Optical Bistability in a Resonant Two-Photon Absorber.

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(ricevuto il 17 Giugno 1978)

Optical bistability as a kind of first-order phase transition has been suggested and experimentally observed in an optical feedback system (*) as well as in a resonant absorber within an interferometer (†). A multiple-pass interferometer with a suitable nonlinear dispersive or absorptive medium can behave as a bistable optical system. The first suggestion was made by Szoke et al. (†), and different versions have been shown to work with the nonlinearity due either to an atomic medium at resonance with the impinging field (‡) or to an external feed-back system (§).

We are here interested in the optical bistability arising from the collective behavior of an atomic system.

A general theory has been formulated in a series of papers by Bonifacio and Luigiato (§) to obtain absorptive and dispersive atomic bistability. As is well known from the first approaches (‡) a difficulty of atomic bistability consists in inhomogeneous broadening due to the Doppler contribution of atom in a cell.

We show here evidence of a bistable behavior of an interferometer filled with a two-photon absorber and working in a travelling wave mode. This is the first step to derive, afterwards, a Doppler free atomic bistability (‡) by absorbing photons from opposite directions and getting rid of the inhomogeneous line.

We consider an n-level atom (fig. 1) where two relevant levels a and b with the same parity can be coupled by two-photon processes via all the other intermediate level's of suitable parity to satisfy the selection rules for one-photon processes.

We introduce an interaction with a classical field that we write in the slowly varying envelope approximation as

\[ E(x, t) = E_0(x, t) \cos \left( \omega t - ks + \varphi(x, t) \right). \]


(‡) F. T. ABECCHI and A. POLITI: to be published.
Starting from Schrödinger equation for the atomic amplitudes $C_a$, $C_b$, $C_c$ eliminating intermediate atomic amplitudes $C_j$ as done by Narducci et al. (4), we obtain Bloch-type equations for three collective variables considered as the three components of a vector in an isospin space

$$\begin{align*}
\dot{u} &= -N(C_a C_b^* \exp[-i\alpha] + C_a^* C_b \exp[i\alpha]), \\
\dot{v} &= iN(C_b C_c^* \exp[i\alpha] - C_b^* C_c \exp[-i\alpha]), \\
\dot{w} &= N(|C_b|^2 - |C_a|^2),
\end{align*}$$

where $N$ is the atomic density and

$$\alpha = (2\omega - \omega_{ba})t - 2ks + 2p(s, t).$$

The above quantities satisfy the following equations:

$$\begin{align*}
\ddot{u} &= -(\Omega + 2p - G_y E_y^2) v, \\
\ddot{v} &= (\Omega + 2p - G_y E_y^2) w + G_y E_y^2 w, \\
\ddot{w} &= -G_y E_y^2 v,
\end{align*}$$

where $\Omega = 2\omega - \omega_{ba}$, and

$$\begin{align*}
G_y &= \frac{k_y}{4\hbar} = \frac{1}{2\hbar s} \sum_j \left[ \frac{\mu_{y}\omega_{y}}{\omega_{y}^{2} - \omega^{2}} - \frac{\mu_{y}^2\omega_{y}}{\omega_{y}^{3} - \omega^{3}} \right], \\
G_z &= \frac{k_z}{2\hbar} = \frac{1}{2\hbar s} \sum_j \frac{\mu_{z}\mu_{z}}{\omega_{z} + \omega}.
\end{align*}$$

$\mu_{ij}$ being the matrix elements of the dipole-moment operator between levels $i$ and $j$.

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Let us write now the induced polarization as the sum of a component \( P_\alpha \) in-phase and a component \( P_\phi \) in quadrature with the field

\[
P = P_\alpha \cos(\omega t - ks + \varphi) + P_\phi \sin(\omega t - ks + \varphi)
\]

At variance with one-photon processes, \( P_\alpha \) and \( P_\phi \) are not directly proportional to \( \alpha \) and \( \sigma \), respectively, but we rather have \(^{(1)}\)

\[
\begin{align*}
P_\alpha &= -2\kappa E_0 G_4 w + G_1 w - Ng \kappa, \\
P_\phi &= -2\kappa E_0 G_1 v,
\end{align*}
\]

where

\[
G_3 = \frac{F_{ba} + F_{bb}}{\kappa}.
\]

Before writing the coupled Maxwell-Bloch equations, we introduce phenomenological loss terms which take into account relaxation of atomic variables. They were not considered in ref. \(^{(1)}\), hence they require some further considerations. A general density matrix approach, given in detail elsewhere \(^{(2)}\), justifies on physical grounds the introduction of separate transverse and longitudinal decay rates \( \gamma_T \) and \( \gamma_L \) for the atomic variables.

With the above assumptions the atomic equations are

\[
\begin{align*}
\dot{u} &= -(\Omega + 2\kappa - G_1 E_0^2) u - \gamma_T u, \\
\dot{v} &= (\Omega + 2\kappa - G_1 E_0^2) v + G_2 E_0^2 w - \gamma_L v, \\
\dot{w} &= -G_2 E_0^2 v - \gamma_L (w + N).
\end{align*}
\]

Coupling, as usual \(^{(3)}\), amplitude and phase with \( P_\alpha \) and \( P_\phi \), respectively, we obtain the following field equations for a wave travelling in the atomic medium.

\[
\begin{align*}
\frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} &= \frac{\omega K}{\epsilon_0} (G_4 u + G_1 w - Ng \kappa), \\
\frac{\partial E_\alpha}{\partial t} + c \frac{\partial E_\alpha}{\partial x} &= \omega K G_4 v.
\end{align*}
\]

We consider now the resonant absorber within a ring cavity of length \( L \) (fig. 2) and introduce stationary boundary conditions.

\[
\begin{align*}
E_\alpha^L \sqrt{T} + BE(L) = E(0) \cos(\varphi(L) - \varphi(0)), \\
E_\alpha^L \sqrt{T} = E(0) \sin(\varphi(L) - \varphi(0)), \\
E_{\phi L} = E(L) \sqrt{T},
\end{align*}
\]

where

\[
E_{\phi L} = E_{\phi L}^0 \cos(\omega t - ks + \varphi(L)) + E_{\phi L}^0 \sin(\omega t - ks + \varphi(L))
\]

and \( T = (1 - B) \) is the transmission coefficient of the mirrors 1 and 2 (mirrors 3 and 4 are 100% reflecting).

Now we look for a stationary solution of eqs. (7) and (8). We eliminate the space dependence making a mean field approximation which is equivalent to integrating the field equations over the cavity length with $P_{s}$ and $P_{a}$ taken as constants, plus assuming $|\varphi(0) - \varphi(L)| \ll 1$.

![Diagram of a ring cavity](image)

Fig. 2. – Ring cavity. $E_{i}$ is the incident-field amplitude, $E_{p}$ and $E_{R}$ are the transmitted and reflected field, respectively.

Under these assumptions we can rewrite the boundary conditions as

\begin{align}
E_{i} \sqrt{T} + RE_{i}(L) &= E(0), \\
E_{i} \sqrt{T} &= E(0)(\varphi(L) - \varphi(0)).
\end{align}

Let us calculate the relations between the transmitted field normalized as follows:

\begin{equation}
\alpha = \frac{E_{T}}{T \left(\gamma_{1} \gamma_{4}\right)^{1/2}}
\end{equation}

and the incident field projections in-phase ($y_{1}$) and in quadrature ($y_{2}$) with respect to $E_{T}$, normalized as $\alpha$.

These relations are

\begin{align}
y_{1} &= \alpha \left[1 + \frac{c_{2} \alpha^{2}}{1 + (\Delta - 2\alpha^{2})^{2} + \alpha^{4}}\right], \\
y_{2} &= c_{1} \alpha \left[\frac{\alpha^{2}(\Delta - 2\alpha^{2}) - \beta(1 + (\Delta - 2\alpha^{2})^{2})}{1 + (\Delta - 2\alpha^{2})^{2} + \alpha^{4}} G_{1}\right].
\end{align}

In these equations the parameters have the following meaning: $\Delta = \Omega / \gamma_{e}$ is the off-resonance between incident field and atomic frequency, while the field frequency has been taken resonant with the cavity (no cavity detuning).

We have further introduced the parameters

\begin{align}
\delta &= \sqrt{\frac{\gamma_{1}}{\gamma_{e} G_{1}}} G_{1}, \\
\beta &= \sqrt{\frac{\gamma_{e} G_{1}}{\gamma_{4} G_{2}}}, \\
\theta &= \sqrt{\frac{\gamma_{e} G_{2}}{\gamma_{4} G_{4}}}.
\end{align}
Finally

$$c_1 = \sqrt{\frac{\gamma_1}{\gamma_L}} \frac{\hbar \omega G_L \nu N}{\gamma_L \epsilon_0 \phi T}$$

is a density-dependent co-operative parameter.

Let us consider the simple case in which there is an intermediate level quasi-resonant with a single-photon transition and further take $\Delta = 0$ and $\gamma_1 \approx \gamma_L$. In such a case it is easily seen that $G_1 \ll G_2, G_4$. Furthermore, a suitable field-cavity mistuning changes the boundary conditions, and hence yields another term in eq. (126) which is proportional to $\alpha$. Then, with a suitable choice of the mistuning, we can neglect the $\theta_1$ term and obtain

$$y_1 = x \left(1 + \frac{c_1 \alpha^4}{1 + \alpha^4}\right), \quad y_2 \approx 0.$$  

Fig. 3. – Qualitative plot of the normalised transmitted field $x$ vs. the incident field $y$ for two different values of the $c_1$ parameter. The dashed line represents the unstable branch.

These equations are very similar to those obtained by Bonifacio and Lugiato (*) for the absorptive case in the corresponding one-photon process. Evaluating the extremes of $y_1$ vs. $x$, we get the condition for a bistable behaviour

$$c_1 > 5.42,$$

as shown in fig. 3.