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Spiking control by means of "phase feedback" in FitzHugh–Nagumo circuits

R. Meucci [a,*], S. Euzzor [a], A. Geltrude [a], K. Al-Naimee [a,b], S. De Nicola [a,c], F.T. Arecchi [a,d]

[a] Istituto Nazionale di Ottica, CNR, Largo E. Fermi 6, 50125 Firenze, Italy
[b] Dept. of Physics, College of Science, University of Baghdad, Baghdad, Iraq
[c] INFN, Sezione di Napoli, Napoli, Italy
[d] Dipartimento di Fisica, Università di Firenze, 50019 Sesto Fiorentino, Italy

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ABSTRACT

We investigate the feedback control of a periodically driven FitzHugh–Nagumo circuit (FHN), which displays both spiking and non-spiking behaviors in chaotic or periodic regimes. The simple high pass filter (washout filter) is compared with an all pass filter which only affects the phase characteristics of the input signal. Experimental measurements performed on the electronic implementation of the FHN are in good agreement with numerical simulations. Apart a small difference in their amplitude corrections that remain in both cases within a few percent with respect to the unperturbed spiking signal, we prove the key role of the phase characteristics of the two filters in achieving control.

1. Introduction

The well-known FitzHugh–Nagumo model (FHN) [1], has been the subject of intensive studies in engineering science [2] due to its relative simplicity for analytical study, numerical simulations, and electronic realization. In spite of its simplicity, many mechanisms responsible for generating complex patterns in the neural information processes [3] are contained in it. Moreover it has been used to investigate the effects of noise. Stochastic resonance effects, that optimize information transmission have been investigated in Ref. [4] and noise enhanced stability phenomena due to the correlation time of the noise [5] have been investigated. A periodically driven FHN can easily provide access to chaotic spiking regimes characterized by random inter-spike intervals. Chaotic spiking has been also theoretically investigated in Hodgkin–Huxley model and FHN introducing a third slow refractory variable [6]. In these chaotic cases it is crucial to investigate how such regimes can be controlled.

Recently [7] it was shown that harmonic perturbations can be used to control different aspects of the dynamics of the driven FHN, that is, to tame or enhance the spiking activity as well to control chaos [8]. This control technique (Phase Control of Chaos) is based on the phase difference φ between the periodic driving and a small harmonic perturbation added to it. The phase control scheme relies on an appropriate selection of φ once the perturbation frequency (usually corresponding to a subharmonic of the driving) and its amplitude are selected, in order to lead the system to the desired dynamical regime. This limitation, which is typical of the non-feedback methods to control chaos, can be overcome by suitable filters inserted in a feedback loop. The dynamic state feedback control incorporating washout filters (High Pass Filter (HPF) and notch filter) had been proposed in Refs. [9–11]. Implementations of this control method were realized in CO_2 [12,13] and Nd-YAG [14] lasers and in a chaotic electronic circuit [15].

In this Letter we present a comparison between two linear filters, that is, an HPF and an All Pass Filter (APF). The latter one only affects the phase of the frequency components leaving unchanged the amplitude components. The effectiveness of the APF with respect to the HPF, due to the double phase shift introduced, is here experimentally demonstrated. This fact highlights the role of phase in controlling spiking behavior by means of filtered feedback signals. Such a strategy is different from Phase Control of Chaos which relays on narrow phase resonances [7,8].

The Letter is organized as follow: in Section 2 we introduce the FHN and its electronic implementation together with the characteristics of the two filters. Section 3 presents experimental results showing that the feedback control can be used both to tame or enhance the spiking regimes. The better performances of the APF with respect to the HPF are numerically and experimentally demonstrated. Conclusions are discussed in Section 4.

2. Model and its implementation

The driven FHN is ruled by the following equations:

$$\begin{aligned} \dot{x} &= x - y - \frac{x^3}{3} - V_d \\ \dot{y} &= 0.08(x - 0.8y + 0.7) \end{aligned} \tag{1}$$

where $x(t)$ is the voltage variable, $y(t)$ is the recovery variable and $V_d = A\sin(2\pi\nu t)$ is an external driving term with amplitude A and frequency ν.

* Corresponding author. Tel.: +39 05523298.
E-mail address: riccardo.meucci@ino.it (R. Meucci).

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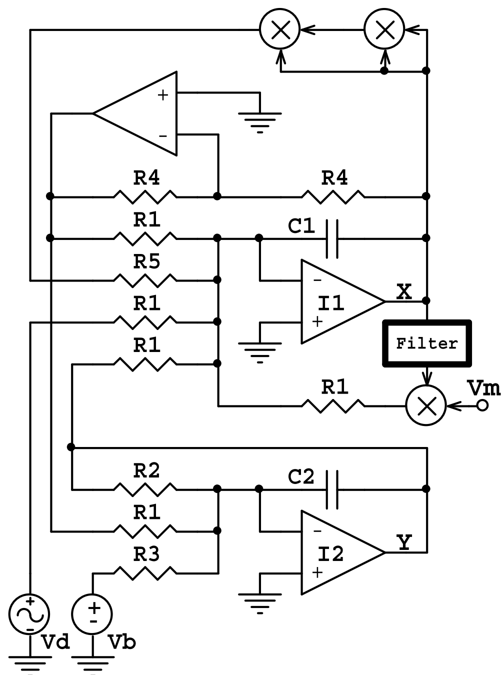

Fig. 1. Electronic scheme of the FHN oscillator with the feedback loop. I1 and I2 are operational amplifiers, R resistors and C capacitors. V_d is the sinusoidal driving signal, V_b is the bias. V_x and V_y are the voltage signals representing the x and y variables respectively. The feedback loop consists in a passive filter (HPF or APF) whose output $F_{1,2}$ modulated by the signal V_m and attenuated by R_1. $R_1 = 100$ K, $R_2 = 125$ K, $R_3 = 143$ K, 1 K, 48 K, $C_1 = 3n$, $C_2 = 37.5n$.

The circuit implementing the FHN model (1) is shown in Fig. 1, without considering the feedback filter. It consists of an electronic analog simulator implemented by commercial semiconductor devices. Increasing A the system, after a few period doubling bifurcations, reaches a small amplitude chaotic regime and for $A = 570$ mVpp a spiking regime appears, as shown in the bifurcation diagram (Fig. 2). During the spiking regime trains of small amplitude irregular oscillations are randomly interrupted by large spikes (Fig. 3). Such a behavior is also known as Mixed-Mode Oscillations (MMOs). These oscillations were first discovered in the Belousov–Zhabotinsky reaction [16] and, since then, have been frequently observed in experiments and models of chemical and biological rhythms [17].

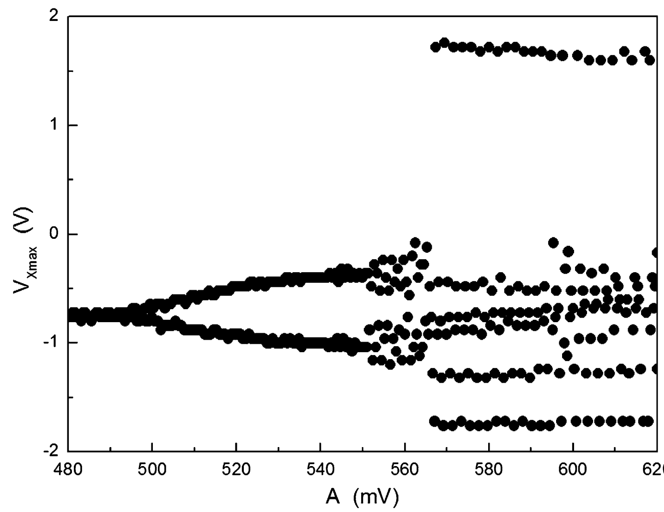

Fig. 2. Stroboscopic bifurcation diagram of the driven FHN circuit.

Fig. 3(b) shows the related power spectrum where it is possible to observe the relevant contribution of the low frequencies (below $f/4$) that are responsible for the spiking behavior. Since our pur-

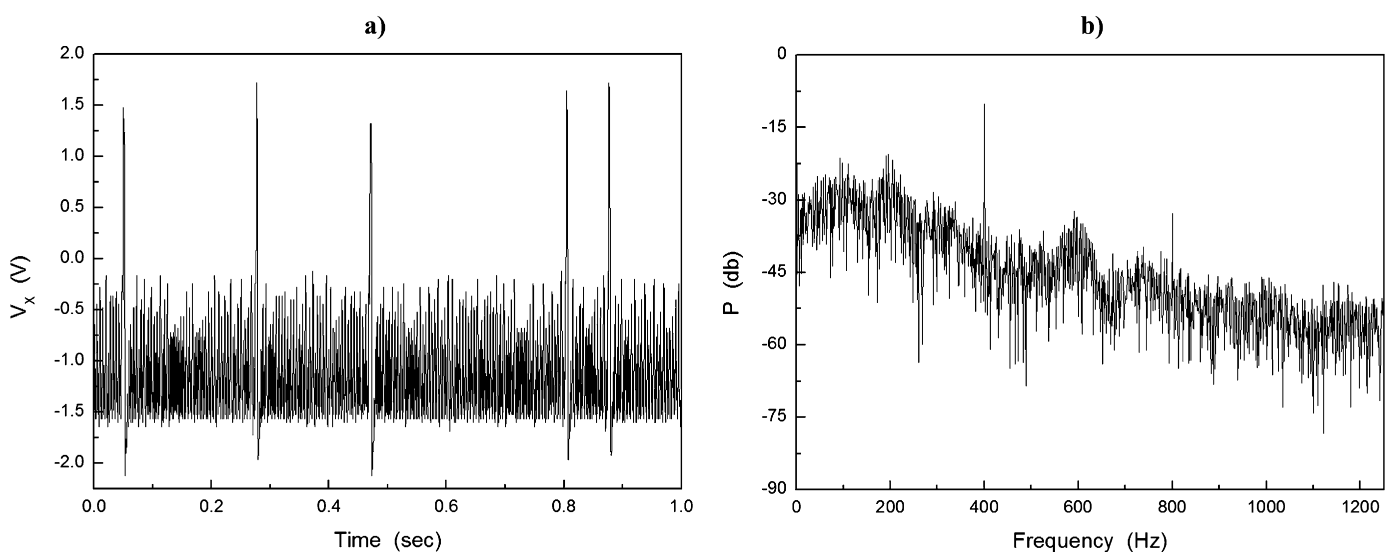

Fig. 3. (a) Voltage amplitude time series of the x variable in open loop conditions. (b) Power spectrum of the signal in (a).

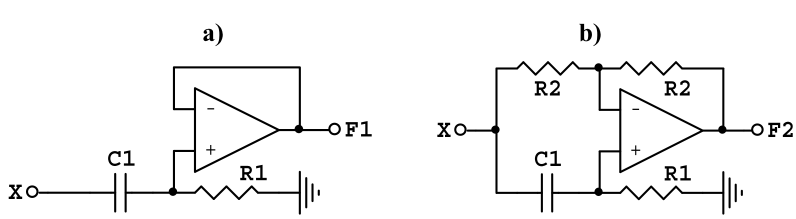

Fig. 4. Electronic schemes of (a) high pass filter and (b) all pass filter.

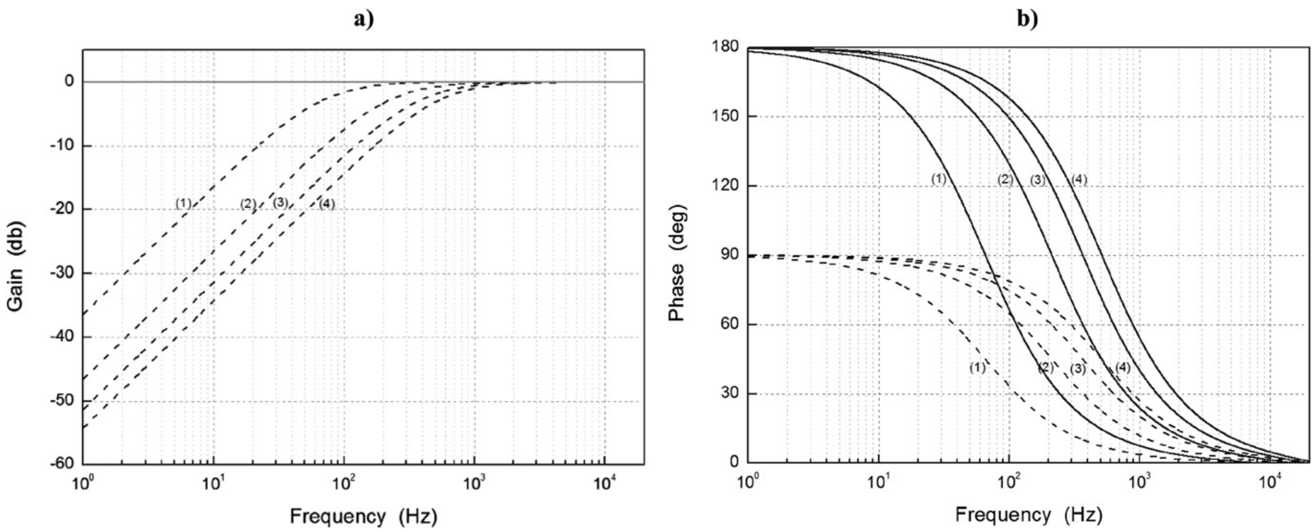

Fig. 5. (a) Gain plot and (b) phase plot for the two filters, HPF (dashed lines) and APF (solid lines), for different cut-off frequency values: 1) 65.8 Hz; 2) 212.6 Hz; 3) 367.2 Hz; 4) 514.0 Hz.

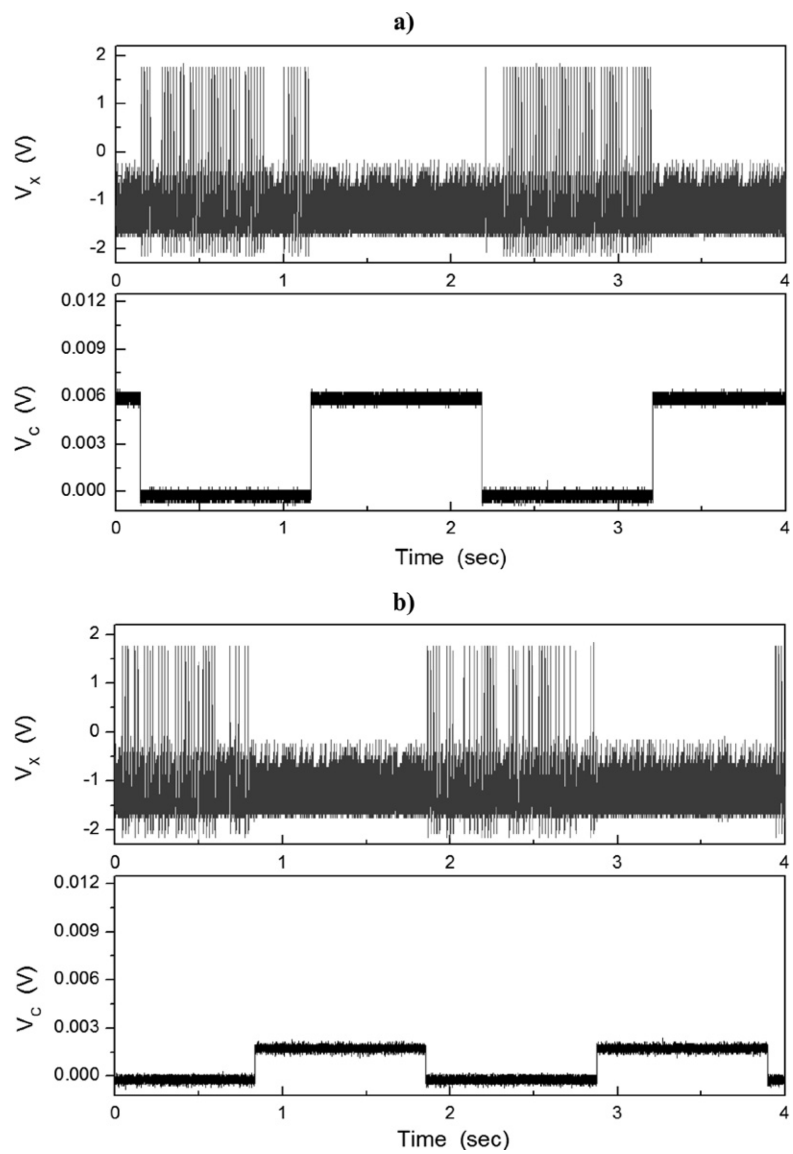

Fig. 6. Voltage signal V_x and the modulated feedback control signal V_C in the case of the HPF (a) and in the case of the APF (b). It's possible to observe that using the APF in the feedback loop, the spiking control is achieved by means of a lower feedback strength.

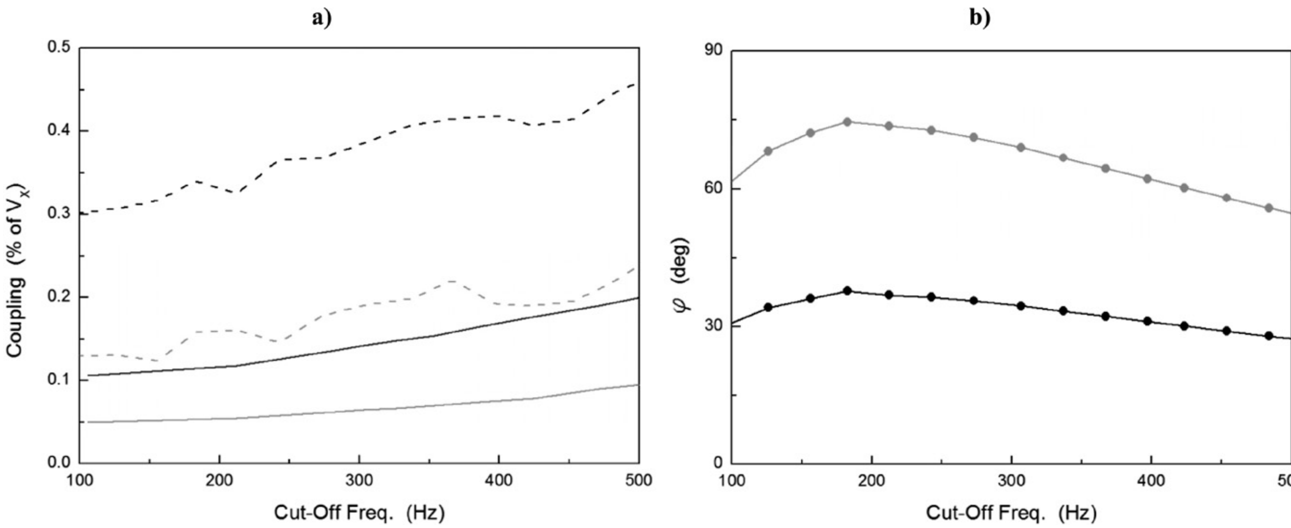

Fig. 7. (a) Coupling strength values as a function of cut-off frequency values for numerical simulations (continuous lines) and experimental acquisitions (dashed lines). These results confirm the better performance of the APF (gray lines) with respect of the HPF (black lines) in controlling bursting. (b) Phase difference φ between the driving frequency at 400 Hz and the frequency component at 100 Hz, one of the components of the spiking regime. Phase values are extracted from Fig. 4(b) for both filters, APF (gray line) and HPF (black line).

pose is to control the spiking regime we consider two filters, that is, an HPF and an APF. Eqs. (1) are modified as follows and the related electric scheme is shown again in Fig. 1 but considering the feedback loop including the two filters:

$$\dot{x} = x - y - \frac{x^3}{3} - V_d - \alpha F_{1,2}$$
$$\dot{y} = 0.08(x - 0.8y + 0.7) \tag{2}$$

where $F_{1,2}$ are the control signals for HPF and APF respectively and α is the coupling strength.

The dynamical evolution F_1 of the HPF shown in Fig. 4(a), is ruled by the following first order differential equation:

$$\dot{F}_1 = \dot{x} - \frac{F_1}{\tau} \tag{3}$$

where τ is the time constant.

In the case of the APF shown in Fig. 4(b), the dynamical evolution of the control signal $F_2(t)$ is given by:

$$\dot{F}_2 = \dot{x} - \frac{x + F_2}{\tau} \tag{4}$$

In both filters, the time constant $\tau = R_1C_1$ is adjusted in order to choose a suitable cut-off frequency value. The transfer functions of the two filters are given by:

$$H_{HPF}(\omega) = \frac{F_1(\omega)}{x(\omega)} = \frac{i\omega R_1C_1}{1 + i\omega R_1C_1}$$
$$H_{APF}(\omega) = \frac{F_2(\omega)}{x(\omega)} = \frac{1 - i\omega R_1C_1}{1 + t\omega R_1C_1} \tag{5}$$

The gain and the phase characteristics of the two filters are shown in Figs. 5(a) and 5(b) respectively, for different values of the cut-off frequency. In particular, Fig. 5(b) shows the greater phase displacement introduced by the APF (solid lines) with respect to the HPF (dashed lines). This difference is relevant for the frequency components that lead the system dynamics to spiking behavior.

3. Results

By varying the time constant of the filters we measured the attenuation necessary to eliminate the erratic spiking behavior. From Fig. 6 it emerges the better performances of the APF with respect to HPF. There are shown two different time series of 4 s each one, when the high pass filter (Fig. 6(a)) or the all pass filter (Fig. 6(b)) is inserted in the feedback loop and modulated by a square wave of 0.5 Hz frequency. In both cases the filter cut-off frequency is 65.8 Hz.

Increasing the cut-off frequency of the filter, the feedback coupling strength needed to controlling spiking behavior increases, in both cases, HPF and APF. We tested the filters inserted in the FHN circuit for several cut-off frequency values, moreover we numerically simulated their effect in the FHN model; related results are shown in Fig. 7(a) where the coupling strengths for different values of the cut-off frequency are reported. Continuous lines represent numerical simulations, dashed ones refer to experimental acquisitions. These results show the better performances of the APF (gray lines) with respect to the HPF (black lines). In Fig. 7(b) the phase difference φ between the driving frequency and the spiking frequency component at 100 Hz is shown for the two filters. The phase values are extracted from Fig. 5(b) for both filters, considering different cut-off frequency values. The phase difference introduced by the APF (gray line) is double with respect to the HPF (black line), explaining the better performance of the APF in spiking control.

The features of the proposed method clearly emerge when the power spectra of the signals (uncontrolled and controlled) are considered. The controlled signal (a chaotic regime with small amplitudes) and its spectrum are shown in Figs. 8(a) and 8(b) respectively. In Fig. 8(b) the lower frequency components, responsible for bursting, are removed. When the sign of the control signal is reversed the dynamics is converted into a periodic spiking regime at a frequency $f \approx 57$ Hz, which matches to the seventh subharmonic of the driving signal. In Figs. 8(c) and 8(d) the controlled periodic signal and its spectrum are shown. From Fig. 7(b) it emerges that controlling spiking behavior does not rely on specific phase differences as in the case of the Phase Control of Chaos, such phase resonances appear more and more narrow as the applied perturbations occur at high subharmonics of the main driving frequency.

4. Conclusions

In this Letter we have introduced a feedback method to control the dynamics of a periodically driven FHN that can easily display small amplitude chaotic behavior and chaotic or periodic spiking behavior. During chaotic spiking, small amplitude irregular oscillations are randomly interrupted by large spikes (MMOs). In the

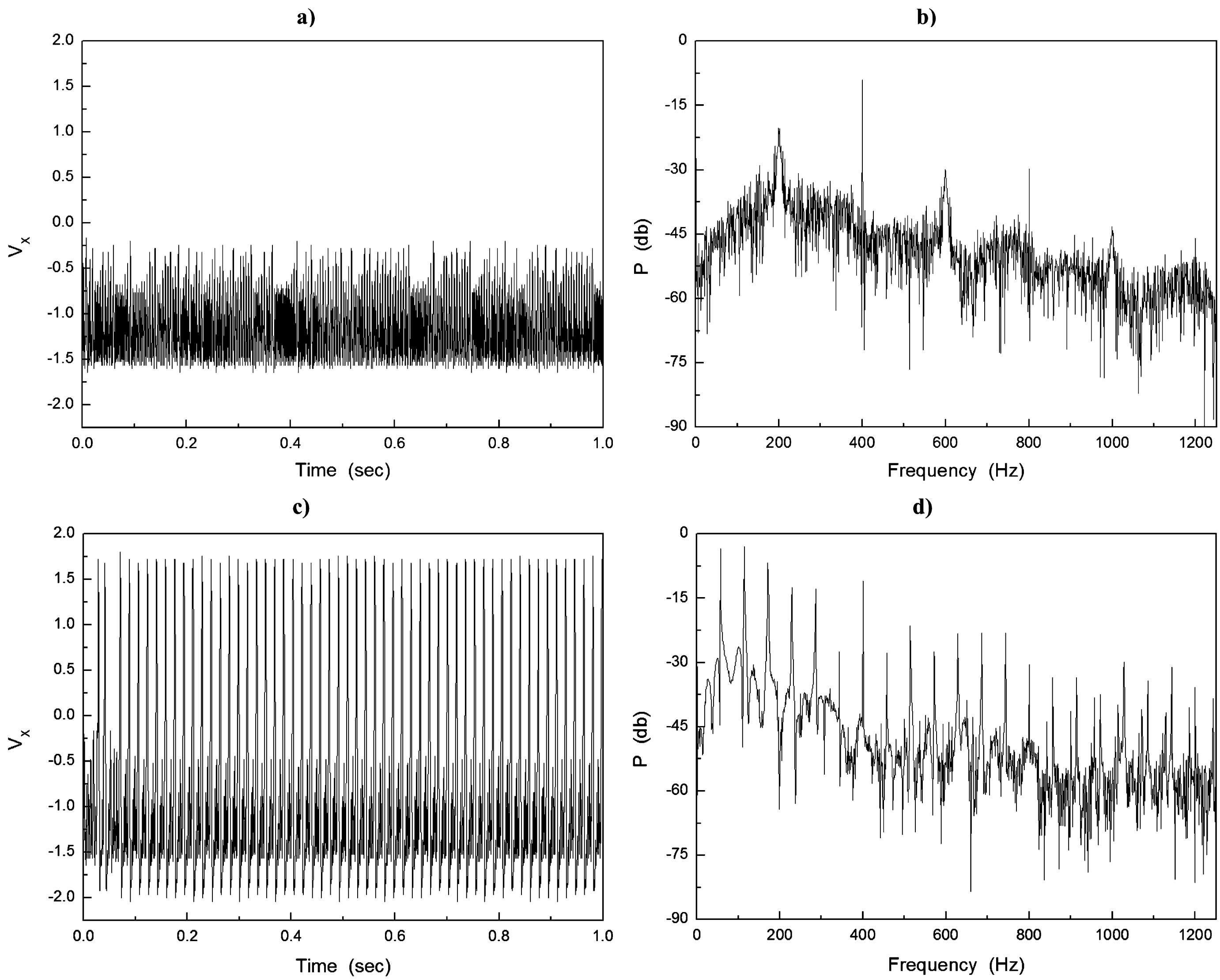

Fig. 8. (a) Negative feedback: the spiking behavior disappeared; (b) power spectrum of the signal in (a): the lower frequency components, indicating spiking behavior, are rejected by the filter. (c) Positive feedback: the spiking behavior becomes periodic; (d) power spectrum of the signal in (c).

chaotic spectrum of this regime we can easily recognize a continuous broadband component around the driving frequency, and a discrete one with peaks centered at low sub-harmonics of the driving frequency. The analysis of the spectrum suggests a feedback strategy to depress or enhance these low frequency components in order to obtain the desired behavior. We have shown that an APS filter is more efficient than the HP filter in achieving suppression of spiking behavior considering its phase properties.

The advantage of this feedback method with respect to the open loop method [7,8] is related to the fact that it is not necessary to scan the relative phase up to when the desired behavior is obtained. On the contrary, the feedback case requires knowledge of the output frequency spectrum, in particular, of the frequency components responsible of the spiking behavior. Once the spiking frequencies are detected they can be easily controlled in a fast way by the technique here introduced. This is particularly relevant in the case of neuron dynamics, where pathological diseases are associated with the neuron spiking behavior [18].

Acknowledgements

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