THRESHOLD EVALUATIONS FOR AN X-RAY LASER

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Photoelectric pumping of inner shells of intermediate Z materials is shown to be efficient for inverting the population of radiative transitions around 1 keV. Non-radiative processes are shown to be essential for achieving a stationary inversion. Reported intense X-ray bursts from plasmas generated by laser radiation suggest the use of a Nd laser as exciting source. Threshold evaluations show that a mirrorless X-ray laser is possible by use of nowadays available Nd lasers.

Several approaches to an X-ray laser have been published or discussed recently [1–7]. Furthermore, amplification schemes without a detailed consideration of the threshold problem have also been proposed [8, 9].

The above schemes may be classified as inner shell or outer shell approaches. In the first approach, pumping is done by resonant excitation of an inner shell, either by photoelectric absorption [1] or by resonant atomic collisions [9]. In the second approach, depletion of the outer shells provides multiply charged ions either isoelectronic to atomic species already known [3] or recombining within a plasma through a chain of radiatively coupled levels [10]. These latter schemes have been worked out only qualitatively, since they rely on plasma processes not completely explored. On the other hand, the inner shell scheme photoelectrically pumped seems to be bottlenecked by the fast decay rate of the upper laser level. To clarify this point we refer to a KL_{III} ($1s_{1/2}$–$2p_{3/2}$) transition. A K electron is stripped leading to a K hole excitation which decays to a lower energy level, e.g. L_{III}. It is well known that the total decay rate of the K level is in general larger than the decay rate of any other level, hence population inversion could be achieved only on a transient basis. This "self-terminating" laser would work for a time of the order of the upper level lifetime, which in the 1 keV region is of the order of 10^{-15} sec [1, 5]. A resonant atomic collision scheme associated with a travelling wave excitation [9] seems to avoid the above bottle neck. However energy density considerations rule out the feasibility of a 1 keV laser with presently available ion accelerators.

In this letter we show how the presence of a non-radiative (Auger or Coster–Kronig) part in the decay of the upper level makes it possible to achieve stationary population inversion on inner shell (e.g. KL_{III}, or $L_{II}M_{IV}$) transitions. Furthermore, we show that by pumping with a visible laser within the reach of present technology, it is possible to achieve sufficient population inversion to overcome threshold in a mirrorless laser around 1 keV.

Let us consider a $KL_{III}(K\alpha_1)$ transition, and call γ_2^t and γ_1^t the total decay rates of the K and L_{III} level, respectively. $\gamma_2^t = \gamma_2^r + \gamma_2^{nr}$ is made of a radiative (γ_2^r) part and a non-radiative (γ_2^{nr}) part due to Auger processes. Most of the γ_2^r channel couples with the L_{III} level, indeed such radiative transition rate is [11, 12],

$$\gamma_{21}^r \sim 0.66\gamma_2^r .$$

Even though $\gamma_2^t > \gamma_1^t$, it is possible to achieve stationary inversion, using the fact that an Auger process leads to a doubly ionized atom where the $K\alpha_1$ line is replaced by a satellite line out of resonance [11]. As a result, the doubly ionized atom does not contribute to re-absorption, and the stationary population ratio between K and L_{III} levels is given by

$$N_2/N_1 = \gamma_1^t/\gamma_{21}^r \, . \tag{1}$$

This ratio may be larger than 1 even though $\gamma_2^t > \gamma_1^t$.

Before showing quantitative details, we give some scaling considerations. For $Z \gtrsim 20$, γ_1^t changes weakly with Z, while γ_{21}^r scales like the frequency squared and hence (for hydrogenic atoms) increases as Z^4. There will be a maximum, $Z_{\max}$, beyond which the inequality $\gamma_{21}^r < \gamma_1^t$ no longer holds. Above $Z_{\max}$ no population inversion is possible. On the other hand, for $Z \lesssim 15$, γ_1^t decreases steeply because of the small total number of electrons leaving no available channels for non-radiative processes. Hence there is also a $Z_{\min}$ which limits the range of available Z's.

Eq. (1) should be refined by two further considerations. First, the gain formula implies the state populations N_i/g_i (g_i being the degeneracy of level i) hence we must impose $N_2/g_2 > N_1/g_1$, or

$$N_2/N_1 = \gamma_1^t/\gamma_{21}^r > g_2/g_1 . \tag{2}$$

Secondly, part of γ_2^{nr} can contribute to fill up the lower level. Indeed, a small fraction of the Auger decays can contribute a second hole on a shell far away from the L shell, giving a satellite line still within the homogeneous width of the Kα_1 line. This problem has not received direct attention in the literature. However experimental data on chemical shifts [13] at least for light elements as C, N, O, show that the screening factor of the 1s level is sensitive to charge perturbations in all outer shells. Henceforth we consider eq. (2) as enough consolidated to allow inversion calculations.

Once an inversion density $N = N_2 - (g_2/g_1) \times N_1 (\mathrm{cm}^{-3})$ has been achieved by photoelectric pumping, the active material has a gain per unit length σN, where the radiative cross section σ is given by

$$\sigma = \frac{\lambda^2}{8\pi} \frac{\gamma_{21}^r}{\gamma_2^t + \gamma_1^t} \, . \tag{3}$$

In order to overcome absorption losses at the wavelength of the lasing transition, provided N is a small fraction of the total density N_T of the material, calling σ_a the absorption cross section, it is necessary that $\sigma N > \sigma_a N_T$, that is, the net gain per unit length is given by

$$\beta = \sigma N - \sigma_a N_T . \tag{4}$$

Proposed X-ray resonators [14–16] allow only a few passes in the cavity, and so far do not contribute to a reduction of threshold requirements. This preliminary study refers to a mirrorless system, for which the standard threshold formula for a cavity laser can not be used.

We can define as threshold length l_t for an amplifying medium that length at which the number of amplified photons within an angle Ω is larger than the number of available electromagnetic modes within the linewidth, so that an atom at a distance l_t be induced to emit a stimulated rather than a spontaneous photon. The angle Ω is defined by the geometry of the material. It is convenient to have a rod-like geometry with a cross size d such that the Fresnel number $N_F = d^2/l_t\lambda$ of the rod be no smaller than 1, to avoid diffraction losses. It may be easily seen that the above threshold criterion gives

$$\exp(\beta l_t) > 4\pi/\Omega . \tag{5}$$

Combining eq. (5) with the above equations, it is easy to evaluate the hole density N_2^* necessary at threshold and hence the power per unit volume $\gamma_2^t N_2^* \hbar\omega_P$ to be absorbed by the K shell in order to have laser action. Here $\hbar\omega_P$ is the energy of the K-edge. In practical terms, we may think of a side irradiation at frequencies above ω_P, with a penetration depth $1/\sigma_T N_T$ (σ_T = total absorption cross section at the K edge) equal to the cross size d of the rod. By the above considerations, the power flux $\phi(\mathrm{W/cm^2})$ at or above the ω_P frequency necessary to pump at threshold is given by

$$\phi = \gamma_2^t N_2^* \hbar\omega_P \frac{1}{\sigma_T N_T} \frac{\sigma_T}{\sigma_P} \, . \tag{6}$$

Here, $\sigma_P < \sigma_T$ denotes the absorption cross section of the K level at the frequency ω_P (the difference $\sigma_T - \sigma_P$ being the contribution to photoelectric absorption at the K edge due to L, M, ... shells). Taking into account that the dependence (5) of l_t on Ω is logarithmic and calling $N_{\min} = \sigma_a N_T/\sigma$ the minimum value of inversion for which gain overcomes losses, it follows that ϕ and l_t depend on $X = N^*/N_{\min}$ (N^* = population inversion associated with N_2^*) as $\phi \sim X$,

Table 1
Numerical data of the K L_{III} transitions.

	$\hbar\omega$ (keV)	γ_2^t a) (eV)	γ_1^t b) (eV)	fluorescence yield c)	σ (10^{-20} cm^2)	σ_a d) (10^{-20} cm^2)	σ_p d) (10^{-20} cm^2)	N_{min} (10^{20} cm^{-3})	β (cm^{-1})	l_t (μm)	ϕ (10^{16} W/cm^2)	P (10^{12}W)
^{16}S	2.31	0.57	0.057	0.082	567.6	1.445	10.66	0.369	209.5	490	2.1	1.02
^{18}A	2.95	0.63	0.166	0.122	448.2	1.231	8.06	0.680	305.2	310	3.6	1.12
^{20}Ca	3.70	0.68	0.194	0.163	390.7	1.076	6.24	0.636	248.5	400	6.6	2.66
^{22}Ti	4.51	0.74	0.239	0.221	332.1	0.895	4.97	1.525	506.5	160	11.3	1.82
^{24}Cr	5.41	0.84	0.330	0.283	279.7	0.761	4.04	2.269	634.6	130	19.5	2.53
^{26}Fe	6.40	1.00	0.418	0.342	238.6	0.650	3.34	2.307	550.5	140	34.8	4.87
^{28}Ni	7.48	1.19	0.586	0.414	200.0	0.605	2.82	2.761	552.3	140	64.2	8.99
^{30}Zn	8.64	1.40	0.806	0.479	164.4	0.507	2.41	2.025	333.0	280	103.6	29.02
^{32}Ge	9.88	1.67	0.954	0.540	142.2	0.439	2.09	1.367	194.4	540	172.8	93.33

a) See ref. [17], b) see ref. [20], c) see ref. [18], d) see ref. [23].

$l_t \sim (X-1)^{-1}$. In a side irradiation of the rod, the total pump power requested is

$$P = \phi d l_t \sim X(X-1)^{-1}.$$

It would seem convenient to increase N_2^* up to N_t in order to reduce P. This however would require a larger brilliance ϕ of the source, with only a mild reduction in P. Once the positive gain condition $X \geqslant 1$ has been reached, it is convenient to work with a moderate X, say $X = 2$, to avoid a strong requirement on the brilliance. At wavelengths $\lambda \simeq 10$ Å and for $d \simeq 10\ \mu$m (right orders of magnitude for the transitions considered in the one keV region) the single-mode condition $N_F \sim 1$ would give a length $l \sim 10$ cm, much longer than the threshold length ($\sim 400\ \mu$m) resulting from eq. (5).

This shows that around the threshold length the laser would be many-mode. A consistent calculation for $N_F \gg 1$ makes use of the geometric relation $\Omega \sim (d/l_t)^2$ in (5), from which a set of values for l_t follows. In table 1 we have collected the relevant data for KL_{III} transitions. The total decay rate of the K level is known experimentally [17] and by using best-fitted values of fluorescent yields [18] it has been possible to obtain values for γ_{21}^I. Experimental values on the decay rate γ_1^t of the L_{III} level are rather scattered [19]. Detailed theoretical calculations are available [20–22]. We have chosen a consistent set of data from the same author [20].

In fig. 1 we plot γ_{21}^I and γ_1^t versus the atomic number Z for the KL_{III} transition. The bar in each γ_1^t point denotes the spread in the available (either experimental [19] or theoretical [20–22]) values. As can be seen, inversion can be obtained between $Z = 16$ and $Z = 47$. By using known cross-section values [23] we have reported in table 1 the values of β for $X = 2$; threshold lengths l_t are then obtained from (5) with

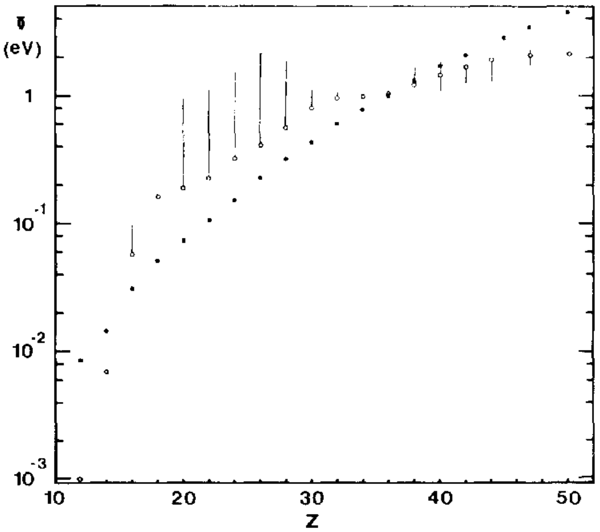

Fig. 1. Values of γ_1^t (circles) and γ_{21}^r (dots) versus atomic number Z. The bar on each γ_1^t point indicates the spread of the values measured or evaluated by different authors. The circles represent the consistent set of data used in the present work.

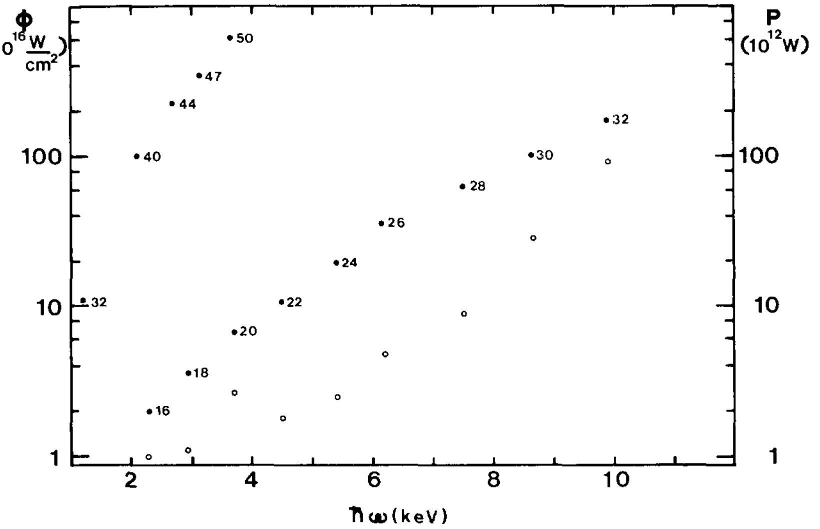

Fig. 2. Threshold values of input power and brilliance versus X-ray laser energy. Dots denote brilliance ϕ and circles power P. The number at each dot denotes the atomic number of the active material. The dots at the upper left of the diagram with Z from 32 to 50, refer to the L_{II}–M_{IV} transition, whereas the lower ones with Z from 16 to 32 refer to the K–L_{III} transition.

$d = 10$ μm; power fluxes and total pump powers are also reported in table 1 and plotted versus the transition frequency in fig. 2.

We have performed similar calculations for $L_{II}M_{IV}$ transitions. The corresponding pump data are reported in fig. 2. For the same transition frequency, the required power densitites are consistently larger. By comparing two lines at about the same frequency (e.g. the $K\alpha_1$ of ^{20}Ca and the $L\beta_1$ of ^{50}Sn) we see a drastic increase in the pump requirements, due mainly to an increase in γ_2^1 (which is also responsible for a decrease in σ) and to an increase in the losses at the lasing frequency.

Even for the most favourable KL_{III} line, the minimum X-ray flux necessary for photoelectric pumping is 2×10^{16} W/cm^2. Such large flux could be produced by converting the radiation of a visible laser to a soft X-ray spectrum on a high Z target [24] with an integrated efficiency which can be as large as 30%, then filtering the low frequency part of the spectrum, which would otherwise contribute spurious excitations of the L shell, and coupling the filtered radiation with the active material. The target would then be a sandwich of converter plus filter plus laser material. The energy conversion efficiencies hold also for fluxes, provided the radiant surface coincides with the surface illuminated by the visible laser, and the radiating time be of the order of the laser pulse duration. Referring to laser pulses on massive ($Z \geqslant 20$) targets, as in Mallozzi experiments [24], the recombinations of the various ionized species occur on the 100-psec time scale. Furthermore the expansion times for massive ions are so long [25] that the radiating plasma surface is not drastically different from the focal spot of the 100 psec laser. We may take therefore a 30% conversion efficiency from visible to X-ray flux over the whole spectrum. Considering furthermore a 30% efficiency in filtering and 40% in optically coupling the X radiation to the active material, we may assume that a neodymium laser with a power density of 5×10^{17} W/cm^2 would be necessary to lase on the $K\alpha_1$ line of ^{16}S. Since the cross area hit by the pump flux is (see table 1) $dl_t \simeq 10 \times 490$ (μm)^2 which implies a laser power of 2.5×10^{13} W. Lasers with such a power may be available in a short time in laser-fusion laboratories[‡]

For a feasibility experiment it is sufficient to produce two small plasma spots, of 10×10 (μm)^2 each, which provide incoherent X-ray radiators, the first exciting a "source" area, and the second an amplifying

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[‡] As, for example, the laser under construction at the Lawrence Livermore Laboratories (10 kJ, 100 ps).

area whose gain would give an experimental check of the above evaluations. Indeed from eq. (4) and the data of table 1, the gain over a 10-μm length is around 50%, beyond the sensitivity limits of an experiment as that of Jaegle's group [26]. This feasibility experiment implies a 25-fold reduction of the power requirement, which makes the experiment possible with commercially available lasers.

If a traveling wave light pulse could be focused successively on $10 \times 10\ (\mu\text{m})^2$ spots synchronized with the stimulated X-ray radiation, an X-ray laser could be operated at the low power levels required for the feasibility experiment. However, besides the difficulty of an ultra-fast light deflection system, a fast pumping process (on the 1-psec scale) would be necessary, and very little is known on how the efficient X-ray emission processes (which are mainly bound-bound transitions [24]) scale down from the 100 psec to the 1 psec time scale.

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