Conversion of laser radiation into x-rays in the subnanosecond region

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Abstract. X-ray radiation from the plasma generated on a solid target by a subnanosecond laser pulse is investigated. The analysis is a semiquantitative one, based upon the energy equation with the relevant microscopic processes taken into account. For the simple case of a rectangular pulse, there are three different phases: a heating transient stage, a stationary regime in which all absorbed radiation is reradiated as soft x-rays and a cooling phase. The conditions to be fulfilled in order that the expansion be neglected are investigated in detail. These analytical estimates are compared with the results of a computer code. In a model where energy propagates from the laser absorption layer into the target by heat conduction, the conversion efficiency at moderate fluxes is drastically reduced by the expansion which sets on before the regime is reached. Moreover the emission is delayed. For large power fluxes, following a recent suggestion by Brueckner, we consider also an energy transfer by fast (10–50 keV) electrons, which thermalize by collisions in a region larger than the layer directly heated by conduction. This reduces the time lag and yields a good power efficiency. Results are presented for absorbed laser fluxes in the range $10^{12}$–$10^{16}$ W cm$^{-2}$.

1. Introduction

Good conversion efficiency into x-rays of high power laser radiation focused upon a solid slab has been reported by several authors (Mallozzi et al 1973, Nagel et al 1974, Eidmann et al 1976). The resulting radiation fluxes in the spectral range 1–4 keV, of the order of $10^{14}$–$10^{15}$ W cm$^{-2}$, are at present unobtainable by any other means. Practical uses of this powerful source have already been proposed (Basov et al 1964, Dawson 1964, Peacock and Pease 1969, Ehler and Weissler 1966, Fawcett et al 1966).

The feasibility of an x-ray laser using inner shell excitation in high-Z elements requires also a very large x-ray flux (Arecchi et al 1974). Along this line we have started an investigation of an ‘x-ray flash light’ to produce population inversion. It seems clear that, in order to make full use of such a flash, its initial geometrical shape should be kept unchanged during the relevant part of the experiment, making thereby possible, for example, the use of x-ray filters. For this reason we have concentrated our attention on ultrashort laser pulses, so that the flash is completed before the irradiated solid has expanded appreciably. Measurements of the reflected energy in the range 10–20 ps at a laser power above $10^{16}$ W cm$^{-2}$ have been performed (Salzmann 1973) and have shown that the absorption coefficient is high, perhaps because of nonlinear effects. The role of non-
linear processes has been stressed very recently (Mead et al 1976) to explain an increase of absorption for increasing incident power (27% at $2 \times 10^{15}$ W cm$^{-2}$; 41% at $3 \times 10^{16}$ W cm$^{-2}$).

The details of the absorption process will not be considered in the present work, but only the conversion of the absorbed energy into x-rays and other losses.

Caruso and Gratton (1969) have considered the heating process of a solid at very short times by electron conduction and described the propagation into the undisturbed material of a sharp, nonlinear heat wave (Zel'dovich and Raizer 1967). The competing process of expansion of the heated layer becomes important after a time $\tau_0$ which turns out to be of the same order of magnitude as the ion-electron energy equipartition time $\tau_{eq}$. Both the processes of heating and of ion expansion should therefore be taken into account after $\tau_0$.

Several theoretical papers and numerical codes are available to describe laser-plasma interactions (Colombant and Tonon 1973, Whitney and Davis 1974a, b). In particular Colombant et al (1975) have developed a very thorough programme including expansion, magnetic effects, three-dimensionality and spectroscopic transitions. We believe that these very complex numerical experiments are usefully supplemented by semiquantitative theoretical analysis, which gives more insight in the physical processes at work and provide analytical expressions for the observed quantities.

We are mainly concerned here with the early stage in which expansion does not play an important role; the laser flux is assumed to be large enough to make the ionization and the radiative losses relevant. The main question we try to answer in the different cases we consider is, is this high conversion efficiency phase reached before the time $\tau_0$ at which expansion becomes important? Also, what is the mechanism responsible for, and the duration of the emitted x-ray flash?

X-ray production at long times, when the expansion process is fully developed, has been investigated by Bernstein and Comisar (1970) and Colombant and Tonon (1973). In both papers the emission is assumed to occur mainly from the expanding and absorbing region, giving a rather low efficiency at short times, when a substantial amount of radiation comes from the high-density, conductively heated region. The latter paper contains also a rather complete discussion of the different radiation and collisional processes.

In the next sections, after having recalled the theory of the heating process, we discuss in detail the loss mechanisms and find that a high efficiency régime is reached before expansion only for large powers. The semiquantitative answer we give is supported by the results of a numerical calculation. In §6 we investigate the role of fast electrons in transferring energy away from the boundary, faster than the heat conduction mechanism. Finally in §7 we report some computer results to test our analytic conjectures.

2. Initial heating phase

We recall in this section known results concerning the first stage, in which all losses of the electron gas are negligible. cgs units are used throughout unless otherwise indicated.

We confine our model to one-dimensional effects; this is possible when the focal spot is larger than the thickness $d$ of the heated region.

A semi-infinite solid slab absorbs at its surface $x=0$ a normal flux $\Phi(t)$, becoming quickly ionized and hot within a thin layer, with a mean charge $Z(x,t)$. Its electron
temperature $T$ (in energy units) is determined by the heat equation
\[
\frac{\partial}{\partial t} \left( \frac{3}{2} ZNT \right) - \frac{\partial}{\partial x} \left( \chi \frac{\partial T}{\partial x} \right) = -P \quad (\text{total loss})
\]  

(1)

and the boundary condition
\[
\Phi(t) = - \left( \chi \frac{\partial T}{\partial x} \right)_{x=0}.
\]

(2)

$N$ is the number density of atoms, taken here as constant and uniform for $x>0$ and vanishing for $x<0$;
\[
\chi = aT^{5/2} = 6.386 \times 10^{61} \frac{e\delta Z}{\ln \Lambda Z^2} T^{5/2}
\]

(3)
is the plasma thermal conductivity. In $\Lambda$ is the Coulomb logarithm, the coefficients $\epsilon$ and $\delta$ take into account respectively a produced secondary electric field which acts to reduce the flow of heat and the difference between the thermal conductivity of an actual gas and a Lorentz gas (Spitzer 1962), $\overline{Z^2}$ is the mean square charge. The dependence of $Z$ from the temperature is determined by the balance between collisional ionization, radiative and three-body recombination (figure 1). In our high-density case, however, three-body recombination often prevails, in which case Saha’s equation holds. Since it is difficult to give a reasonable analytic approximation for $Z(T)$, we shall solve our differ-

![Figure 1. Mean ionic charge $\overline{Z}$ as a function of electron temperature $T$ for Be and Al at the solid density. The ion densities are calculated by the corona model modified to include three-body recombination.](image)

† Equation (3) gives the electron thermal conductivity for a maxwellian plasma. In a real experiment local suppression of the heat conductivity may occur due to the induced azimuthal magnetic field $B$, leading to ‘hot spots’ off the axis of symmetry of the light beam (Craxton and Haines 1975). When the expansion is neglected the electron density $n$ is a function of temperature and $\nabla \times \mathbf{n} \nabla T = 0$, so that $B=0$. 

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ential equation assuming $Z$ to be constant, but insert its appropriate value in the final formulas. Our main concern, in fact, is the regime in which high-$Z$ atoms are almost completely stripped and $Z$, becoming almost $Z$, does not change much.

In this case equations (1) and (2) with $P=0$ can be solved exactly (Zel’dovich and Raizer 1964) when $\Phi=\text{constant}$ and $T(x, 0)=0$

$$T(x, t) = T_0(t) \left(1 - \frac{x}{d(t)}\right)^{2/5} \text{ for } x \leq d$$

(4)

$$T(x, t) = 0 \text{ for } x > d.$$ $d(t)$, the abscissa of the leading edge of the heat pulse, is given by

$$d(t) = a^{2/9} c^{-7/9} \Phi^{5/9} t^{7/9}$$

(5)

where

$$c = \frac{3}{2} Z N.$$ (6)

The temperature $T_0(t)$ at $x = 0$ is

$$T_0(t) = \frac{1.5 \Phi t}{cd(t)} = 1.5 c^{-2/9} a^{-2/9} t^{2/9} \Phi^{4/9}.$$ (7)

There is a generalized solution corresponding to a variable flux:

$$d(t) = a^{2/9} c^{-7/9} \int_0^t \left[ \int_0^{t'} \Phi(t')^2 \right]^{1/2} dt'$$

(5')

$$T_0(t) = \frac{1.5}{cd(t)} \int_0^t \Phi(t')^2 dt'.$$

(7')

Physical considerations, however, show that this solution is meaningful only if the time scale of the flux is longer than the time taken by the heat front to cross the heated layer, namely

$$\frac{\Phi}{|d\Phi/dt|} > \frac{cd^2}{aT^{5/2}} = \tau_c.$$ (8)

To see this, notice that the absorbed flux is given explicitly in terms of the functions $T_0(t)$ and $d(t)$ by

$$\Phi = \frac{3}{2} \frac{c}{dt} (T_0 d) \frac{42 a T_0^{7/2}}{50 d}.$$ (9)

In other words, a change in $\Phi$ implies a simultaneous change in the whole temperature profile, even for an instantaneous jump; in particular the heated layer suddenly disappears ($T_0 \rightarrow 0$) if the flux drops to zero instantaneously.

In reality a fast change in $\Phi$ will generate a heat wave over the pre-existing temperature profile which reaches the other end after a time of order $\tau_c$; for example, when $\Phi$ drops to zero the temperature profile after a time $\tau_c$ must go over into the profile

$$T = T_{01} \left(1 - \frac{x^2}{d_1^2}\right)^{2/5}$$

(10)

corresponding to the instantaneous deposition of an energy $Q$ per unit surface (Zel’dovich and Raizer 1967). Here

$$T_{01} \sim Q/cd_1.$$ (10')
where
\[ d_1 \sim Q^{5/9} / t^{3/9} a^{2/9} c^{-1/9}. \] (11)

We can expect our solution (4) to be valid only for a slower time dependence of the flux.

3. Power losses

The heating process will stop if the energy absorbed by the electron gas is dumped into the various losses, namely: recombination radiation \( P_{\text{rb}} \) (free–bound), Bremsstrahlung radiation \( P_{\text{B}} \) (free–free), bound–bound radiation \( P_{\text{bb}} \), ionization \( P_{\text{ion}} \) and ion heating \( P_{\text{ion}} \). In our order-of-magnitude analysis on electron dynamics we can neglect ion heating, which will take up at most an energy \( \frac{3}{5} NT \) per unit volume, always smaller than the electron thermal energy \( \frac{3}{5} NT \). Furthermore, the energy required to obtain a Z ion is smaller than the energy \( \frac{3}{2} ZT \) of the liberated electrons. Indeed, to get out the last electron one needs an energy \( \sim T \), but to get out the other \( Z-1 \) electrons one needs much less, hence we may also neglect ionization. The power loss for bound–bound transitions is computed for hydrogen and helium like ions only, because we consider laser pulses sufficient to rapidly ionize through the L-shell electrons and excite K-shell line radiation. The power loss for free–free transitions is given by
\[ P_{\text{B}} = 1.2 \times 10^{-8} Z N^2 T^{1/2} = h T^{1/2}. \] (12)

This relation is valid only when the electron sees an atomic field \( es/r^2 \), corresponding to the total charge \( es \) of the species ionized \( s \) times; at high energies a correction is needed. Recombination radiation is given by
\[ P_{\text{rb}} = 1.1 \times 10^{-8} Z N^2 T^{1/2} \sum_{s=1}^{s_{\text{max}}} \frac{f_{s+1}}{\omega_s(s+1)} \chi_s \wedge_n^{s_n \gamma_n} \] (13)

![Figure 2. Radiated power density as a function of the temperature \( T \) for (a) Be and (b) Al.](image-url)
(see the Appendix for explanation of the symbols). $P_{\text{tt}}, P_{\text{fb}}, P_{\text{ob}}$ and $P = P_{\text{tt}} + P_{\text{fb}} + P_{\text{ob}}$ are shown in figure 2 as functions of $T$ for Be and Al.

4. Stationary régime

When the laser flux is constant the steady-state régime is given by the ordinary differential equation

$$\frac{d}{dx} \left( a T^{5/2} \frac{dT}{dx} \right) = P(T)$$

with the boundary condition

$$\Phi = - \left( a T^{5/2} \frac{dT}{dx} \right)_{x=0}. \quad (15)$$

This equation can be easily reduced to quadrature introducing the dependent variable $\int_0^x a T^{5/2} \, dT'$. We obtain

$$x = \int_T^{T_*} \frac{dT'' a T''^{5/2}}{2 \int_0^T dT' a T'^{5/2} P(T')^{1/2}}. \quad (16)$$

The relationship between $T_0$ and the absorbed flux $\Phi$ is

$$\Phi = \left( 2 \int_0^{T_*} a T^{5/2} P(T) \, dT \right)^{1/2}. \quad (17)$$

The finite extension of the heated layer depends upon the behaviour of the quantities $a(T)$ and $P(T)$ near $T=0$; if

$$\lim_{T \to 0} P(T) = O(T^s)$$

and

$$\lim_{T \to 0} a(T) = O(T^r)$$

when $s - r < \frac{3}{2}$ the integral (16) converges to

$$d = \int_0^{T_*} \frac{dT'}{2 \int_0^{T'} dT'' a T''^{5/2} P(T'')^{1/2}}. \quad (18)$$

near $x = d$ the temperature behaves as $(1 - x/d)^{s/2}$. When $s - r = \frac{3}{2}$ the temperature profile is exponential, while when $s - r > \frac{3}{2}$, $T \sim x^{s/2 - (r - s)}$.

At very high temperatures, when the ionization is nearly complete, the Bremsstrahlung loss prevails (equation 12) and $a$ can be regarded as practically constant; the solution has then an explicit form

$$T = T_0 \left( 1 - \frac{x}{d} \right)^{2/3} \quad \text{for} \quad x \leq d$$

$$T = 0 \quad \text{for} \quad x > d \quad (19)$$

with

$$T_0 = 1.2 \, \Phi^{1/2} \, a^{-1/4} \, b^{-1/4}$$

$$d = 0.8 \, \Phi^{3/4} \, a^{1/8} \, b^{-7/8}. \quad (20)$$
Of course the solution breaks down near \( x = d \) where the temperature is too low to neglect other losses. While the temperature of the heated layer depends upon \( \Phi \) roughly in the same way for Be and Al, note that the thickness is an order of magnitude smaller for Al (figures 3 and 4); this is due to the fact that the power density for Al is an order of magnitude larger at the temperatures which we consider (see figure 1).

![Graph](image)

**Figure 3.** The temperature \( T_0 \) as a function of the absorbed laser flux \( \Phi \) for the stationary regime for a Be and an Al target.

![Graph](image)

**Figure 4.** The thickness of the heated layer \( d \) as a function of the absorbed laser flux \( \Phi \) for a Be and an Al target.

The duration \( \tau \) of the transient régime can be estimated by equating the thickness of the heated layer for the transient and the asymptotic solution (equations (5) and (18)) (figure 5). In this situation the power loss is sufficiently large to balance the conduction term.

In the case of prevailing Bremsstrahlung loss \( \tau \) has an analytical expression

\[
\tau = 0.74 \ a^{-1/8} \ c b^{-9/8} \Phi^{1/4}.
\]  (21)
Figure 5. The transient duration $\tau$ and the times $\tau_e$ at which expansion becomes important as a function of the absorbed laser flux $\Phi$ for (a) a Be and (b) an Al target.

5. Ion expansion and conversion efficiency

Caruso and Gratton (1969) have pointed out that the effect of ion expansion becomes important roughly at the same time as the energy equipartition between the ion and electron times $\tau_{eq}$. In view of the strong temperature dependence of the different coefficients it is important to discuss this problem more precisely. Neglecting, for simplicity, spatial inhomogeneities, the ion temperature $T_i$ is determined by the equation

$$\frac{dT_i(t)}{dt} = \frac{1}{\tau_{eq}} [T_0(t) - T_i(t)] = \frac{D}{T_0(t)^{3/2}} [T_0(t) - T_i(t)]$$

(22)

with

$$D = 10^{-18} \frac{N}{Z^3} \ln \Lambda/\Lambda \text{ s}^{-1} \text{ keV}^{3/2}$$

where $\Lambda$ is the mass number of the target, and from the transient equation (7)

$$T_0(t) = Bt^{2/3},$$

(23)
This equation (with constant $D$) is solved by

$$\frac{T(t)}{T^*} = G \left( \frac{T_0}{T^*} \right) = G(u)$$

(24)

where

$$G(u) = \exp \left( -u^2 \right) \int_0^{a_3} dv \, v^{1/3} c^v, \quad T^* = (\frac{a_3}{3})^{1/3} \, D^{-1/3} B^{3/2}. \quad (25)$$

The function $G(u)$ behaves like $\frac{1}{4} u^4$ for $u \ll 1$ and like $u + O(1/u^2)$ for $u > 1$. Therefore $T^*$ gives, to an order of magnitude, the electron temperature at which equipartition is reached. Its value is

$$T^* = \frac{2 \times 10^5 A^{1/3} \Phi^{1/3}}{N^{2/3} Z^{2/3}} \, \text{keV.} \quad (26)$$

For $A = 10$, $N = 10^{23} \, \text{cm}^{-3}$, $Z = 5$, $\Phi = 10^{14} \, \text{W cm}^{-2}$, $T^* = 0.15 \, \text{keV}$. From equations (23) and (25) $T_0(t) = T^*$ at a time equal to $0.5 \, \tau_{eq} = 0.5 \, T^{3/2} / D$.

It is interesting to compare this critical temperature $T^*$ with the regime temperature $T_0$ (figure 3); the ratio $T^*/T_0$ is a weakly increasing function of $\Phi$, so that, if the flux is sufficiently large, $T^* > T_0$ and the equipartition is never reached during the transient.

Expansion becomes important when the thickness $d$ of the heated layer is of the same order as the thermal expansion distance scale

$$\delta(t) = \int_0^t dt' \left( \int Z T_0(t') \right)^{1/2} (2AM)^{1/2} \quad (27)$$

where $M$ is the proton mass. Using equations (5) and (7) this condition is equivalent to $t = \tau_0 = 27 \, Z^{-3/2} \, \tau_{eq}(T^*)$.

$$= (2.4 \times 10^{34} A^{3/2} \Phi) / (Z^2 \ln L N^2 Z^{1/2}) \, \text{ps.} \quad (28)$$

Equation (27) is an order of magnitude estimate; we have taken the temperature $T_0/2$ as a mean temperature within the heated layer. $Z$ is evaluated for simplicity at the final temperature $T_0(\tau_0)$. When $\tau > \tau_0$ the stationary régime is never reached; the condition $\tau < \tau_0$ is equivalent to stating $\Phi < \Phi_c$ given by figure 5.

It is obvious that the time-integrated conversion efficiency

$$\eta_E = \frac{\int_0^\infty dt \int_0^d P \, dx}{\int_0^\infty \Phi(t) \, dt} \quad (29)$$

is smaller than unity only because some energy is used up in the plasma expansion. Even when the latter is neglected, however, it is important to know the time dependence of the emitted radiation. During the stationary régime $\int_0^d P \, dx = \Phi$: the plasma behaves as a perfect machine converting radiation from the optical region to the UV and X regions. During the transient period (with constant $\Phi$) we have from equation (1)

$$P_T = \int_0^d P \, dx = \frac{5d(t)}{2T_0(t)^{5/2}} \int_0^{T_0} T^{3/2} P(T) \, dT. \quad (30)$$

However this is only an upper limit because the radiated power slows down the heating.

After the end of the pulse of duration $\tau_1$, a transient is set up, leading from the solution (4) or (16) to (10). This transient lasts a time of order $\tau_0$, given by equation (8). Note that if $\tau_1 < \tau$ (régime never reached) from (5) and (7) it follows $\tau_0 \approx 0.3 \, \tau_1$. Once
the condition of equation (10) is set up, with \( Q = \Phi r_1 \) the cooling is due to two causes the thermal conduction, which increases \( d \), and the radiative loss which keeps \( d \) constant.

The complexity of the phenomenon, in which several different time scales occur, makes an analytical discussion un rewarding; numerical analysis is necessary.

6. Fast-electron heating

Thus far, we have calculated the emitted x-ray power by assuming electron heating at the boundary layer and a spread of the energy there deposited by electron heat conduction. Because of the rather slow diffusion process, it takes some time before a consistent energy deposition spreads out over the bulk of the target, giving rise to an output power. The time lag \( \tau \) (see figure 5) between the peak of the laser pulse and the full x-ray power is at variance with some preliminary time resolved x-ray measurements (Key et al 1976) which show an almost instantaneous energy conversion.

A possible faster mechanism for energy conversion may be the following. Brueckner (1976) (see also Eidmann et al 1976) has suggested that for absorbed fluxes of the order of \( 10^{14} - 10^{15} \) W cm\(^{-2}\) nearly all the absorbed energy is converted in fast electrons. Since the rates of energy loss both for radiation processes and for electron–electron collision are inversely proportional to the electron energy, these fast electrons can travel a relatively long distance \( l \) before they thermalize.

With their relativistic speed the thermalization time \( l/c_{\text{rel, el.}} \) is in general smaller than the other time scales occurring in the problem and the process can be regarded as instantaneous. Provided \( l \) is of the same order as \( d \), or at any rate consistently larger than the boundary layer over which EM radiation is absorbed, the fast electron transport replaces the slow conduction mechanism, leading to a thick energized layer over which thermal electrons are smoothly distributed.

The characteristic time for x-ray emission is then determined only by the emission rate of the thermal electrons. Note that thefast electrons mainly thermalize rather than radiating, as seen experimentally from the ratio between the emitted soft and hard x-rays (see figures 1 and 2 of Brueckner 1976).

Let us now evaluate the thermalization length \( l \). The rate of energy loss by electron–electron collision is

\[
\frac{d\varepsilon}{dx} = -\frac{4\pi Z Ne^4 \ln \Lambda}{\varepsilon}.
\]

(31)

We introduce a scale of length

\[
\frac{1}{L} = 8\pi Z N \ln \Lambda \left( \frac{e^3}{mc^2} \right)^2
\]

(32)

so that equation (31) can be rewritten as

\[
\frac{1}{mc^2} \frac{d\varepsilon}{dx} = -\frac{1}{2L} \frac{mc^2}{\varepsilon}
\]

(31')

whose solution is

\[
\frac{\varepsilon}{mc^2} = \left[ \left( \frac{\varepsilon_0}{mc^2} \right)^2 - \frac{x^2}{L} \right]^{1/2}
\]

(33)

\( \varepsilon_0 \) being the initial energy of the fast electrons (assumed to be mono-energetic).
The length over which fast electrons thermalize is

\[ l = L \left( \frac{\epsilon_0}{mc^2} \right)^2. \] (34)

For an Al target with \( Z = 10 \) and \( N = 6 \times 10^{22} \text{ cm}^{-3} \), equation (32) yields, putting \( \Lambda \sim 5 \),

\[ L = 0.16 \text{ cm}. \]

Hence for the two energies considered in Brueckner, \( \epsilon_0 = 10 \) and 30 keV, the thermalization length is respectively \( l \sim 0.7 \) and \( 7 \mu \text{m} \), with thermalization times of order \( 10^{-3} \) and \( 10^{-2} \) ps respectively. If \( \Phi^c \) is the critical flux above which a substantial number of fast electrons are generated and \( \alpha \Phi \) the fraction of flux which goes into fast electrons (in the experimental cases considered in Brueckner \( \alpha \) is near to 1), then the boundary condition (2) has to be modified as follows:

\[ \Phi(t) (1 - \alpha) = - \left( \alpha T^{3/2} \frac{\partial T}{\partial x} \right)_{x=0}. \] (35)

Calling \( S \) the rate of generation of fast electrons we have further

\[ S = \alpha \Phi / \epsilon_0. \] (36)

The total rate of production of thermal electrons is then given by equations (31) and (36):

\[ S \frac{d\epsilon}{dx} = \frac{\alpha \Phi}{2l[1 - (x/l)]^{1/2}} \] (37)

so that the energy balance (1) determining the electron temperature becomes

\[ \frac{\partial}{\partial t} \left( \frac{3}{2} Z N T \right) = \frac{\partial}{\partial x} \left( \alpha T^{3/2} \frac{\partial T}{\partial x} \right) - P + \frac{\alpha \Phi}{2l[1 - (x/l)]^{1/2}} H(l - x) \] (38)

where \( H \) is the Heaviside function.

For a slab thickness much less than \( l \) the fast electrons do not thermalize, and rather than the conversion into EM radiation one should see most of the absorbed energy as kinetic energy of the electron emitted by the thin film. When the slab thickness is much greater than \( l \) and the conduction thickness \( d < l \), there is a steep temperature gradient of thickness \( d \) at the surface, beyond it the conduction can be neglected. Averaging equation (38) from 0 to 1 we get

\[ \frac{d}{dt} \left( \frac{3}{2} N Z T \right) = -P - \frac{\Phi}{l}. \] (39)

Note that this equation cannot be integrated unless \( \epsilon_0(t) \) is known. When \( \Phi \) is constant the heating takes place during a transient phase of duration (in order of magnitude)

\[ \tau_r = \frac{2NZ(T_0)T_r}{\Phi/l}. \] (40)

where \( T_r \) is the regime temperature at which input and losses balance, that is,

\[ P(T_r) = \frac{\Phi}{l}. \] (41)

The evaluation of these quantities requires experimental knowledge of \( \epsilon_0 \); for \( \epsilon_0 = 30 \) keV (Brueckner 1976), \( \Phi = 10^{18} \text{ W cm}^{-2} \) and the Al target considered we have \( P(T_r) = \)
$1.43 \times 10^{19}$ W cm$^{-3}$, $T_{r} = 320$ eV (from figure 2b), $Z(T_{r}) = 11$ (from figure 1b) and $\tau \approx 5$ ps which is an order of magnitude less than $\tau$ for the same value of $\Phi$ (figure 5b). For a Be target with $N = 10^{28}$ cm$^{-3}$ and $Z = 2$, $\Phi = 10^{14}$ W cm$^{-2}$ we have $P(T_{r}) = 7 \times 10^{16}$ W cm$^{-3}$, $T_{r} = 50$ eV (from figure 2a), $Z(T_{r}) = 2$ (from figure 1a) and $\tau \approx 40$ ps which has to be compared with the value of $\tau \approx 100$ ps for the same flux $\Phi$ (figure 5b).

We can see that fast electrons speed up energy conversion, but the temperature is lower.

7. Computer results

In order to test the analytic conjectures previously discussed we have developed a computer code (Bertotti et al 1976) to solve the slab problem including the following effects: heat conduction, energy transfer from electrons to ions, Bremsstrahlung radiation, recombination radiation and bound-bound radiation as described with the LTE model. Only the ground state of all but the last two ionization stages are included in the calculation. For the hydrogen and helium like ions, the principal level structure is added by including several members of the $np$ ($n \leq 5$) series. Ion dynamics has been neglected and the absorption is taken into account by the boundary condition (2).

The agreement with the results computed analytically is generally satisfactory (see figure 6 and 7).

![Figure 6. The surface temperature $T_0$ for Al at $\Phi = 10^{14}$ W cm$^{-2}$. The cross denotes the time $t = \tau$ (end of the transient).](image)

In figure 7 we show the time dependence of the radiated loss. Notice that the duration of the transient as computed analytically beforehand, was obtained by comparing the physical quantities of the transient solution with the physical quantities of the stationary solution; we see from figure 7 that the agreement is in this sense satisfactory. The time $\tau$ should not be viewed as the time taken to reach the stationary state which is indeed longer.

The computer code has also evaluated the time at which expansion is important, denoted by a circle in figure 7. This time agrees in general with the theoretical value.
To study the effect of a more realistic laser pulse and the duration of the emitted radiation the computer code has been run for the pulse

$$\Phi = \frac{\Phi_0}{2} \left[ \sin \left( \frac{\pi t}{\tau_1} - \frac{\pi}{2} \right) + 1 \right] \quad 0 \leq t \leq 2\tau_1$$

$$\Phi = 0 \quad \text{otherwise}$$

whose total energy is $\Phi_0 \tau_1$.

For an Al target and $\tau_1 = 25$ ps, $\Phi_0 = 3.2 \times 10^{16}$ W cm$^{-2}$ the maximum temperature on the surface is $T_{\text{max}} = 2.840$ keV, the duration of the emitted pulse, defined as the time at which the emitted radiation is 30% of the laser energy, is $\tau = 39$ ps and the expansion becomes important at the time $\tau_1 \approx 60$ ps. In figure 8 we show the surface temperature $T_0$, the thickness $d$ of the heated layer, the total radiated power flux $P_\nu$ and the conversion efficiency $\eta_B$ as a function of time for this case.

We find $\eta_B$ almost 100%. As previously stated $\eta_B$ is the conversion efficiency of the absorbed energy into radiation. A very high value of $\eta_B$ means that the plasma cools down radiatively re-emitting the absorbed energy before expansion becomes important. The overall conversion efficiency $\eta$ of the incident laser energy into radiation depends on the fraction of the absorbed energy. If we take the latter of the order of 40% (Mead et al 1976) we have in this case $\eta \approx 40\%$.

8. Conclusions

The main limitations of our calculation are the one-dimensionality and the conventional expression for the heat transport coefficient (Salzmann 1972). Moreover since we have neglected ionization losses, the present calculation cannot be applied to high-$Z$ elements. The very short conduction distances we find justify the former for usual spot sizes, but when heat is transported by fast electrons three-dimensional effects may be
relevant. The existence of a regime where energy is transported by fast electrons is supported by experimental investigation (Brueckner 1976, Kolodner and Yablonovitch 1976) and should be the main object of the research in the future. In particular, one would like to know if it occurs only above a threshold flux $\Phi^*$, as it is to be expected theoretically. The evaluation of $\Phi^*$ as well as of the electron energy spectrum hinges upon a detailed theoretical and experimental investigation of the nonlinear plasma acceleration and absorption processes taking place within the very thin absorption layer. Our phenomenological description (equation (37)) will provide information about $\epsilon_0$ and $\alpha$ from the experimental results.

In our paper this problem has been by-passed and its results can be used only if the reflected flux is measured experimentally; but it would be very important to understand how it is determined by the time scale of the laser pulse, its peak power and the nature of the material. Recent experiments on light backscattered from a target show a dramatic increase of absorption about 20–30 ps after the beginning of the pulse suggesting a
threshold process (private communication from J Forsyth, Laboratory for Laser Energetics, University of Rochester, NY; Mead et al 1976).

For \( \Phi < \Phi^* \) our calculations should be correct, but predict rather low efficiencies for the pulse duration and fluxes currently available; in fact as figure 5 shows the expansion very soon becomes important.

We have discussed also possible effects due to opacity on the bases of the formulas given by Cooper (1966). For the radiation emitted backwards, we find that the optical depth for the free–bound and free–free radiation is always much larger than the thickness \( \delta \) of the heated layer. The forward radiation can be absorbed if the thickness of the target is sufficiently large. For bound–bound radiation the plasma is optically thin at temperatures at which the term \( P_{lb} \) is negligible compared with \( P_{rr} \) and \( P_{ff} \).

Finally, a better theoretical and experimental understanding of the different and complex atomic processes taking place in the plasma—including the opacity of the material—should throw light upon the basic problems of deviations from thermal equilibrium and population inversion within the plasma, which could lead to x-ray laser schemes within the heated plasma itself.

Thus far, there is not sufficient information to say whether the scheme of Arechchi et al (1974), of a three-layered sub-millimetre rod with the first layer acting as an x-ray flash lamp, the second as a filter, and the third as the lasing material excited at quasi-resonance on the K–L transition, is more favourable than schemes achieving population inversion directly in the plasma. These latter ones seem more promising at \( \lambda \approx 180 \) Å, the former one is the only quantitative suggestion for \( \lambda \approx 10 \) Å.

Appendix. Discussion of equation (13) for the recombination power

To get equation (13) we have used the quantum defect method (Seaton 1958). The effective quantum number \( n^* \) is defined by

\[
\chi_{s-1, n^*} = \frac{13.6}{n^*} = \chi_{s-1, n, l, j}^\ast.
\] (A1)

Here \( \chi_{s-1, n, l, j}^\ast \) is the ionization energy of a real ion in the level \( (n, l, j) \) to be got from spectroscopic data.

The power emitted in the recombination of a species \( s \) to a level \( n^* \) has been computed according to the theory of Menzel and Pekeris (1935), Burgess (1938) and Seaton (1959).

We have taken the Gaunt factor equal to unity but we have also estimated the corresponding correction factor as a function of temperature \( T \).

In the limit \( n^* \to \infty \) with \( n^2 \varepsilon \) finite the Gaunt factor \( g_s(n^*, \varepsilon) \) has the expansion

\[
g_s(n^*, \varepsilon) = 1 + 0.1728 n^*^{-2/3} (1 + n^2 \varepsilon)^{-2/3} (n^2 \varepsilon - 1) - 0.0496 n^*^{-4/3} (1 + n^2 \varepsilon)^{-4/3} + \ldots
\]

where \( \varepsilon = (h\nu - \chi_{s, n^*})/\chi_{s, n^*} \).

The total percentage error

\[
\varepsilon = \frac{\chi_{s, n^*}}{T} \int_0^\infty \exp\left(-\chi_{s, n^*}/\varepsilon/T\right) \left[0.1728 n^*^{-2/3} (1 + n^2 \varepsilon)^{-2/3} (n^2 \varepsilon - 1) - 0.0496 n^*^{-4/3} (1 + n^2 \varepsilon)^{-4/3} (1 + \frac{2}{3} n^2 \varepsilon + n^4 \varepsilon^2)\right] d\varepsilon
\]

is given in figure A1.
In the work we have computed the recombination power for the ground level of the recombined ion only.

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