mutual phase synchronization.

### longitudinal case with delay

A nonlinear system with delayed feedback, as the delay time $\tau$ is much longer than the zero-delay correlation time $Tc$, displays two widely separated time scales. In such a case, a two-dimensional representation shows close analogies between delayed chaos and space-time chaos in extended systems. These phenomena have been dealt with in A.188,102,214 (http://www.inoa.it/home/arecchi/Papers.php); in particular, A.214 (http://www.inoa.it/home/arecchi/Papers.php), provides evidence of phase jumps in chaotic time sequences that, once folded in a two-dimensional representation, look like phase singularities in a two-dimensional fabric, to be discussed extensively in the next sub-section. When both non-delayed and delayed feedback are applied to the same laser, the combination of the two effects provides what has been called DSS(Delayed Self-Synchronization) and that is equivalent to the unidirectional coupling of a large array (virtually infinite number) of lasers.

### transverse case

Inserting a photorefractive crystal in a cavity, the crystal provides high optical gain if pumped by a laser beam. As the gain overcomes the cavity losses, we have a coherent light oscillator. Due to the narrow linewidth of the crystal, a single longitudinal mode is excited; however, by an optical adjustment we can have a large Fresnel number, and hence many transverse modes (Fig.1 (iii)). The dynamic interplay of a few modes for $F$ below 10, gives rise to an ''itinerancy phenomenon'' whereby each separate mode shows up for a finite time, followed by a next one and so on. All modes alternate sequentially and the same sequence then repeats for ever, either with a regular (periodic) or irregular (chaotic) timing A.177 (http://www.inoa.it/home/arecchi/Papers.php). For larger Fresnel numbers, many modes are simultaneously above threshold giving a chaotic pattern. If we had the random superposition of different waves, the phase relations would give rise to large peaks along directions where the random phase relations provide maximum field summation, intercalated by directions of zero field, wherever the random phases cancel the overall field. Such patterns are called *speckles*. Let us consider a (2-D) complex field, as the solution of generic 2-D wave problem. The real ( imaginary) part of the field crosses zero at a point, as it continuously varies positive to negative. As we unite all zero points for the real ( imaginary) part we have a network of two families of lines. At each crossing point where the two families intersect the field amplitude is zero and its phase not defined, so that the circulation of the phase gradient (line integral of the phase gradient along a closed path sorrounding that point) is non-zero (either $\pm 2\pi$ ) yielding phase singularities. A phase gradient circulation $\pm 2\pi$ is called a topological charge of $\pm 1$ respectively. In general two lines on a plane cross at least twice,

yielding two singularities of opposite $\pm 1$ charge. The outlined feature is generic of a 2-D field. In the linear case, by the central limit theorem, we expect a gaussian statistics of the singularities (speckle behaviour). In nonlinear mode-mode coupling, we observe similar phase singularities, but with a statistics different from speckles. How to visualize phase singularities: a photodetector responds to the modulus square of the field amplitude. To have phase information, we superpose a plane wave light to the 2D pattern, so that the modulus squared of the resulting field includes an interference term, as illustrated in Fig.2.

For a high Fresnel number we have a number of singularities scaling in average as the square of the Fresnel number [A.197]. When both intersections of the two zero lines for real and imaginary part are within the mirror boundary, we expect a balance of opposite topological charges. However, for small Fresnel numbers, it is likely that only one intersection is confined within the boundary; this corresponds to an inbalance, as shown in Fig.2, upper right. The scaling with the Fresnel number is purely geometric and does not imply dynamics. The statistics of zero-field occurrences can be predicted on purely geometric considerations, as done for random speckles. If instead we look at the high intensity peak in between the zeros, the high fields in a nonlinear medium give a strong mode-mode coupling which goes beyond speckles and hence display a non-Gaussian statistics. A 1-D experiment in an optical fibre has
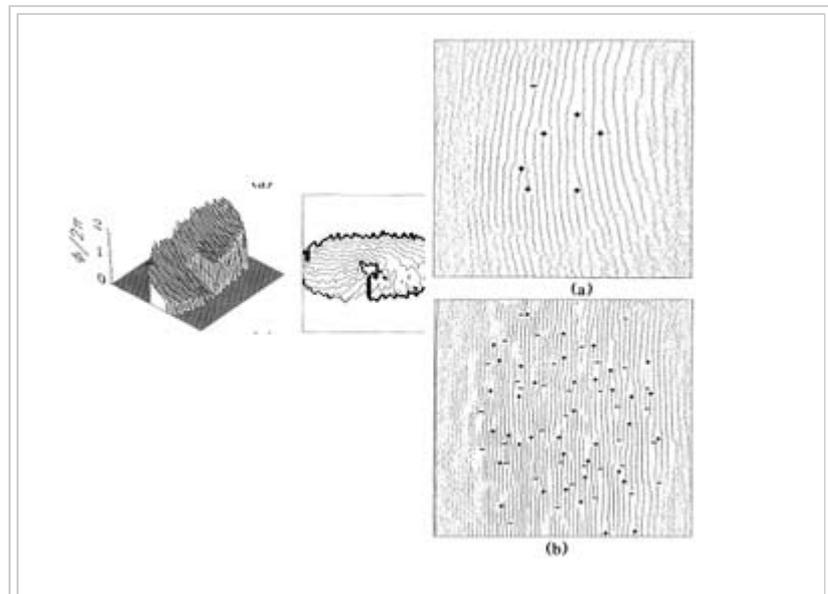


Figure 2: Left: a phase singularity is visualized by superposing an auxiliary coaxial plane wave to the optical pattern of the photorefractive oscillator; reconstruction of the instantaneous phase surface: perspective and equi-phase plots. Right: if the auxiliary beam is tilted, we obtain interference fringes, interrupted at each phase singularity ( $\pm$ correspond to $\pm 2\pi$ circulation, respectively). The digitized fringe plots correspond to : upper plot (Fresnel number about 3), 6 defects of equal topological charge against 1 of opposite charge; lower plot (Fresnel number close to 10), almost 100 singularities with balanced opposite charges, besides a small residual inbalance A.183 (http://www.inoa.it/home/arecchi/Papers.php)

produced giant optical spikes with non-Gaussian statistics [Solli et al.,(2007)]. The authors draw an analogy with the so called "rogue" wave in the ocean which represent a frequent problem to boats, since satellite inspection has shown that they are more frequent than expected on a purely linear basis. We consider the anomalous statistics of giant spikes as a case of complexity, because the mutual coupling in a nonlinear medium makes the number of possible configurations increasing exponentially with the Fresnel number, rather than polynomially.

## optical chaos and patterns in passive devices

Nonlinear dynamics does not necessarily require gain and self-oscillation as in a laser device. In fact, a nonlinear distortion of an impinging field can occur in course of propagation in a passive medium. If the geometry imposes a Fresnel number close to 1, then we have a single mode propagation (that of the distorted incoming wave) and no chaos can occur. In such a case, a well established phenomenon is that of optical bistability, whereby the output field can be at either a low level, having undergone an absorption in the medium, or at a high level, having propagated through the medium with practically zero absorption. This second scenario occurs beyond a critical field amplitude, as the two level transition of the absorbing transition saturates and the medium has become transparent. If the geometry allows for a high Fresnel number, then the impinging plane wave is scattered over many directions, and several modes propagate being mutually coupled by the medium nonlinearities. Once the dynamics imply more than 3 coupled equations, chaos becomes

generic. If the field propagates in a high Fresnel number geometry, and the medium can be approximated by a fast cubic nonlinearity (class A dynamics in a so called Kerr medium) then the wave equation in the eikonal approximation is given by:

$$\frac{\partial E}{\partial \tau} = -E + E_i + i\eta(|E|^2 - \theta)E + ia\nabla_\perp^2 E,$$

where $E$ is the field amplidute depending on both time $t$ and coordinate $r_\perp$ transverse to the direction of propagation and $E_i$ is the coherent injected field; the last term accounts for diffraction, and $\tau$ is a scaled time $t/t_c$, $t_c$ being the mean cavity lifetime for photons, a=1/4$\pi$ $F$, $F$ being the Fresnel number of the cavity, $\eta = \pm 1$ determines whether the nonlinearity is focusing or defocusing, $\theta$ is the detuning between the field and the cavity frequency. A survey of optical chaos in extended media, both active and passive, is given in A.266 (http://www.inoa.it/home/arecchi/Papers.php).
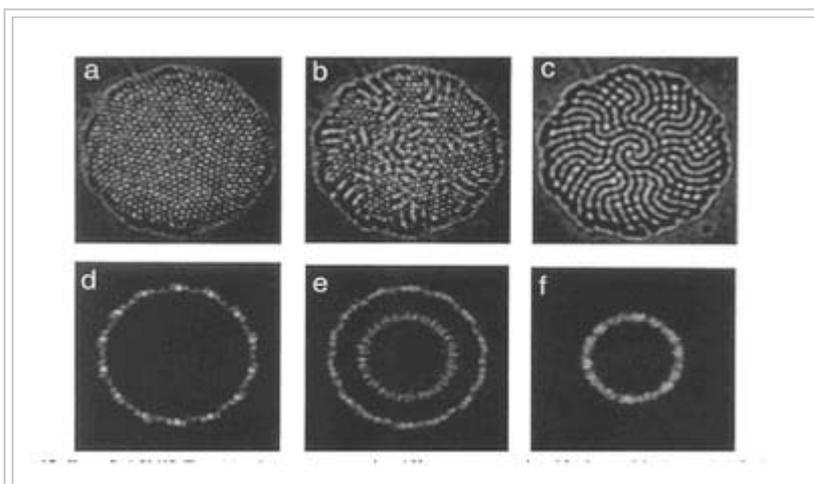


Figure 3: Chaos in passive optics-transition from patterns on band II to patterns on band I, observed at an object plane by increasing the input intensity from left to right. a) to c): near field patterns in a passive medium with diffractive feedback rotated by $\Delta = 2\pi/7$. For perfect cylindrical symmetry, all patterns with k-vectors lying on a circle are equally likely, but a small amount of imperfection gives smallest threshold to a given mode characterized by 14 Fourier peaks, as can be seen in fig. d) to f) (far field images collected in the focal plane of a lens). In d) the lowest threshold pertains to a band with a larger radius, in f) we observe the transition to a band with smaller radius. In between (e)we observe the coexistence of the two bands; this implies a dynamical competition which induces chaotic patterns, as can be seen in the object plane b)A.237 (http://www.inoa.it/home/arecchi/Papers.php).

## Mutual synchronization of coupled chaotic lasers

### general

Chaotic synchronization is a process wherein two or many chaotic systems adjust a given property of their motion to a common behaviour due to a coupling or to a forcing
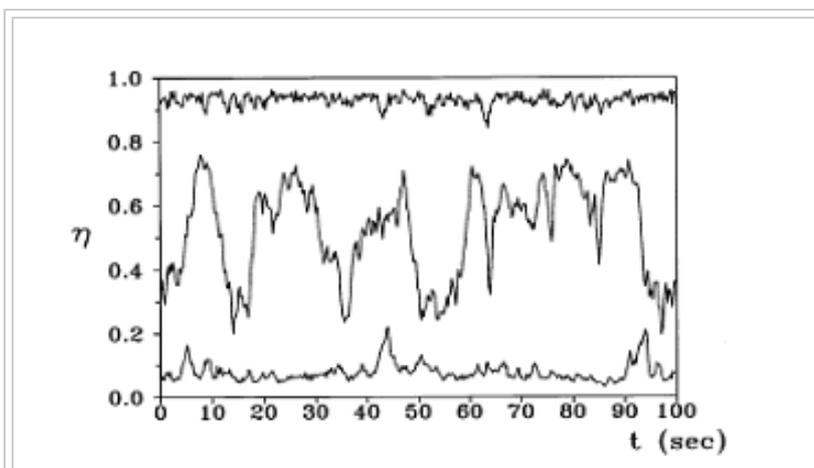
[Boccaletti et al.,(2002)].

Synchronization of oscillations in int...

Synchronization of chaotic lasers wa...

with a saturable absorber inside the o...

synchronization of chaotic erbium do...

al., (2000)] lasers were experimentall...

phenomena in semiconductor lasers in view of their impact in telecommunication systems using chaotic carriers [Heil et al. (2001); Widekind et al. (2002); Shahverdiev et al. (2002); Koumou et al., 2004; Argyris et al. (2006)]. Synchronisation of chaotic CO2 lasers with intracavity electro-optic modulator has been also extensively investigated during these years considering their analogies with neurodynamics when they emit spiking and bursting patterns A.334 (http://www.inoa.it/home/arecchi/Papers.php). Synchronizing several chaotic lasers is easy if the single system is compact and low cost; it is a problem in the case of CO2 lasers. For these lasers, we report two kinds of coupling: i) unidirectional, in which a laser at site $i$ is coupled to the same variable at site $i-1$. This corresponds to modifying the feedback term proportional to a suitable variable $x_1$ by an extra term as follows:

$$x_1^i + \epsilon(x_1^{i-1} - \langle x_1^i \rangle).$$

ii) bidirectional: each site is coupled to the nearest neighbors in a one-dimensional chain, with a modified feedback term as

$$x_1^i + \epsilon(x_1^{i+1} + x_1^{i-1} - 2\langle x_1^i \rangle).$$

### unidirectional coupling: single laser with delayed feedback

Adding a double feedback loop, as shown in Fig.5 we realize a DSS (Delayed Self-Synchronization). As the delay time is longer than the time over which information has been lost (reciprocal of the positive Liapunov exponent), the second feedback acts on a system which has lost memory of its initial conditions. It is as coupling a laser to an independent one, though with the same parameters. Let the laser without delayed feedback be set in a HC regime; it will emit a chaotic train of spikes. Let then activate the delayed loop with a delay $T_d$. If during $T_d$ the HC system has emitted a certain number of spikes, such a spike train will be repeated once and once again (Fig.6). If $\tau$ is the time lag between an input spike and a response spike, then DSS is equivalent to the unidirectional coupling of $N$ systems where $N\tau = T_d$, (see Fig.5 on the right).
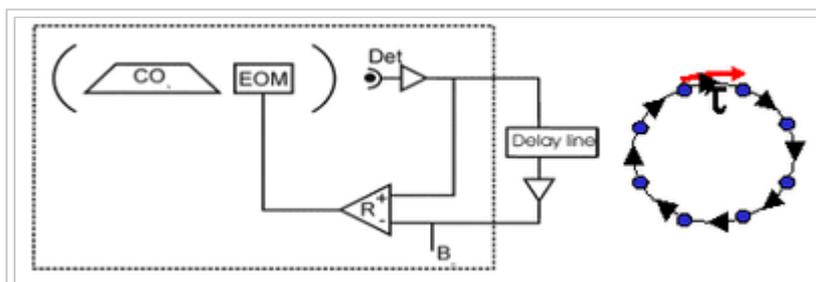


Figure 5: Left: Set up of a CO2 laser with double feedback, instant and delayed. Right:Numerical simulation by an array of *N* unidirectionally coupled HC systems with closed boundary conditions; if $\tau$ is the time lag in the transfer from the *i*-th to the *(i+1)*-th system (so called refractory time), then $N_\tau = T_d$, where $T_d$ is the delay time. However, the effectively measured delay is $T_\gamma = T_d + \tau = (N+1)\tau$, as shown in the next figure, A.296,305 (http://www.inoa.it/home/arecchi/Papers.php).

### bidirectional coupling and

### transition to collective synchronization

Study of large arrays of bi-directionally coupled lasers is costly, since one must do it on separate physical systems. One has to recur to cheap single Class B lasers, as e.g. diode lasers. Thus far, this investigation has been carried only numerically. The space-time plots of Fig.7 refer to 40 HC systems on line with bi-directional coupling. Each dot of the space–time plot reports the time of a spike
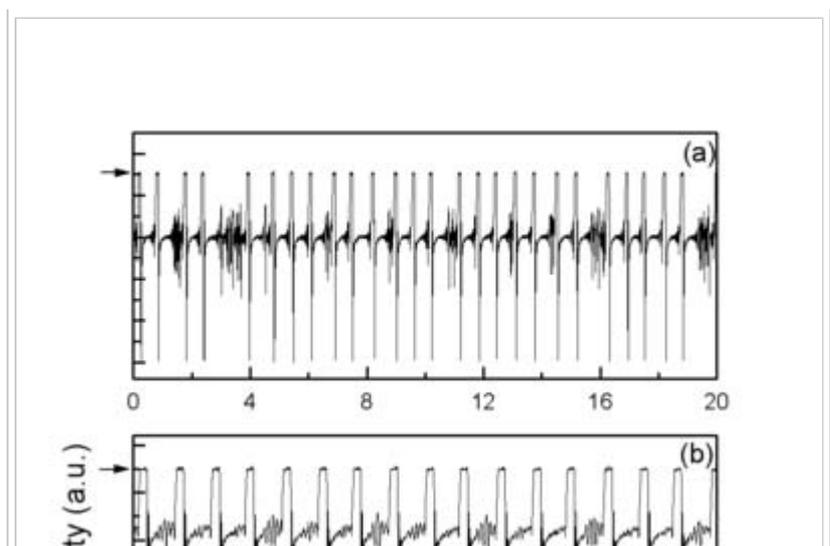
occurrence on each site. For zero
coupling, the dots are erratically distr
more correlated, until they get fully s
chronism: in fact a fixed time lag sep
of nearest spikes on two adjacent site
entropy of the probability distribution

$$S(t_r) = -\sum p(t_r) log p(t_r)$$

For HC systems, this entropy display
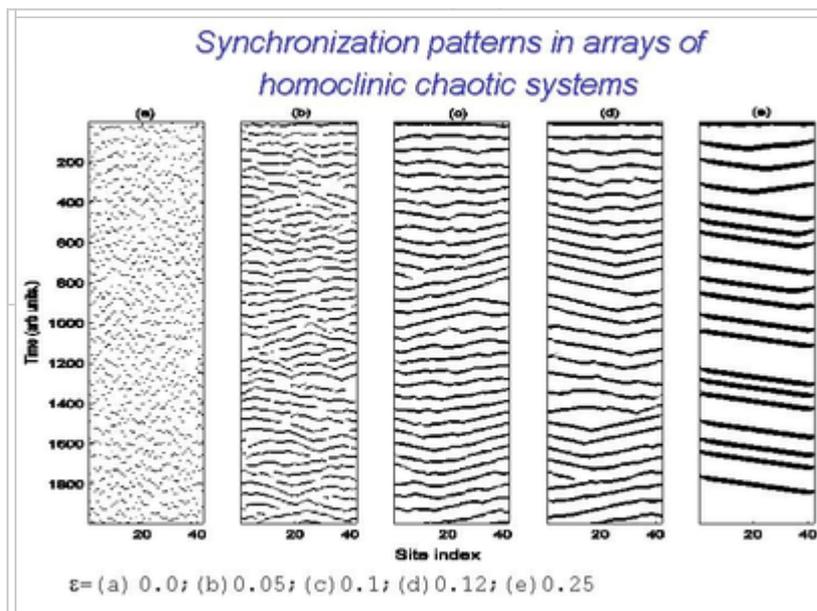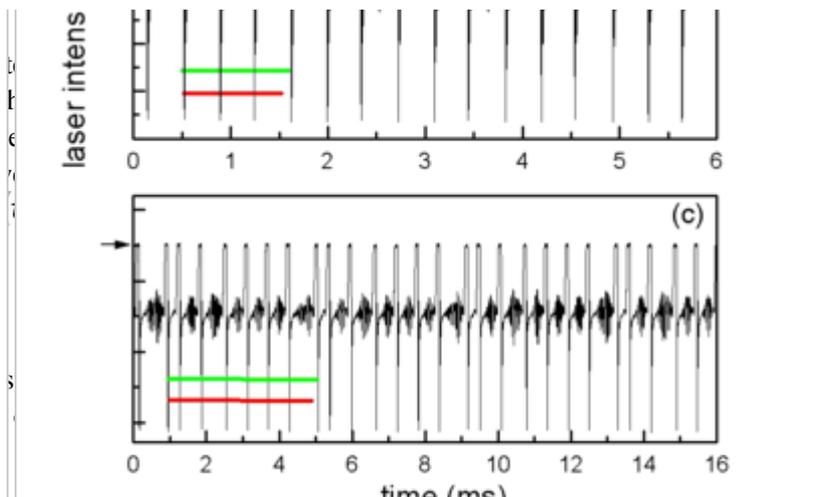synchronization, in fact if we investig
high to low entropy is smooth (Fig.8)



Figure 7: Space–time plots of the spike occurrences on 40 HC systems
bidirectionally coupled; the coupling strength increases from left to right,
A.312 (http://www.inoa.it/home/arecchi/Papers.php).

An array of coupled HC systems
displays a *semantic* property, which
suggests a possible modelling of
coupled neurons in the brain.
Precisely if we set the coupling
below the critical value (high
entropy regime) and apply a short
signal of amplitude A to the first
system of the chain, for a suitable
A, a transient state of collective
synchronization arises (Fig. 9). The
limited time duration of the low
entropy state is crucial for a
cognitive agent: indeed a minimal
time duration is necessary to take
decisions based on the input; on the
other hand, the collective state must
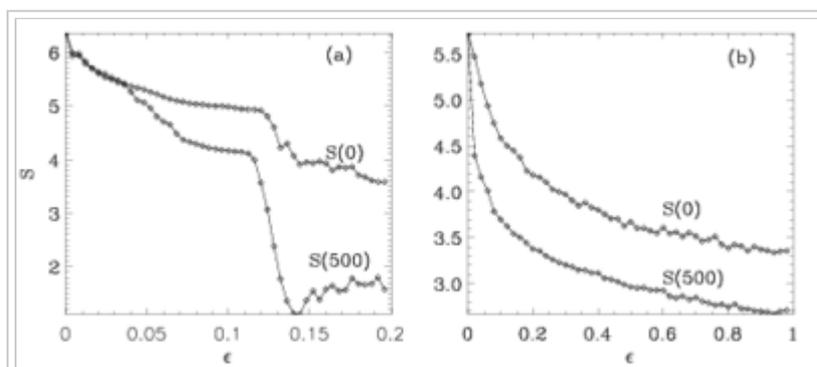fade out and leave room for other tasks.



Figure 8: Degree of synchronization in terms of entropies of nearest site
spike separation $t_r$, $S(t_r) = -\sum p(t_r) \log p(t_r)$, where $p(t_r)$ is
the distribution of $t_r$. a)array of 100 coupled HC systems; b)array of 40
coupled Roessler systems, A.356
(http://www.inoa.it/home/arecchi/Papers.php).

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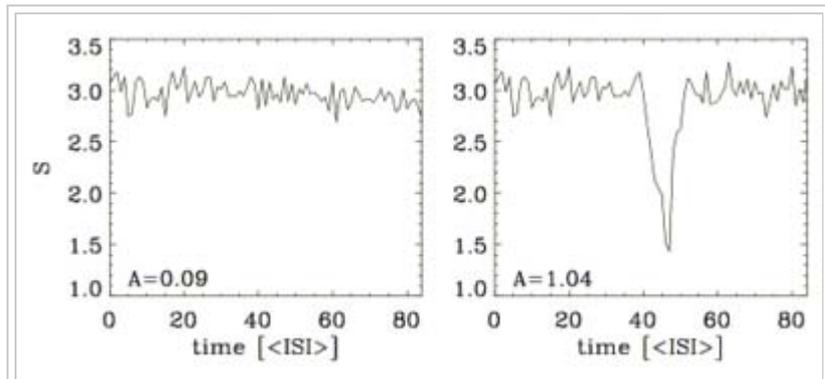


Figure 9: Mutual coupling $\epsilon = 0.104$ (less than critical coupling for spontaneous synchronization); entropy vs time for two inputs of short duration (around one <ISI>),and increasing amplitude. Left: input stimulus amplitude A insufficient to trigger an entropy change. On the right, response to a stimulus amplitude A=1.04, it displays two features namely: i) collective excitability to a short trigger; and ii) excitation lasting a finite time slot, around 5 <ISI>. Feature ii) represents a collective synchronization state lasting for a time depending on A (bottom-up input) and $\epsilon$ (top-down adjustment of the mutual coupling), A.356 (http://www.inoa.it/home/arecchi/Papers.php).

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## See also

Chaos, Laser, Synchronization

Categories: Dynamical Systems | Chaos