Control of transient synchronization with external stimuli

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A network of coupled chaotic oscillators can switch spontaneously to a state of collective synchronization at some critical coupling strength. We show that for a locally coupled network of units with coexisting quiescence and chaotic spiking states, set slightly below the critical coupling value, the collective excitable or bistable states of synchronization arise in response to a stimulus applied to a single node. We provide an explanation of this behavior and show that it is due to a combination of the dynamical properties of a single node and the coupling topology. By the use of entropy as a collective indicator, we present a new method for controlling the transient synchronization. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3080195]

Current views see the brain as a self-organizing dynamical system operating close to instability points which allow flexible switching between different states.[1] It has been experimentally demonstrated that an external stimulus can induce oscillatory and synchronous neuronal activity with the predominant frequency in the 40 Hz range (gamma wave activity). In sensory systems, phase synchronization of oscillation links neurons functionally together to groups that respond to identical stimulus features.[2] Through synchronization, the neurons which perceive the stimulus characteristics amplify their activity and contribute to a temporal increase in a local field potential (firing rates)[3] which gives rise to the attention and conscious perception processes. The principal properties of such a collective oscillatory activity is the simultaneity and very short onset latency.[4] Exploring a wider frequency band reveals an intrinsic dynamics of long lasting activity patterns of spatial extent, initiated by a sensory event. Essential results of numerous studies show that brain activity undergoes a sudden transition from one stable state to another at some critical stimulus parameter. This phenomenon suggests that the universal property of complex physical systems operating in a metastable dynamics can be attributed to the brain.[5] The global neuronal workspace hypothesis[6] considers a phase transition in the metastable dynamics of the brain. This implies that only one single global representation can be sustained at each time. Under this hypothesis the occurrence of conscious states would be a sudden transition in brain space activity which has been linked to bifurcations and phase transitions.[7]

I. INTRODUCTION

Synchronization processes were intensively studied by Wiener[8] who first argued that frequency adjustment was a universal mechanism for self-organization operating everywhere in nature. Later studies revealed that many processes—specially the self-organized ones—work, thanks to the mutual cooperation of many constituents. In biology, at the biochemical level, such cooperation is a way to accomplish the sophisticated tasks in living organisms. Synchronization is of great importance not only in biology[9] but also in physics,[1] chemistry,[10] neuroscience,[11] and medicine.[12,13] The problem of controlled synchronization processes has attracted great attention recently due to the new discoveries of a possible role of synchronization in conscious perception processes.[4]

Numerous physical processes are caused by pulsatile stimuli and are transient,[14] e.g., transient short-term brain responses evoked by sensory stimuli play a key role in the study of cerebral information processing. In particular, various experiments have confirmed that transient synchronization is used by neural systems for encoding the olfactory stimuli[15] and for spatiotemporal integration in visual system.[16]

In view of the experimental observations, in this paper we present possible mechanism for the control of stimulus-dependent transient synchronization in an array of oscillators in the presence of internal noise (chaos). The synchronization, as well as the global complex dynamics in neural networks, has been widely studied, e.g., see Ref. 17 for review. However, there are no reports concerning the transient synchronization mechanism and its dependence on sensory stimuli. In view of the actual debate concerning the brain functionality, consideration of how sensory stimuli induce synchronization is of crucial importance.

Our model is made of interacting nodes, each one exhibiting chaotic spiking dynamics. This dynamics can be produced by single neurons or by columns of neurons, each of them being described by a simple equation like that of FitzHugh–Nagumo (FHN). The use of a column as a node is due to a recent results on chaotic spiking dynamics which can arise through an interaction of simple FHN units.[18] Moreover, it has been proposed in Ref. 19 that the network of coupled FHN neurons can lead to an appearance of heteroclinic connections between saddle regions (fixed points or limit cycles), giving a network the possibility for efficient encoding of inputs.

Our model provides a novel dynamical mechanism of transient synchronization. Application of an external stimulus switches the system to a metastable synchronization state which lasts for a few interspike intervals (ISIs). Such a transient synchronization looks as a collective excitability inter-

val in an order parameter. As order parameter we introduce entropy, which describes the degree of order between interacting dynamical systems. If the spike timings are correlated, then the entropy reaches a low value, otherwise it remains large. We show that, depending on the values of the coupling strength between neurons and on the amplitude of the external stimulus, the overall system exhibits entropy oscillations as well as bistable behavior. We demonstrate the existence of an excitability threshold in the system since at a fixed coupling the transient synchronization appears at a critical value of the perturbation amplitude. After the transient synchronization interval during which the system is insensitive to another stimulus (refractory period), the dynamics switches again to a disordered state. We explain this phenomenon in terms of the internal dynamical properties of the coupled nodes. An external stimulus applied at one site can change the dynamics of that node; that is, it can either be in chaotic autonomous spiking regime or in a steady fixed point state. As the input node is "switched off" into the steady state for a short time or permanently, the global collective properties of the array change.

II. MODEL

The model consists of N mutually coupled units $\mathbf{x}_i$,

$$\dot{\mathbf{x}}_i = \mathcal{F}(\mathbf{x}_i) + \mathcal{P}(t) + \epsilon(\mathbf{x}_{i-1} + \mathbf{x}_{i+1} - 2\mathbf{x}_i). \tag{1}$$

The coupling consists of mutual excitatory and inhibitory interactions. Since the neural system is an open system (with many inputs), we consider no-flux boundary conditions; thus the last term in the equation reduces to $\epsilon(\mathbf{x}_{i-1} - \mathbf{x}_i)$ and $\epsilon(\mathbf{x}_{i+1} - \mathbf{x}_i)$ at $i = N$ and $i = 1$, respectively. $\mathcal{P}(t)$ is an external stimulus, being nonzero only at one site. The nodes $\mathbf{x}_i$ are bistable, that is, either in chaotic spiking or a fixed point. The effect of having bistable nodes in an array is such that in the case of large coupling strengths all sites will finally settle in a fixed point state. Our model however operates far away from such a high value of the coupling strength. We fix the network topology and concentrate on the dependencies of the synchronization on the internal dynamics of the perturbed site.

The nodes $\dot{\mathbf{x}} = \mathcal{F}(\mathbf{x})$ describe the dynamics of single neurons or a column of neurons. In our paper we concentrate on irregular spiking produced by a column of neurons for which global dynamics can be represented by a set of equations exhibiting irregular spiking (see, e.g., Ref. 20),

$$\dot{x}_1^i = k_0 x_1^i (x_2^i - 1 - k_1 \sin^2 x_6^i),$$

$$\dot{x}_2^i = -\gamma_1 x_2^i - 2k_0 x_1^i x_2^i + g x_3^i + x_4^i + p,$$

$$\dot{x}_3^i = -\gamma_1 x_3^i + g x_2^i + x_5^i + p,$$

$$\dot{x}_4^i = -\gamma_2 x_4^i + z x_2^i + g x_5^i + z p, \tag{2}$$

$$\dot{x}_5^i = -\gamma_2 x_5^i + z x_3^i + g x_4^i + z p,$$

$$\dot{x}_6^i = -\beta\{x_6^i - b_0 + r[f(x_1^i) + \epsilon(x_1^{i-1} + x_1^{i+1} - 2\eta^i(t))]\},$$

where $f(x_1^i) = x_1^i / (1 + \alpha x_1^i)$ and $\eta^j(t)$ is a variable obeying the filter equation $\dot{\eta}^j = -d(\eta^j - x_1^i)$ with $d = 10^{-3}$. The index i denotes the ith site position for $i = 1, \ldots, M$. The values of parameters are $k_0 = 28.5714$, $k_1 = 4.5556$, $\gamma_1 = 10.0643$, $\gamma_2 = 1.0643$, $g = 0.05$, $p = 0.016$, $z = 10$, $\beta = 0.4286$, $\alpha = 32.8767$, $r = 160$, and $b_0 = 0.1032$. Chaos in these equations is due to the homoclinic return to a saddle focus, thus it implies a high sensitivity to an external perturbation in the neighborhood of the saddle. There is also a heteroclinic connection between a saddle focus and a fixed point which gives a contribution to a coexistence of two states: chaotic spiking and quiescence (bistability). At each pseudoperiod or ISI, equations yield the alternation of a regular large spike and a small chaotic background. The chaotic background is the sensitive region where the activation from the neighbors occurs, while the spike provides a suitable signal to activate the coupling. Coupling of N HC (homoclinic chaos) equations corresponds to a network containing heteroclinic connections between N saddle focus points. Despite the oscillating regular or irregular regimes, also bistability is a common feature found experimentally in neural networks.[21] We show that introducing the bistable features into a network, i.e., coexistence of a saddle focus with a fixed point in a system, adds a new dynamical characteristics to a network and allows the efficient control of synchronization.

The degree of order of the system is measured by the entropy S of the generation times T_g. The generation time T_g is defined as the shortest time difference between spike occurrence at neighboring sites starting from the first. Then the entropy S is, $S(t) = -\Sigma_{T_g} p(T_g, t) \ln p(T_g, t)$. The probability function $p(T_g, t)$ is a discrete and normalized probability distribution of a continuous variable T_g evaluated at time t and is defined as the ratio of the number of elements with given T_g to the total number of elements. We choose entropy to characterize synchronization because, at variance with other statistical quantities, it allows us to measure also nonisochronous coherence in a precise way.

The synchronization transition for the systems under consideration is sharp and occurs spontaneously at the critical value ϵ_c of the coupling strength.[22] As the coupling strength is set slightly below ϵ_c, a coupled array displays low intersite correlation. An external stimulus of amplitude A in the form of the modulation of some system parameter b_0, e.g., $b_0 \to b_0' = b_0 + A$ for a time Δt, shorter than, or equal to, $\langle \text{ISI} \rangle$ (the average ISI) is applied at the first site $i = 1$. Depending on the stimulus amplitudes, the array responds in various manners. After the stimulus application, it can either not respond at all, remaining in the uncorrelated (high entropy) state, or it can switch to the synchronized state remaining in it permanently. Such a behavior indicates the existence of a bistability. Moreover, for a limited range of the stimulus amplitudes, it can synchronize for a short time and then come back again to the uncorrelated state. This behavior, shown in Fig. 1, is reminiscent of excitable dynamics.

The collective dynamical regimes mentioned above are shown in the upper panel of Fig. 2, where by changing the coupling strength ϵ and with fixed stimuli features, we report the fixed point [Fig. 2(a)], excitable [Fig. 2(b)], oscillatory [Fig. 2(c)], and bistable [Fig. 2(d)] regimes. The use of en-
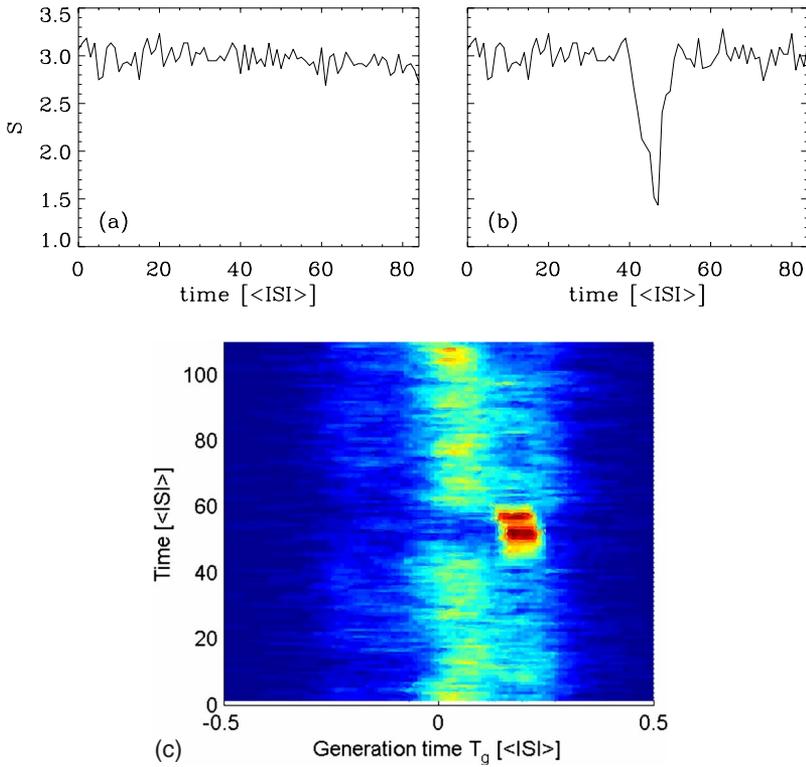
FIG. 1. (Color online) Time evolution of entropy S for an array of $N=30$ coupled HC systems perturbed at the first site at time $t=40$ ISI by a pulse of amplitudes of (a) $A=0.09$ and (b) $A=0.104$. In both cases the pulse duration is $\Delta t=\langle ISI \rangle$ and the coupling strength $\epsilon=0.104$. (c) Histograms of generation times during time evolution. At time $t=40$ the external stimulus is applied which causes the sharpening of the generation times distribution (with a shift to the right of their maxima). This sharpening corresponds to the temporal lowering of entropy.

tropy as the indicator of the collective properties of the array establishes an analogy between the collective response of an array and a dynamical system. Excitable and oscillatory responses follow cycles in a phase space of the system variables. In our case we have only one variable, but we can reconstruct a phase space by an embedding method[23] (lower panel). A standard definition of excitable system says that the form of the system response, beyond a particular excitability threshold, is determined by the system parameters and not by the external signal features. There is however a crucial difference between excitability in dynamical systems and collective excitable behavior of an array. In the latter case, the excitability exists for some range of the stimulus amplitude, whereas the classical excitability refers to the fixed response for all stimuli amplitudes. In Figs. 3(a) and 3(b) we plot the average entropy values in the parameter space of the coupling strength ϵ and the stimulus amplitude A. The average entropy is calculated from the time series far beyond the stimulus was applied. All collective dynamical regimes are seen: excitability, oscillations, and bistability. An interesting phenomenon can be noticed from the figures: For different initial conditions, different configurations of the excitability regions are formed. It is a kind of a memory of the initial state.
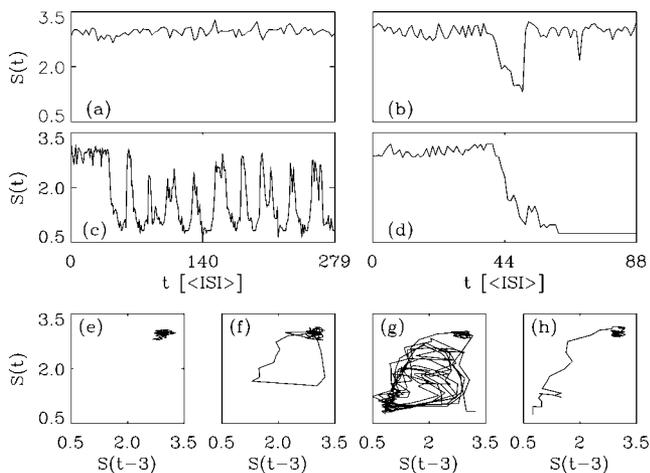

FIG. 2. Upper panel: entropy S vs time for $N=30$ weakly coupled HC units with different coupling strength values: (a) $\epsilon=0.1015$, (b) $\epsilon=0.1045$, (c) $\epsilon=0.105\,795$, and (d) $\epsilon=0.106$, perturbed with a stimulus of amplitude $A=1.04$ and duration $\Delta t=\langle ISI \rangle$ applied at time $t=40$. Lower panels (e)–(h) show phase spaces for the embedded variable S corresponding to figures (a)–(d), respectively.
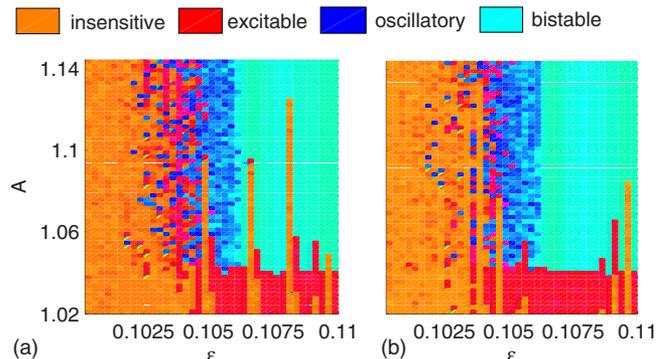

FIG. 3. (Color online) The stimulus of duration $\Delta t=\langle ISI \rangle$ and amplitude A applied to the first of $N=30$ weakly coupled systems induces excitable, bistable, and oscillatory responses. (a) and (b) are produced for two different sets of the initial conditions.
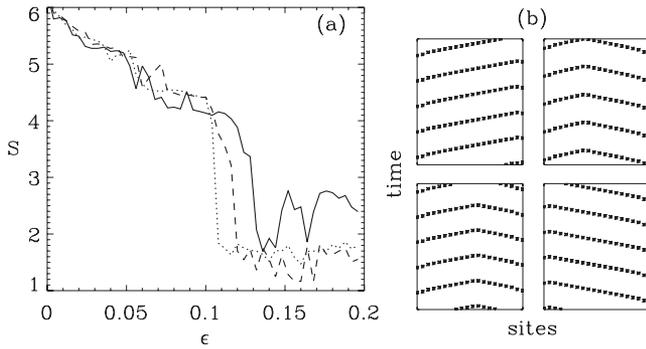
FIG. 4. (a) Spontaneous synchronization transitions in $N=40$ interacting systems for different boundary conditions at site $i=1$ determined by the following coupling forms: $\epsilon(\mathbf{x}_{i+1})$ (solid line), $\epsilon(\mathbf{x}_{i+1}-2\mathbf{x}_i)$ (dashed line), and $\epsilon(\mathbf{x}_{i+1}-3\mathbf{x}_i)$ (dotted line). (b) Raster plots for $N=20$ coupled HC systems with $\epsilon=0.136$ far beyond the initial transient. Various final synchronization states arise from different sets of initial conditions.

III. DYNAMICAL MECHANISM OF COLLECTIVE BEHAVIORS

The mechanism underlying collective excitability and bi-stability is related to the internal characteristics of the single nodes. As the stimulus is applied to one site, for a particular value of the stimulus amplitude, the node collapses to a fixed point. Before the stimulus is applied, the coupling at $i=1$ is $\epsilon(\mathbf{x}_{i+1}-\mathbf{x}_i)$ for the first site, whereas for the second site it is $\epsilon(\mathbf{x}_{i+1}+\mathbf{x}_{i-1}-2\mathbf{x}_i)$. Now, if the dynamics of the first site is switched off, then the new coupling for the second site is $\epsilon(\mathbf{x}_{i+1}-2\mathbf{x}_i)$, that is different from the previous one.

In order to elaborate the above observations, let us look closer on the synchronization features in the case of various types of boundary conditions. For the nearest neighbor and no-flux boundary conditions, we notice that as the nodes at the boundaries are more strongly coupled to the array, the spontaneous transition to synchronization occurs for smaller coupling strengths (decreasing of ϵ_c). In Fig. 4(a) the synchronization transition for three different cases are shown. Entropy is calculated from the distribution of the generation times T_g for the following coupling configurations at site $i=1$: $\epsilon(\mathbf{x}_{i+1})$, $\epsilon(\mathbf{x}_{i+1}-2\mathbf{x}_i)$, and $\epsilon(\mathbf{x}_{i+1}-3\mathbf{x}_i)$. Thus, besides the single node dynamics, also the boundary conditions are responsible for setting the collective synchronization. The typical patterns of spontaneous synchronization which arise above ϵ_c are shown in Fig. 4(b). Synchronization patterns are formed by various clusters of oscillators enslaved in a unison motion which are determined by the initial conditions and by the leading sites at boundaries. The number of such states increases linearly with the number of sites N in an array. The synchronization states can exist because above ϵ_c, nodes in the array enter a regular oscillatory regime. As was demonstrated in Ref. 22, the value of ϵ_c remains the same for any system size N. However, as shown in Fig. 4(a), the synchronization transition occurs for a smaller value of the coupling strength as the ending sites are tied more strongly to the array. Thus, if the temporal change in boundary conditions is such that the array synchronizes for $\epsilon_c' < \epsilon_c$, then the excitability and bistability can be observed. When the spiking stops at the first site, the array's boundary conditions change and the value of ϵ_c for the synchronization decreases until
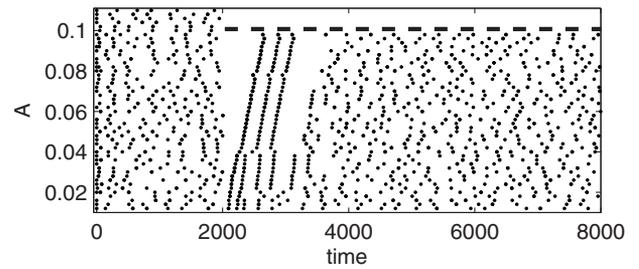

FIG. 5. Response characteristics for a single HC system to external stimuli of various amplitudes A and fixed duration $\Delta t = \langle \text{ISI} \rangle$. Dots represent spiking events. External stimulus is applied at time $t=2000$. The dashed line marks a threshold for A at which the system is pushed to a steady fixed point state (bistability).

the first site recovers its chaotic dynamics again. The characterization of the responses of the single node to external stimuli is presented in Fig. 5. A single system equation is integrated in time starting from different initial conditions. At time $t=2000$, an external stimulus is applied, causing the delayed reappearance of spiking. As the amplitude of the stimulus increases, the delay time becomes longer and finally becomes infinite (spiking does not reappear). The critical value of A at which the spiking is suppressed marks the bistability regime for an array of coupled systems. On the other side, values of A, which cause the delayed spiking (temporal switching off of the system dynamics), allow excitable regimes in an array. Finally, when no delay in spiking is present, the array is insensitive to an external stimulus.

IV. CONCLUSIONS

In conclusion, we have shown that below the critical point of the coupling strength at which synchronization arises spontaneously, a coupled array responds collectively to a stimulus. The collective behavior arises even though the stimulus is applied at a single site. We have shown that in the response to a single perturbation, an array exhibits bistable-, oscillatory- and excitablelike behavior. That is, the array switches from a disordered state into the perfectly synchronized one permanently or for a short time. Such a transition appears at a critical value A_c of the perturbation amplitude. In the excitablelike regime, after a transient synchronization, the system goes back to its previous disordered state. As we decrease the amplitude A, the system does not respond at all to the external stimuli, thus there exists an excitability threshold.

We have here introduced the sufficient general conditions for the control of collective behavior which may be summarized as follows: *population of coupled bistable units (coexistence of fixed point and chaotic attractor) satisfying the condition for a sharp transition to synchronization*[22] *with no-flux boundary conditions.* The collective phenomena appear when an external stimulus applied at one site changes the dynamics of that bistable node. As the dynamics of that node is temporally or permanently switched off into the steady state, the global collective properties of the array change. This is due to the boundary effects on the critical value of the coupling strength needed to induce array synchronization. Thus the collective behavior of an array can be

controlled by application of an external stimulus at one site inducing transient or permanent synchronization. The behavior here discussed persists for the systems which have a bistable dynamics with the coexistence of autonomous spiking (not necessary chaotic) and fixed point state.[24,25] Since in animals excitable,[26] as well as bistable,[21] cells are well known to constitute the brain columns and networks, the model presented here may play an important role during the interactions of the neural assemblies in response to sensory stimuli. Moreover the mechanism presented here may be applied in other areas of science to control systems, e.g., temporal or permanent global switching on-off of large networks by a one-point input.

In the absence of definite experimental evidence on the role of collective synchronization as outlined here, in brain processes, we ask the question: Where in the cognitive ladder starting with sensorial detectors and ending in a decision (motor or linguistic)[27] collective synchronization might be crucial? The first step of the ladder, called "encoding" is the buildup of sequences of spikes in coupled neuron arrays that code the input information. Two recent strategies have been discussed at length, namely, "*chaotic itinerancy*" by Tsuda *et al.*[28] and "*stable heteroclinic chains*" by Rabinovich *et al.*[19] The next crucial question is how the brain utilizes the encoded information in selecting specific motor decisions. We draw a tentative boundary between two relevant time scales, a short one within which decisions are almost automatic and based on probabilistic (Bayes) inference[29] and a long one, where there is enough time to introduce top-down perturbations from memorized learning to the bottom-up signals resulting from encoding.[30]

A comparison of the cognitive role of different time windows has been done by Poppel[31] and we refer to those papers for further details. It is sound to hypothesize that decisions are based on the recruitment of large brain areas acting coherently for a sizable time span. In view of this, we are exploring how the collective synchronization outlined here might be a workable strategy to interface encoding to decisions.

ACKNOWLEDGMENTS

[1] H. Haken, *Advanced Synergetics* (Springer, Berlin, 1983).
[2] M. G. Gray and W. Singer, Proc. Natl. Acad. Sci. U.S.A. **86**, 1698 (1989); R. Eckhorn, R. Bauer, W. Jordan, M. Broisch, M. Kruse, M. Munk, and H. J. Reitboeck, Biol. Cybern. **60**, 121 (1988).
[3] F. Varela, J. P. Lachaux, E. Rodriguez, and J. Martineri, Nat. Rev. Neurosci. **2**, 229 (2001).
[4] P. R. Roelfsema, A. K. Engel, P. Koening, and W. Singer, Nature (London) **385**, 157 (1997).
[5] S. Bressler and J. A. S. Kelso, Trends Cogn. Sci. **5**, 26 (2001); W. J. Freeman and M. D. Holmes, Neural Networks **18**, 497 (2005).
[6] J.-P. Changeux and S. Dehaene, Cognition **33**, 63 (1989).
[7] C. Sergent and S. Dehaene, Psychol. Sci. **15**, 720 (2004).
[8] N. Wiener, *Cybernetics*, 2nd ed. (MIT, Cambridge, 1961).
[9] A. T. Winfree, *The Geometry of Biological Time* (Springer, Berlin, 1980).
[10] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence* (Springer, Berlin, 1984).
[11] H. Steriade, E. G. Jones, and R. R. Llinas, *Thalamic Oscillations and Signaling* (Wiley, New York, 1990).
[12] P. A. Tass, *Phase Resetting in Medicine and Biology. Stochastic Modelling and Data Analysis* (Springer, Berlin, 1999); P. A. Tass, Europhys. Lett. **57**, 164 (2002).
[13] K. H. Chiappa, *Evoked Potentials in Clinical Medicine* (Raven, New York, 1983).
[14] P. A. Tass, Europhys. Lett. **59**, 199 (2002).
[15] R. W. Friedrich and G. Laurent, Science **291**, 889 (2001).
[16] J. J. Hopfield and C. D. Brody, Proc. Natl. Acad. Sci. U.S.A. **98**, 1282 (2001).
[17] M. V. Ivanchenko, G. V. Osipov, V. D. Shalfeev, and J. Kurths, Phys. Rev. Lett. **93**, 134101 (2004); D. Battaglia, N. Brunel, and D. Hansel, *ibid.* **99**, 238106 (2007).
[18] T. Yanagita, T. Ichinomiya, and Y. Oyama, Phys. Rev. E **72**, 056218 (2005).
[19] M. Rabinovich, A. Volkovskii, P. Lecanda, R. Huerta, H. D. I. Abarbanel, and G. Laurent, Phys. Rev. Lett. **87**, 068102 (2001).
[20] A. N. Pisarchik, R. Meucci, and F. T. Arecchi, Eur. Phys. J. D **13**, 385 (2001); F. T. Arecchi, Physica A **338**, 218 (2004).
[21] C. Monteiro, D. Lima, and V. Galhardo, Neurosci. Lett. **398**, 258 (2006).
[22] M. Ciszak, A. Montina, and F. T. Arecchi, Phys. Rev. E **78**, 016202 (2008).
[23] N. H. Packard, J. P. Crutchfield, J. D. Farmer, and R. S. Shaw, Phys. Rev. Lett. **45**, 712 (1980).
[24] H. R. Wilson, *Spikes, Decisions, and Actions: The Dynamical Foundations of Neuroscience* (Oxford University Press, New York, 1999).
[25] E. M. Izhikevich, Int. J. Bifurcation Chaos Appl. Sci. Eng. **10**, 1171 (2000).
[26] M. A. Segraves and M. E. Goldberg, J. Neurophysiol. **58**, 1387 (1987).
[27] W. Freeman, *How brains make up their minds* (Columbia University Press, New York, 2000).
[28] I. Tsuda, Chaos **19**, 015113 (2009).
[29] J. M. Beck, W. J. Ma, R. Kiani, T. Hanks, A. K. Churchland, J. Roitman, M. N. Shadlen, P. E. Latham, and A. Pouget, Neuron **60**, 1142 (2008).
[30] S. Grossberg, Behav. Brain Sci. **21**, 473 (1998).
[31] E. Poppel, Trends Cogn. Sci. **1**, 56 (1997); Acta Neurobiol. Exp. (Warsz) **64**, 295 (2004).