Tomographic Reconstruction of a Squeezed Laser Field: Experiment and Reconstruction Algorithm

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Abstract

We present a self-homodyne tomography experiment for the reconstruction of the Wigner of an amplitude squeezed state generated by a semiconductor laser. We discuss in particular the topic of the reconstruction algorithm and the problem related to the adequate choice of the transformation kernel.

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1. Introduction

The possibility to reconstruct the quantum state of electromagnetic fields has been extensively demonstrated in the optical frequencies domain during the last years. In particular, following the idea of Vogel and Risken [1], one is able to deduce the Wigner function $W(x,y)$ or the elements of the density matrix, using a tomographic reconstruction from a set of probability distributions $P_q(x)$ of the field quadratures detected by standard homodyne technique.

Since the first work of Smithey et al. [2] homodyne tomography has been performed on different types of coherent and squeezed states mainly generated by non linear processes involving optical parametric amplifiers [3]. These experiments are characterized in general by a very weak field and by the availability of a strong local oscillator.

Following the work of Galatola et al. [7] a different scheme of self-homodyne detection has been recently demonstrated [8]. In this case, self-homodyning is achieved by a high finesse Fabry Perot analysis cavity which introduces a frequency-dependent phase shift. In this way, the noise ellipse is rotated with respect to the mean field and the rotation angle can be controlled by changing the cavity detuning. Such a scheme can be applied if a local oscillator, in addition to the field to be analyzed, is not available.

In [9] we have presented a tomographic reconstruction of the quantum state of the field emitted by a semiconductor laser. The analysis is here successfully applied to amplitude squeezed laser emission. An improved detection apparatus allows to appreciate the non-classical features of the reconstructed Wigner function.

The quantum state generated by our laser presents indeed a small amount of amplitude squeezing ($-0.5 \text{ dB measured at the detector}$) and a phase noise about 30 times larger than the shot noise level. In order to reconstruct the Wigner function we apply an inverse Radon transform. With respect to previous works on quantum tomography, the field analyzed here has a very strong mean value, high phase fluctuations and weak non-classical properties. These features yield different problems concerning the signal acquisition and elaboration.

In this work we focus in particular on the algorithm for the reconstruction and on the problems related to the adequate choice of the transformation kernel.
2. Experiment

The experimental apparatus is described in details in [9] and in [10].

The laser source is a semiconductor laser emitting at 830 nm, in extended-cavity configuration. This kind of source provides squeezed radiation thanks to a high-impedance pump ([4]) and to the depression of the longitudinal side-modes fluctuations ([5, 6]). Moreover, the feedback from an external grating ensures single mode operation and reduces the strong phase noise of the laser [8].

The beam is sent to a Fabry-Perot analysis cavity after a polarizing beam splitter and a quarter-wave plate which serve as optical circulator. The reflected beam from the cavity is directed towards a balanced detection.

The two detectors for homodyning are based on high quantum efficiency pin photodiodes followed by home-made low-noise amplifiers. The two amplified signals are summed or subtracted by passive power combiners. In this way the sum provides the amplitude noise fluctuations, and the difference signal gives the shot-noise fluctuations when the cavity is out of resonance.

As the analysis cavity is scanned the sum/difference signal is mixed with a rf local oscillator at 58 MHz, then is recorded by an acquisition board after a low pass filter with a bandwidth of 2 MHz. The resolution of the acquisition board is particularly important to appreciate the amplitude squeezing of the quantum state produced by the laser. Indeed, to measure 0.5 dB of squeezing it is necessary to resolve the shot noise level with about 50 levels. On the other hand, the necessity to cover the strong fluctuations on the phase quadrature implies at least 14 bits of resolution on the whole dynamic range of the signal.

During each scan of the Fabry Perot cavity (with a duration of 800 ms), $4 \cdot 10^6$ samples with a resolution of 21 bits are acquired. To infer the phase associated to each sample, we have performed a best fit of the variance of the signal with the response function of the cavity calculated by Fiorentino et al. [9] and by Zhang et al. [11] who extend the results of [7] to a cavity with losses.

In order to reduce the statistical uncertainty on the quadrature histograms, we have acquired about 300 consecutive scans of the Fabry Perot cavity, with a total number of $10^9$ samples. The data points from each acquisition are divided into 250 near constant phase intervals of equal width and the intervals from all the acquisitions are then put together and their data points are grouped in histograms with 16000 amplitude bins.

3. Tomographic Reconstruction

The collected histograms $P_\theta(x)$, where $\theta$ is the relative phase and $x$ is the amplitude, are the marginal distributions of the quasi-probability Wigner function, defined as:

$$P_\theta(x) = \int_{-\infty}^{+\infty} W(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta) \, dy.$$  

(1)

To reconstruct the Wigner function we use the Inverse Radon Transform (IRT):

$$W(x, y) = \frac{1}{4\pi^2} \int_{-\pi/2}^{+\pi/2} \int_{-\infty}^{+\infty} \, d\theta \int_{-\infty}^{+\infty} \, dz \, P_\theta(z) \, (z - \xi)$$  

(2)

where $\xi = x \cos \theta + y \sin \theta$ and $K$ is the filter function:

$$K(x) = \frac{1}{2} \int_{-\infty}^{+\infty} \, d\xi \, |\xi| \, e^{i\xi x}.$$  

(3)
The filter function $K$ exists only in the sense of generalized function. To numerically implement the IRT a regularization of the generalized function $K$ is achieved by setting a cut-off $k_c$ in the integration limits of Eq. (3). In this way the integral can be calculated and the result is

$$K(x) = \frac{1}{x^2} (\cos(k_c x) + k_c x \sin(k_c x) - 1).$$  \hspace{1cm} (4)$$

Near the origin (for $|k_c x| < 0.1$) this function is approximates by a fourth-order power expansion [12].

The procedure of the IRT consists, actually, in a convolution of the collected histograms with the regularized filter function followed a back-projection integral over $\theta$. The cut-off parameter $k_c$ has to be adjusted according to the state to be reconstructed. Since the algorithm is strongly influenced by the choice of $k_c$, it is important to use a value which both allows for extracting the crucial informations about the Wigner function and limits the artifacts introduced.

In our case, we choose the cut-off number in order to have a good resolution and a small amount of excess fluctuations due to the algorithm. In Fig. 1 we show the two sections of

![Fig. 1. Phase (left) and amplitude (right) reconstructed section of the Wigner function for different values of the cut-off parameter $k_c$. The horizontal unit corresponds to a bin interval.](image-url)
the reconstructed Wigner function along the phase and amplitude planes (i.e., \( W(0,y) \) and \( W(x,0) \)) for different values of \( k_c \), namely \( \pi/200 \), \( \pi/50 \) and \( 2\pi/25 \). One can see how for small values of \( k_c \), the peak of the filter function is larger than the histogram associated with the amplitude noise and the filter function features are reproduced on the reconstructed amplitude quadrature (low resolution). On the other hand, with large values of \( k_c \) strong fluctuations are present in the reconstructed quadrature due to the large amplitude of the oscillating tails of the filter function (high resolution at the expense of large excess fluctuations).

To better quantify the quality of the reconstruction, we have calculated a distance \( d \) between the above mentioned sections of the Wigner function and the respective best fitted Gaussian functions: 

\[
d = \sqrt{\sum_k (f_k - g_k)^2}
\]

where \( f_k \) are the values of the reconstructed Wigner function and \( g_k \) results from the Gaussian fit. The values of \( d \) as a function of \( k_c \) are plotted in Fig. 2. The optimal cut-off parameter results to be \( k_c = \pi/50 \).

We remark that the resolution required to distinguish the non-classical properties of the Wigner function is of the order of the width of the coherent state. A \( a \) priori choice of the cut-off parameter can be \( k_c = \pi/\sigma_{\text{shot}} \), where \( \sigma_{\text{shot}} \) is the standard deviation of the shot noise. In our case, the measured \( \sigma_{\text{shot}} \) is indeed 52.7.

Finally, the two sections of the reconstructed Wigner function along the phase and amplitude planes are shown in Fig. 3 and compared with the corresponding coherent state Wigner function obtained from the shot noise. The main properties of the Wigner function are well reproduced, even if some artificial features, such as slightly negative or oscillating values on the wings, are still present.

**Fig. 2.** Distance \( d \) (see text) calculated for the amplitude (a) and phase (b) sections of the Wigner function, as a function of \( k_c \).
4. Conclusions

We have presented an experimental demonstration of optical self-homodyne tomography applied to the reconstruction of an amplitude squeezed state. With respect to previous experimental works, our method does not require an external local oscillator. On the other hand, some feature of the analyzed field such as high phase noise and strong mean value imply suitable data acquisition and elaboration methods. We have discussed also the problem of the cut-off frequency for the transformation kernel and we have suggested a criterion for the choice of its optimal value.

References