Optical Bistability in a Resonant Two-Photon Absorber.

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Optical bistability as a kind of first-order phase transition has been suggested and experimentally observed in an optical feedback system (1) as well as in a resonant absorber within an interferometer (1 2). A multiple-pass interferometer with a suitable nonlinear dispersive or absorptive mechanism can behave as a bistable optical system. The first suggestion was made by SZÖKE *et al.* (1), and different versions have been shown to work with the nonlinearity due either to an atomic medium at resonance with the impinging field (2) or to an external feed-back system (3).

We are here interested in the optical bistability arising from the collective behaviour of an atomic system.

A general theory has been formulated in a series of papers by BONIFACIO and LUGIATO (4) to obtain absorptive and dispersive atomic bistability. As is well known from the first approaches (2) a difficulty of atomic bistability consists in inhomogeneous broadening due to the Doppler contribution of atom in a cell.

We show here evidence of a bistable behaviour of an interferometer filled with a two-photon absorber and working in a travelling wave mode. This is the first step to derive, afterwards, a Doppler free atomic bistability (5) by absorbing photons from opposite directions and getting rid of the inhomogeneous line.

We consider an n-level atom (fig. 1) where two relevant levels a and b with the same parity can be coupled by two-photon processes via all the other intermediate levels of suitable parity to satisfy the selection rules for one-photon processes.

We introduce an interaction with a classical field that we write in the slowly varying envelope approximation as

$$E(z, t) = E_0(z, t) \cos\big(\omega t - kz + \varphi(z, t)\big) . \tag{1}$$

(1) A. SZÖKE, V. DANEU, J. GOLDHAR and N. A. KURNIT: *Appl. Phys. Lett.*, **15**, 376 (1969).

(2) S. L. GIBBS: *Phys. Rev. A*, **9**, 1515 (1974), H. M. GIBBS, S. L. MCCALL and T. N. C. VENKATESAN: *Phys. Rev. Lett.*, **36**, 1135 (1976); T. N. C. VENKATESAN and S. L. MCCALL: *Appl. Phys. Lett.*, **30**, 282 (1977).

(3) P. W. SMITH and E. H. TURNER: *Appl. Phys. Lett.*, **30**, 280 (1977); P. W. SMITH, E. H. TURNER and P. J. MALONEY, *IEEE Journ. Quant. Elect.*, QE-**14**, 207 (1978).

(4) R. BONIFACIO and L. A. LUGIATO: *Opt. Comm.*, **19**, 172 (1976); *Lett. Nuovo Cimento*, **21**, 505 510, 517 (1978); *Phys. Rev. A*, to appear.

(5) F. T. ARECCHI and A. POLITI: to be published.

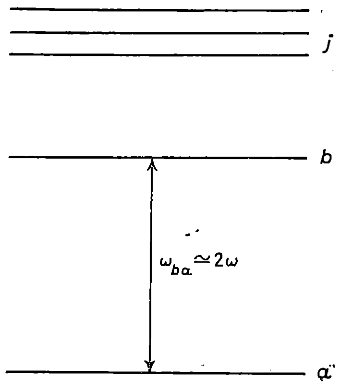

Fig. 1. – Schematic energy-level diagram for an absorbing atom. The energy separation between states a and b is approximately twice the energy of an incident photon.

Starting from Schrödinger equation for the atomic amplitudes C_a, C_b, C_j eliminating intermediate atomic amplitudes C_j as done by NARDUCCI *et al.* (6), we obtain Bloch-type equations for three collective variables considered as the three components of a vector in an isospin space

$$\left\{\begin{aligned} u &= -N(C_a C_b^* \exp[-i\alpha] + C_a^* C_b \exp[i\alpha])\,, \\ v &= iN(C_a^* C_b \exp[i\alpha] - C_a C_b^* \exp[-i\alpha])\,, \\ w &= N(|C_b|^2 - |C_a|^2)\,, \end{aligned}\right. \tag{2}$$

where N is the atomic density and

$$\alpha = (2\omega - \omega_{ba})t - 2kz + 2\varphi(z, t)\,.$$

The above quantities satisfy the following equations:

$$\left\{\begin{aligned} \mathring{u} &= -(\Omega + 2\mathring{\varphi} - G_1 E_0^2)v\,, \\ \mathring{v} &= (\Omega + 2\mathring{\varphi} - G_1 E_0^2)u + G_2 E_0^2 w\,, \\ \mathring{w} &= -G_2 E_0^2 v\,, \end{aligned}\right. \tag{3}$$

where $\Omega = 2\omega - \omega_{ba}$, and

$$\left\{\begin{aligned} G_1 &= \frac{k_{aa} - k_{bb}}{4\hbar} = \frac{1}{2\hbar^2}\sum_j \left[\frac{\mu_{ja}^2 \omega_{ja}}{\omega_{ja}^2 - \omega^2} - \frac{\mu_{jb}^2 \omega_{jb}}{\omega_{jb}^2 - \omega^2}\right], \\ G_2 &= \frac{k_{ab}}{2\hbar} = \frac{1}{2\hbar^2}\sum_j \frac{\mu_{ja}\mu_{jb}}{\omega_{jb} + \omega}\,, \end{aligned}\right. \tag{4}$$

μ_{ij} being the matrix elements of the dipole-moment operator between levels i and j.

(6) L. M. NARDUCCI, W. W. EIDSON, P. FURCINITTI and D. C. ETESON: *Phys. Rev. A*, **16**, 1665 (1977).

Let us write now the induced polarization as the sum of a component P_c in-phase and a component P_s in quadrature with the field

$$P = P_c \cos(\omega t - kz + \varphi) + P_s \sin(\omega t - kz + \varphi) .$$

At variance with one-photon processes, P_c and P_s are not directly proportional to u and v, respectively, but we rather have [6]

$$\left\{\begin{aligned} P_c &= -2\hbar E_0(G_2 u + G_1 w - NG_3) ,\\ P_s &= -2\hbar E_0 G_2 v ,\end{aligned}\right. \tag{5}$$

where

$$G_3 = \frac{k_{aa} + k_{bb}}{4\hbar} . \tag{6}$$

Before writing the coupled Maxwell-Bloch equations, we introduce phenomenological loss terms which take into account relaxation of atomic variables. They were not considered in ref. [6], hence they require some further considerations. A general density matrix approach, given in detail elsewhere [5], justifies on physical grounds the introduction of separate transverse and longitudinal decay rates $\gamma_\perp$ and $\gamma_\parallel$ for the atomic variables.

With the above assumptions the atomic equations are

$$\left\{\begin{aligned} \mathring{u} &= -(\Omega + 2\mathring{\varphi} - G_1 E_0^2) v - \gamma_\perp u ,\\ \mathring{v} &= (\Omega + 2\mathring{\varphi} - G_1 E_0^2) u + G_2 E_0^2 w - \gamma_\perp v ,\\ \mathring{w} &= -G_2 E_0^2 v - \gamma_\parallel (w + N) .\end{aligned}\right. \tag{7}$$

Coupling, as usual [7], amplitude and phase with P_s and P_c, respectively, we obtain the following field equations for a wave travelling in the atomic medium.

$$\left\{\begin{aligned} &\frac{\partial\varphi}{\partial t} + c\frac{\partial\varphi}{\partial z} = \frac{\omega\hbar}{\varepsilon_0} E_0 (G_2 u + G_1 w - NG_3) ,\\ &\frac{\partial E_0}{\partial t} + c\frac{\partial E_0}{\partial z} = \frac{\omega\hbar}{\varepsilon_0} E_0 G_2 v .\end{aligned}\right. \tag{8}$$

We consider now the resonant absorber within a ring cavity of length L (fig. 2) and introduce stationary boundary conditions.

$$\left\{\begin{aligned} &E_I' \sqrt{T} + RE(L) = E(0)\cos(\varphi(L) - \varphi(0)) ,\\ &E_I'' \sqrt{T} = E(0)\sin(\varphi(L) - \varphi(0)) ,\\ &E_T \quad = E(L)\sqrt{T} ,\end{aligned}\right. \tag{9}$$

where

$$E_I = E_I' \cos(\omega t - kz + \varphi(L)) + E_I'' \sin(\omega t - kz + \varphi(L))$$

and $T = (1 - R)$ is the transmission coefficient of the mirrors 1 and 2 (mirrors 3 and 4 are 100% reflecting).

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[7] F. T. ARECCHI and R. BONIFACIO: *IEEE Journ. Quant. Elect.*, QE-1, 169 (1965).

Now we look for a stationary solution of eqs. (7) and (8). We eliminate the space dependence making a mean field approximation which is equivalent to integrating the field equations over the cavity length with P_c and P_s taken as constants, plus assuming $|\varphi(L)-\varphi(0)|\ll 1$.

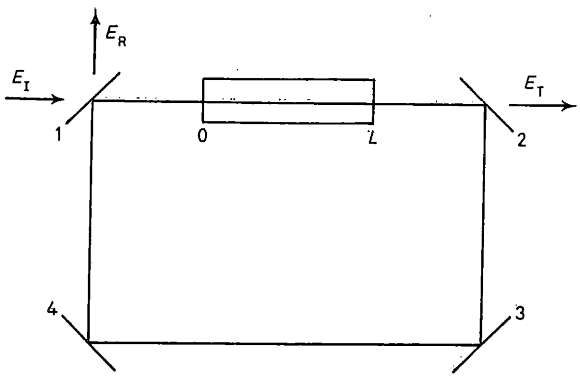

Fig. 2. – Ring cavity. E_I is the incident-field amplitude, E_T and E_R are the transmitted and reflected field, respectively.

Under these assumptions we can rewrite the boundary conditions as

$$(10)\qquad \begin{cases} E_I'\sqrt{T}+RE(L)=E(0)\,,\\ E_I''\sqrt{T}=E(0)(\varphi(L)-\varphi(0))\,. \end{cases}$$

Let us calculate the relations between the transmitted field normalized as follows:

$$(11)\qquad x=E_T\sqrt{\frac{G_2}{T(\gamma_\perp\gamma_\parallel)^{\frac{1}{2}}}}$$

and the incident field projections in-phase (y_1) and in quadrature (y_2) with respect to E_T, normalized as x.

These relations are

$$(12a)\qquad y_1=x\left[1+\frac{c_1x^2}{1+(\Delta-\delta x^2)^2+x^4}\right],$$

$$(12b)\qquad y_2=c_1x\left[\frac{x^2(\Delta-\delta x^2)-\beta\,[1+(\Delta-\delta x^2)^2]}{1+(\Delta-\delta x^2)^2+x^4}-\theta_1\right].$$

In these equations the parameters have the following meaning: $\Delta=\Omega/\gamma_\perp$ is the off-resonance between incident field and atomic frequency, while the field frequency has been taken resonant with the cavity (no cavity detuning).

We have further introduced the parameters

$$\delta=\sqrt{\frac{\gamma_\parallel}{\gamma_\perp}}\frac{G_1}{G_2},\qquad \beta=\sqrt{\frac{\gamma_\perp}{\gamma_\parallel}}\frac{G_1}{G_2},\qquad \theta_1=\sqrt{\frac{\gamma_\perp}{\gamma_\parallel}}\frac{G_3}{G_2}.$$

Finally

$$c_1 = \sqrt{\frac{\gamma_\parallel}{\gamma_\perp}}\,\frac{\hbar\omega G_2 LN}{\varepsilon_0 c T} \tag{13}$$

is a density-dependent co-operative parameter.

Let us consider the simple case in which there is an intermediate level quasi-resonant with a single-photon transition and further take $\Delta = 0$ and $\gamma_\parallel \approx \gamma_\perp$. In such a case it is easily seen that $G_1 \ll G_2, G_3$. Furthermore, a suitable field-cavity mistuning changes the boundary conditions, and hence yields another term in eq. (12*b*) which is proportional to x. Then, with a suitable choice of the mistuning, we can neglect the θ_1 term and obtain

$$y_1 = x\left(1 + \frac{c_1 x^2}{1 + x^4}\right), \qquad y_2 \simeq 0 . \tag{14}$$

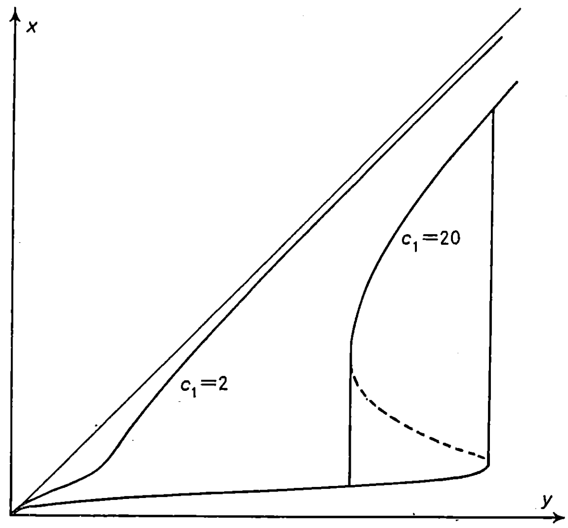

Fig. 3. – Qualitative plot of the normalized transmitted field x *vs.* the incident field y for two different values of the c_1 parameter. The dashed line represents the unstable branch.

These equations are very similar to those obtained by BONIFACIO and LUGIATO [4] for the absorptive case in the corresponding one-photon process. Evaluating the extremes of y_1 *vs.* x, we get the condition for a bistable behaviour

$$c_1 > 5.42 ,$$

as shown in fig. 3.