MEASUREMENTS OF THE FINE STRUCTURE OF LASER INTENSITY CORRELATIONS NEAR THRESHOLD ☆

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The correlation function of the intensity fluctuations of a single mode laser in the threshold region has been measured with a fast digital correlator. The various exponential components have been separated for the first time, by exploiting the nonlinear characteristics of the correlator. The results are in quantitative agreement with the theoretical predictions.

It is well known that the electromagnetic field emitted by a single-mode laser near threshold can be described as a non-linear Markov process. The most remarkable consequence of non-linearity is that the laser–field correlation functions depart from a single-exponential behaviour and are characterized by a series of relaxation rates.* By using a new technique, we have been able to measure for the first time relaxation rates of order higher than one in the intensity correlation function of a cw He–Ne laser near threshold.

The intensity correlation function $K(\tau)$ of a cw laser is defined by

$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle - \langle I(t)\rangle^2, \qquad (1)$$

where $I(t)$ is the laser intensity at time t. The operational meaning of the average denoted by angular brackets is discussed in ref. [1]. The laser theory predicts that, in the threshold region, $K(\tau)$ may by written as [2]

$$K(\tau) = K(0) \sum_{j=1}^{\infty} M_j \exp(-\Lambda_j \tau), \qquad (2)$$

where $\Sigma_{j=1}^{\infty} M_j = 1$. The weights M_j are functions only of a dimensionless quantity a, called the pump parameter.** The pump parameter is proportional to the difference between unsaturated gain and cavity losses, and is, therefore, negative below and positive above threshold. The relaxation rates Λ_j can be expressed as $\Lambda_j = C\lambda_j$, where the dimensionless quantities λ_j are also functions of a only. The proportionality constant C depends on the particular laser under investigation, and will be discussed later on in this paper.

The numerical results obtained in ref. [2] indicate that, for any value of the pump parameter a, M_j is decreasing with the order j, if the λ_j 's are such that $\lambda_j \leqslant \lambda_{j+1}$. The experimental verification of eq. (2) is

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* A detailed description of the statistical properties of the laser fluctuations can be found in the experimental paper by Arecchi and Degiorgio in ref. [1] and in the theoretical papers by Haken [1], M. Sargent III and M.O. Scully [1].

** See Risken and Vollmer in ref. [2]. The connection with the notation of other authors is discussed by Arecchi and Degiorgio in ref. [1].

difficult because M_j is much smaller than 1 for $j \geq 3$ (that is $M_1 + M_2 \simeq 1$), and M_2 becomes comparable to M_1 only when λ_2 is close to λ_1. This explains why the first accurate measurements of the laser dynamics in the threshold region [3], gave only an effective decay rate

$$\Lambda_{\mathrm{eff}} = \left[\sum_{j=1}^{\infty} (M_j/\Lambda_j)\right]^{-1}$$

without resolving the different exponential contributions. Recent experiments [4], performed around $a = 2$, where the predicted deviations from a single exponential are largest, have given evidence of a non-exponential decay of the intensity correlation function. However, no attempt was made to give a quantitive disentangling of the different relaxation rates.

In our experiment the source is a He–Ne laser (Spectra Physics 119) working on a single longitudinal and transverse mode. The output intensity is stabilized by an active feedback, as already described in previous papers [1]. The operating point is chosen by controlling the position of one of the cavity-mirrors through a piezoelectric transducer. The laser intensity is detected by a photomultiplier tube, whose output current is suitably amplified and sent, through an a.c. coupling, to a fast digital correlator [5].

The correlator samples the input signal periodically, with a sample duration of 50 nsec. The sampling frequency is variable, with a maximum value of 4 MHz. The result of each sample is classified sequentially as a digital number of three bits plus sign. In all our measurements the spacing between two successive clipping levels is kept fixed at 60 mV. The correlation function is measured simultaneously at 1024 different delays going from 0 to 1023 times the sampling period. The correlator works in real time for sampling frequencies less than 1 kHz. For higher frequencies the correlator is less efficient, however its performance is still far better than that of the wave analyser used in ref. [3] and of the delayed coincidence method of ref. [4].

Even with this improved technique, it would be very difficult to obtain accurate values of the relaxation rates λ_j, with $j > 1$, except for λ_2 in a small region around $a = 2$, without the "help" of the distortions introduced by the correlator itself. Indeed a correlator which quantizes an analog signal by a finite number of clipping levels yields a measured correlation function $K'(\tau)$ which is distorted with respect to the correlation function of the input signal [6]. For a fixed number of clipping levels (15 in our case), the amount of the distortion depends on the statistical properties of the input signal and on the ratio r between the spacing of two successive clipping levels and the root mean square input voltage. A full statistical analysis of this problem is given elsewhere [7]. There it is shown that $K'(\tau)$ has the same structure of eq. (2), that is

$$K'(\tau) = K'(0) \sum_{j=1}^{\infty} M_j' \exp(-\Lambda_j \tau), \tag{3}$$

where $\Sigma_{j=1}^{\infty} M_j' = 1$, but M_j' is generally different from M_j. The experimental weights M_j' depend on both the pump parameter a and the instrumental parameter r. For given a and r, they are not always monotonously de creasing with j. For a given a, it is possible to choose r in such a way that, for instance, M_2' becomes larger than M_1' even if M_2 is much smaller than M_1. As a general rule, $K'(\tau)$ is very similar to $K(\tau)$ in the range $r = 0.3$ – 1, while large distortions have to be expected for r very small or much larger than 1.

A dramatic example of how the weights M_j' are affected by the clipping is given in the calculated data of table 1 [‡]. A typical experimental plot is given in fig. 1, where we give two experimental correlation functions obtained with the laser in the same working point (slightly below threshold), but with different values of r.

A qualitative understanding of the nonlinear operation of the correlator is attained as follows. The dif-

Table 1
Ratio between the distorted weights M_4'/M_1' of fourth and first decay rate versus r, for $a = 3$.

$r = 0$ (understorted)		1	1.5	2.5
M_4/M_1	4.6×10^{-3}	0.1	0.5	1.5

‡ Since the joint probability density of the laser field is theoretically known, the distorted correlation function $K'(\tau)$ can be computed, once the parameters a and r are given. Risken and Vollmer have kindly performed for us the computation of the weights M_j', up to $j = 4$, for $a = 3$ and for a set of values of r. A sample is given in table 1. Experimentally, we have found that the amount of the distortion is generally lower than that predicted by the theory. The reason is that uncorrelated noise (such as shot noise) in the apparatus is equivalent to a randomization of the clipping level position. A full account is given in ref. [7].

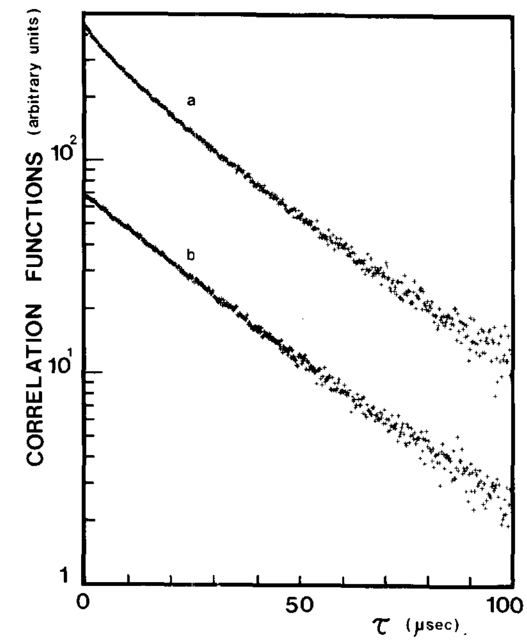

Fig. 1. Experimental results of the intensity correlation measurements for $\langle I\rangle/\langle I\rangle_{th}$ = 0.9 ($a \simeq -0.4$). Curve a) corresponds to r = 2.5, and curve b) to r = 0.8.

ference $\Delta K(\tau) = K'(\tau) - K(\tau)$ between the measured and the actual second-order correlation function can be suitably expressed in terms of two-time correlation functions of higher order. Recent calculations [8] and related measurements [9] show that the third- and higher-order correlations are more sensitive to the faster decay rates of the laser than the second-order correlation. Therefore, the contribution of the faster decay rates is expected to be larger in $K'(\tau)$ than in $K(\tau)$.

The values of the laser time constants have been obtained by measuring the intensity correlation function of the laser beam, at a fixed value of the pump parameter a, for two (in some cases three) different values of the instrumental parameter r. Variations of r have been performed by controlling the gain of the amplifier following the photodetector.

It is worthwhile to point out that, besides the present application to the laser dynamics near threshold, the method can be of interest for the detailed study of any optical field generated by non-linear interactions. The basis of the method is that the distortion due to the correlator does not introduce new relaxation rates in the correlation function, besides those already present in the joint probability density of the field [7].

The experimental problems and the data interpretation procedure can be better understood with reference to a specific example. Let us consider the measurements reported in fig. 1, which correspond to a pump parameter $a = -0.4$. The theoretical values of the weights are $M_1 = 0.95$, $M_2 = 0.05$, M_3 negligible. In the first measurements ($r \simeq 0.8$) the value of r is so chosen as to give little distortion, and hence $K'(\tau)$ contains practically only the first exponential. The relaxation rate is readily determined by a best-fitting procedure. The value of r for the second measurement is made high enough to have the second weight M_2', easily detectable. The relaxation rate Λ_2 is evaluated by choosing $r \simeq 2.5$.

It should be taken into account that there is an upper limit for r dictated by the following practical considerations: (i) By increasing r the number of non-negligible exponential terms in $K'(\tau)$ increases, thus making the best-fitting analysis more difficult. Our data do not allow a meaningful fit with more than five parameters. (ii) The data accumulation process becomes slower and slower by increasing r because only the largest fluctuations give a non-zero contribution to the measured correlation function. This consideration does not apply if the distortion is generated by making r very small compared to 1. Further experimental difficulties may arise however for small values of r [7].

The considerations made above apply to all the measurements below threshold as well. Above threshold the weight M_2 is sufficiently large to allow the determination of Λ_1 and Λ_2 directly from the first run with $r \simeq 1$. The successive runs with larger r gave Λ_3 and Λ_4.

The theoretically predicted values [2] of the dimensionless relaxation rates are plotted in fig. 2 as function of the ratio between the average laser intensity $\langle I\rangle$ at the working point and the average laser intensity $\langle I\rangle_{th}$ at threshold. This ratio is univocally related to the pump parameter a through a known non-linear expression [1]; the values of a are reported in the upper abscissa scale of fig. 2.

The value of $\langle I\rangle_{th}$ has been obtained by an independent measurement of the reduced second factorial moment

$$H_2 = (\langle n(n-1)\rangle/\langle n\rangle^2) - 1$$

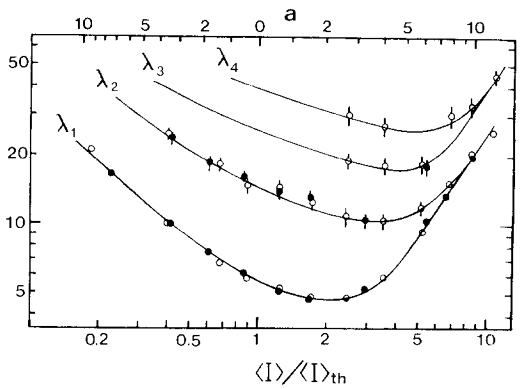

Fig. 2. Plots of the relaxation rates of the laser intensity fluctuations. The full curves represent the theoretically predicted results. The standard deviations for the experimental points are reported only when they exceed the dot size. The two values of the proportionality constant C are 0.97×10^3 (open dots) and 1.03×10^3 (full dots).

of the photocount probability distribution $p(n)$ due to the laser field. At threshold, $H_2 = 0.57$ [1]. The experimentally determined Λ_j's should differ from the theoretical λ_j's by the unknown factor C, which corresponds to a rigid vertical translation in a logarithmic plot.

The experimental points reported in fig. 2 refer to two measurement runs performed in different days. The relaxation rates have been determined by a least-mean-squares fitting procedure. Since the intensity correlation function has a nonlinear dependence on the best-fit parameters, the computation of the standard deviations of the measured relaxation rates is rather involved. Some approximations have therefore been used to determine the error bars reported in fig. 2. The approximations have been checked for a few points by repeating several times the measurements in the same operating conditions.

The two reported values of the constant C have been determined by the vertical translation needed to superpose the experimental plot with the theoretical one. The constant C can be expressed as [2] $C=\sqrt{\beta q}$, where β is the saturation parameter and q is the "diffusion" parameter ‡. The comparison between theory and experiments is straightforward only if a can be varied while keeping C constant. In these measurements, as in the previous experiments of our group [1], the threshold region is scanned by varying the frequency of the laser mode within the Doppler broadened line. When a is changed by this technique, C is also slightly modified. The variation of C is, however, less than 1% in the range $-10 \leqslant a \leqslant 10$ and it can be neglected.

The two values of C are very close to each other and have the same order of magnitude as the values can be derived from the experimental results of refs. [3] and [9]. An accurate comparison is not possible, since β and q depend on many experimental parameters, such as the volume of the laser mode, the filling pressure and the temperature of the gas mixture, the discharge current, and so on

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‡ See footnote ** on p. 329.

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