TURBULENCE AND 1/f NOISE IN QUANTUM OPTICS

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An exciting chapter of physics has been the study of fluctuations and coherence in lasers: how atoms or molecules, rather than radiating e.m. fields independently, decide to "cooperate" to a single coherent field; then, for still higher excitation, how and why they organize in a complex pattern of space and time domains, with small correlations with one another (optical turbulence). Here we discuss these new features of quantum optical systems.

It is generally known[1] that $n \geq 3$ degrees of freedom nonlinearly coupled may lead to multiperiodic or chaotic oscillatory behavior (turbulence). Since quantum optics, in the finite-boundary (single mode) plus semiclassical approximations, is ruled by the 5 Maxwell-Bloch equations, one expects similar behavior in quantum optical devices[2]. Often these instabilities are ruled out by time scale considerations. When the atomic variables have fast damping times, at any instant polarization and inversion are in quasi-equilibrium with the rather slow field amplitude; hence the evolution reduces to a one-equation dynamics (adiabatic elimination of atomic variables). That is why a gas laser beyond threshold assumes a smooth coherent behavior. But make a bad cavity, or add an external modulation as done for Q-switching or mode-locking, then one easily gets a three-variable dynamics, sufficient to yield chaos, for particular values of the coupling constants. What was initially considered as a "bad" or "dirty" behavior (self-pulsing, irregular mode-locking) is nowadays studied as a relevant phenomenon. Furthermore, when many domains of attraction coexist (optical multistability) and an external noise allows for jumps among them, a low frequency power spectrum appears with a shape $f^{-\alpha}$ ($\alpha \sim 1$) like the 1/f noises familiar in many systems[3,4].

Equivalent to the three Lorenz first-order equations[1] is a system of two first-order equations (or one second-order equation) plus an external modulation. An example is the driven Duffing oscillator

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x - \beta x^3 = A \cos\omega t \quad , \tag{1}$$

which can be experimentally realized with an electronic circuit[5,3]. The potential corresponds to a single minimum. For different control parameters μ (either modulation amplitude A or frequency ω), it may give a sequence of subharmonic bifurcations leading eventually to chaos.

Noise is not essential (deterministic chaos), but if we add it, the number of subharmonic bifurcations before chaos becomes smaller and smaller. This can be put in terms of a scaling law where the variance of the external noise appears somewhat as a modification of the control parameter[5].

Let us now change the sign of the potential, getting two stable valleys. Depending on the initial conditions, we have two independent attractors. Increase μ until they both get strange. Now, addition of a random noise may trigger jumps from one to the other. These jumps give a low frequency divergence in the power spectrum[3]. Here random noise is essential to couple the two strange attractors, otherwise independent. After the first evidence of the jumping phenomenon, a similar effect was observed in a Q-modulated CO_2 laser[4]. It corresponds to a set of 2 coupled rate equations, with time dependent cavity losses k(t), that is, calling Δ the population inversion and n the photon number, to

$$\left.\begin{aligned} \dot{\Delta} &= R - 2Gn\Delta - \gamma_{\parallel}\Delta \\ \dot{n} &= Gn\Delta - k(t)n \quad , \end{aligned}\right\} \tag{2}$$

where $k(t) = k_0(1+m \cos\omega t)$.

Figure 1a shows generalized bistability, that is, the simultaneous coexistence of two attractors in the phase space $(\dot{n},n)$. Increasing the modulation depth m, the attractors become strange and the power spectrum displays a low frequency divergence (Fig. 1b).

Experiments[3,4] show that nonlinear driven systems yield power spectra with a low frequency divergence $f^{-\alpha}$, α being around 1 whenever the following conditions are fulfilled: i) the system is multistable, that is, it has 2 or more attractors; ii) the attractors are near to being destabilized or they have just become unstable; and iii) the system is "open" to external fluctuations, i.e., the presence of noise is essential to yield jumps between different basins of

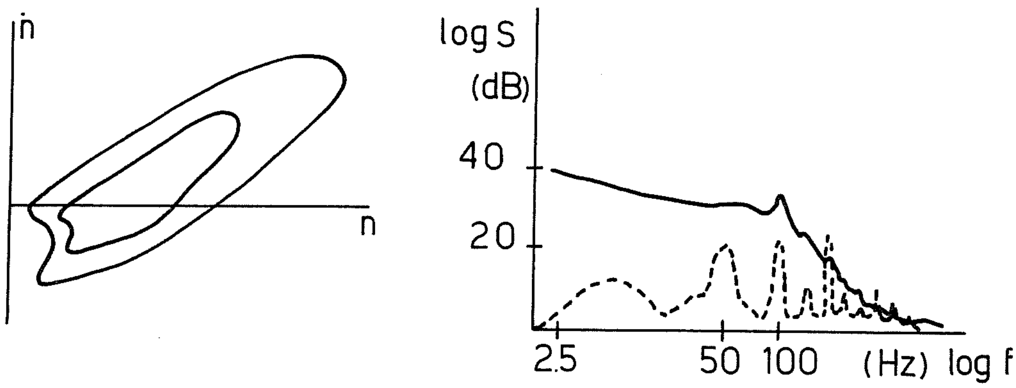

Fig. 1 Bistability and 1/f noise in a CO_2 laser with loss modulation a) coexistence of two attractors (period 3 and 4 respectively). The two superposed spectra correspond to two starts with different initial conditions; b) comparison between the low frequency cut-off (dashed line) when the two attractors are stable and the low frequency divergence (solid line, slope α=0.6), when the two attractors are strange.

attraction. The above conditions show that we are in the presence of a phenomenon which occurs beyond the usual approach to chaos by either one of the current scenarios. A simple jump between two attractors is not sufficient to explain the phenomenology. Indeed, experiments[3,4] show that, when leaving an attractor, the representative point in phase space has a long erratic motion before landing on another attractor. This "transient" regime is made of motions among repulsive orbits.

A dynamics in terms of a recursive map must allow for at least two independent attractors. The simplest one-dimensional map with two attractors must have two extrema[7]. Hence, we study a cubic map in the interval (-1,1)

$$x_{n+1} = (a-1)x_n - ax_n^3 \quad . \tag{3}$$

To account for item iii) the map will be disturbed by white noise with r.m.s. between 10^{-7} and 10^{-5}. Up to a value $a = \bar{a} = 3\sqrt{3}/2 + 1 = 3.598076$ ..., the motion is confined either on the interval (-1,0) or (0,1) with qualitative features like the well-known logistic map. For $a = \bar{a}$, we may still have two independent attractors, whose domains, however, are interlaced in complicated ways over the interval (-1,1).

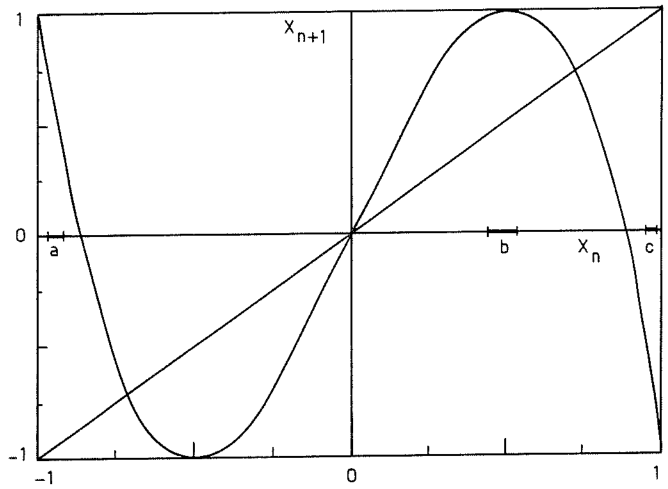

Fig. 2 Cubic map for $\tilde{a}$. a,b, and c show (not in scale) the intervals covered by one of the two period-3 attractors.

The simplest stable pair of attractors A,B above $\bar{a}$ is a pair of period-3 attractors which are superstable for $a_s = 3.981797$ .... These period-3 attractors disappear for $a = \tilde{a} = 3.982000642$ ... For $a_s<a<\tilde{a}$ (Fig. 2) the presence of a small amount of noise makes it easy to leave one attractor and jump toward the other one. Before landing on the other attractor, the representative point wanders on the available space through a long transient because of the complex structure of the two basins of attraction. The corresponding low frequency power specta, for different noise levels, are given in Fig. 3. These spectra show a power law region extending over about one decade with a slope between 0 and 2. They appear qualitatively in agreement with the experimental spectra of references 3 and 4.

A model explanation of the above spectra was given[7] in terms of jump processes among three regions of phase space (the two attractors and the intermediate transient), by taking the jump probabilities

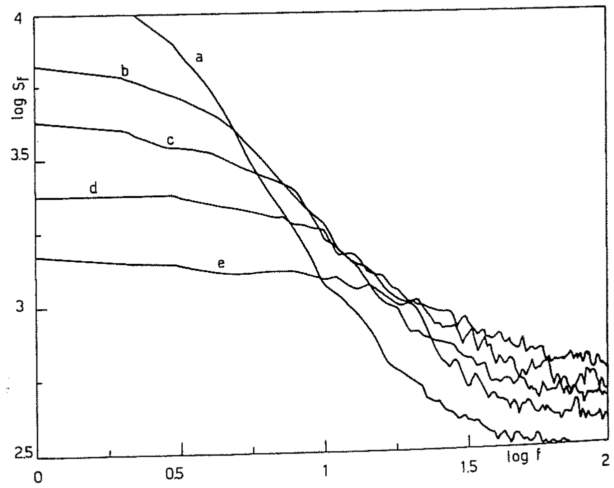

Fig. 3 Power spectra for $a=\tilde{a}$, and for increasing noise levels σ, that is (a) 5×10^{-7}; (b) 10^{-6} (c) 2×10^{-6}; (d) 4×10^{-6}; (e) 10^{-5}. They can be fitted by $f^{-\alpha}$, with α decreasing from 1.5 for (a) to 0.5 for (e).

decorrelated from the internal motions within the three regions. This leads to a power spectrum made of three Lorentzians which fit well the spectra shown in Fig. 3.

Extrapolating the model to a many attractors situation, we formulate the conjecture that for the limit of a highly multistable system (large number of attractors), the low frequency part $f^{-\alpha}$ has the exponent $\alpha=1$. This is akin to the well known model for 1/f noise in equilibrium systems[8], where the 1/f slope is the sum over a large number of Lorentzian curves suitably weighted.

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